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Accelerating models with a hybrid scale factor in extended gravity

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Abstract. Dynamical aspects of an anisotropic cosmological model are investigated in an extended gravity theory. A hybrid scale factor is employed to investigate the cosmic dynamics. The hybrid scale factor simulates a cosmic transit behaviour from a decelerated phase of expansion to an accelerated phase. The parameters of the hybrid scale factors have been constrained from the cosmic transit behaviour and the signature flipping behaviour of the deceleration parameter. The evolutionary aspect of the model has been discussed through the dynamical evolution of the equation of state parameter and the energy conditions. From the present model, scalar fields have been reconstructed using certain reconstruction techniques. Different diagnostic methods have been applied to analyse the viability of the constructed model. The present model almost looks like a cosmological constant for a substantial cosmic time zone and does not show any slowing down feature in near future.

Keywords. $f(R, T)$ cosmology—Bianchi type VI_h—anisotropic universe—cosmic string.

1. Introduction

The astronomical observations revealed two important phenomena on the universe: the dark energy and the dark matter. Dark energy yields a late time acceleration of the cosmological background whereas dark matter behaves as an invisible dust matter favouring the process of gravitational clustering. The concept of the intriguing matter fields such as dark energy has emerged as an alternative to the General Relativity (GR) based description of gravity (Tsujikawa 2010; Capozziello & de Laurentis 2011; Nojiri & Odintsov 2011; Berti *et al.* 2015). The search for alternative theories to GR has become inevitable after its failure to explain the cosmic phenomena occurring at late times. GR can be modified in many distinct directions. Consequently there are a good number of modified gravity theories available in literature (Capozziello *et al.* 2003; Nojiri & Odintsov 2003; Carroll *et al.* 2004; Sotiriou & Faraoni 2010; Nojiri & Odintsov 2011). Different exotic matter fields simulating a positive energy density and negative pressure are commonly used in literature as additional terms to explain the cosmic speed up phenomenon. However, it is possible to settle the issue of late time cosmic acceleration through the suitable geometrical modification in GR without adding any exotic source of matter. The most simple geometrical modification to GR has been proposed is the $f(R)$ theory, where the geometrical action contains an arbitrary function $f(R)$ of the Ricci scalar R in place of R in GR action. Some generalizations of $f(R)$ gravity have been proposed recently. One such generalization is the $f(R, T)$ gravity proposed by Harko *et al.* (2011). In $f(R, T)$ gravity, a weak coupling between the geometry and matter is assumed and accordingly the geometrical part of the gravitational field action is modified by considering an arbitrary function $f(R, T)$ of the Ricci scalar R and the trace of the energy momentum tensor T. It is worthy to mention here that employing the trace of the energy momentum tensor in the new theory may be associated with the existence of exotic

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imperfect fluids or quantum effects such as particle production.

Many researchers have studied different aspects of *f*(*R*, *T*) gravity in various physical background. Sharif and Zubair (2014) have studied the cosmological reconstruction of *f*(*R*, *T*) gravity in FRW universe. Shabani and Farhoudi (2014) obtained the cosmological parameters in terms of some defined dimensionless parameters that are used in constructing the dynamical equations of motion. Dynamics of an anisotropic universe is studied by Mishra *et al.* (2016) in *f*(*R*, *T*) gravity using a rescaled functional whereas Yousaf *et al.* (2016) have shown the causes of irregular energy density in *f*(*R*, *T*) gravity. Velten and Carames (2017) have challenged the viability of *f*(*R*, *T*) as an alternative modification of gravity. Abbas and Ahmed (2017) have formulated the exact solutions of the non-static anisotropic gravitating source in *f*(*R*, *T*) gravity which may lead to expansion and collapse. Baffou *et al.* (2017) have investigated the late-time cosmic acceleration in mimetic *f*(*R*, *T*) gravity with the Lagrange multiplier. Carvalho *et al.* (2017) have shown the equilibrium configurations of white dwarfs in a modified gravity theory. Mishra *et al.* (2018a) developed a general formalism to investigate Bianchi type VI_h universes in an extended theory of gravity and studied the dynamical features of the models (Mishra *et al.* 2018b, c). Tripathy *et al.* (2020a, b) have investigated some bouncing cosmological models in *f*(*R*, *T*) theory. Tripathy and Mishra (2020) have obtained some phantom cosmological models in the framework of the extended gravity theory and studied the possible occurrence of big trip in wormhole solutions. In the framework of the extended gravity theory, Tarai and Mishra (2018) have constructed some anisotropic cosmological models in presence of electromagnetic field. Tarai *et al.* (2020) have investigated the effect of magnetic field on the dynamics of Bianchi type VI_h universes in *f*(*R*, *T*) theory. The investigation of different aspects of the cosmic phenomena concerning the late time dynamics including the isotropic and anisotropic nature in the modified gravity theory requires an involved calculations of the dynamical parameters. Also, the results of the calculations should be in conformity with the lot of information gathered over the years from observations. In view of this, in the present work, we have developed a formalism to investigate the dynamical cosmic features in the framework of *f*(*R*, *T*) gravity. We have considered a Bianchi type anisotropic and homogeneous universe for this purpose.

The paper is organised as follows: in Section 2, the basic formalism of *f*(*R*, *T*) gravity and the field equations for Bianchi type VI_h space time have been derived. In Section 3, anisotropic nature of the cosmological model has been presented. The dynamical parameters of the model have been calculated and analysed in Section 4. A scalar field reconstruction technique has been employed in Section 5 to obtain the behaviour of the reconstructed scalar fields. Keeping in view the dynamically varying nature of the deceleration parameter in passing from a decelerated phase of expansion to an accelerated one, we have employed a hybrid scale factor (HSF) in the proposed formalism to investigate the late time cosmic dynamics in Section 6. The viability of the constructed models have been tested through different diagnostic mechanisms in Section 7 and finally the concluding remarks are given in Section 8.

2. Basic formalism of *f*(*R*, *T*) theory and the field equations for a BVI_h space-time

Within the scope of an extended gravity theory as proposed by Harko *et al.* (2011), the Einstein–Hilbert action is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi}f(R,T) + \mathcal{L}_m\right]. \quad (1)$$

In the above, we have used the natural unit system so that $G = c = 1$, where G and c are respectively the Newtonian gravitational constant and speed of light in vacuum. $\mathcal{L}_m$ is the matter Lagrangian. The above action is different from that of GR where an arbitrary function of R and T (*f*(*R*, *T*)) replaces the Ricci scalar R. This interesting coupling of matter and curvature is motivated from quantum effects and leads to a non vanishing divergence of the energy-momentum tensor $T_{\mu\nu}$. Such a feature of the non-minimal matter-geometry coupling provides a strong ground for cosmic acceleration. Because of this coupling, an additional force comes into play that dispels massive particles away from geodesic trajectories. Since the functional form governing the matter-curvature coupling is arbitrary, different choices of the functional *f*(*R*, *T*) generate different models. In general, there can be three possible functional ways of coupling namely: (i) $f(R,T) = R + 2f(T)$, (ii) $f(R,T) = f_1(R) + f_2(T)$ and (iii) $f(R,T) = f_1(R) + f_2(R)f_3(T)$, where $f(T), f_1(R)$, $f_2(R), f_2(T)$ and $f_3(T)$ are some arbitrary functions of their respective arguments. In fact, there can be infinite number of ways to chose these functions. However, the viability of the constructed models depends on the suitable choice of these functions and their ability to pass certain geometrical and observational tests. Out of these three types, the first choice is more like GR and can be reduced to GR under certain condition. The second term of the first choice in that case can be considered as a small deviation from GR. In the present work, we consider the functional as $f(R,T) = R + 2\Lambda_0 + 2\beta T$, where Λ_0 is the usual time independent cosmological constant and β is a coupling constant. This functional form resembles the first type of coupling as mentioned with R being replaced by $R + 2\Lambda_0$. The GR features with a cosmological constant can be recovered from the model for a vanishing β. It is needless to mention here that the choice of the functional of the type $f(R,T) = R + 2f(T)$ has been widely used in literature to address different cosmological and astrophysical issues.

The field equations for the present model can be obtained by varying the action. One may refer to the procedure followed to obtain the field equations in Harko *et al.* (2011) and Mishra *et al.* (2016). For an arbitrary choice $f(R,T) = f(R) + f(T)$ and $\mathcal{L}_m = -p$, the field equations are obtained as

$$\begin{aligned} f_R(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} &= \left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right)f_R(R) \\ &\quad + [8\pi + f_T(T)]T_{\mu\nu} \\ &\quad + \left[f_T(T)p + \frac{1}{2}f(T)\right]g_{\mu\nu}. \end{aligned} \tag{2}$$

In the above, p is the pressure of cosmic fluid. The partial differentiations for the model become $f_R = \frac{\partial f(R)}{\partial R} = 1$ and $f_T = \frac{\partial f(T)}{\partial T} = 2\beta$, so that we can express the field equations as

$$G_{\mu\nu} = \kappa T^{\text{eff}}_{\mu\nu}, \tag{3}$$

with

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}, \tag{4}$$

$$T^{\text{eff}}_{\mu\nu} = T_{\mu\nu} + \frac{\Lambda_{\text{eff}}(T)}{8\pi + 2\beta}g_{\mu\nu}. \tag{5}$$

Here, we have a redefined Einstein matter-geometry coupling constant $\kappa = 8\pi + 2\beta$ and $\Lambda_{\text{eff}}(T) = (2p + T)\beta + \Lambda_0$. For $\beta = 0$, $\Lambda_{\text{eff}}(T)$ becomes the usual cosmological constant Λ_0 and the above field equation reduces to the Einstein field equations with a cosmological constant. For non vanishing value of β, $\Lambda_{\text{eff}}(T)$ becomes a time dependent quantity.

In order to provide some anisotropic pressure along a specific direction, we consider the universe to be filled with a cloud of one dimensional cosmic strings with string tension density ξ aligned along the x-axis. The energy-momentum tensor for such a fluid is given by

$$T_{\mu\nu} = (p + \rho)u_\mu u_\nu - pg_{\mu\nu} - \xi x_\mu x_\nu, \tag{6}$$

with

$$u^\mu u_\mu = -x^\mu x_\mu = 1 \tag{7}$$

and

$$u^\mu x_\mu = 0. \tag{8}$$

Here ρ represents the energy density and is composed of the particle energy density ρ_p and the string tension density ξ,

$$\rho = \rho_p + \xi. \tag{9}$$

We wish to investigate dynamical aspects of the universe in *f*(*R*, *T*) theory as described above for an anisotropic Bianchi type VI_h (BVI_h) space-time given by

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2x}dy^2 - C^2e^{2hx}dz^2, \tag{10}$$

where the metric potentials are considered only to depend on cosmic time. As has already been discussed in some works of Tripathy *et al.* (2015) and Mishra *et al.* (2016b), the exponent h in the metric can be useful if it assumes the value $h = -1$. This conclusion has been derived from the null total energy concept of the whole universe known as the Tryon's conjecture (Tripathy *et al.* 2015; Mishra *et al.* 2016b). Going along the same line of thought, in the present study, we consider the same value of h. Now, the field equations for BVI_h space-time with $h = -1$ in the extended theory of gravity can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\alpha(p - \xi) + \rho\beta + \Lambda_0, \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\alpha p + (\rho + \xi)\beta + \Lambda_0, \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\alpha p + (\rho + \xi)\beta + \Lambda_0, \tag{13}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{1}{A^2}=\alpha\rho-(p-\xi)\beta+\Lambda_0, \tag{14}$$

$$\frac{\dot{B}}{B}=\frac{\dot{C}}{C}. \tag{15}$$

Here $\alpha=8\pi+3\beta$ and we denote the ordinary time derivatives as overhead dots. The metric potentials of the BVI_h space-time along the y- and z-directions are same implying an almost constant expansion rates in the yz-plane. This expansion rate is usually different from the expansion along the x-axis. This imply there must be some anisotropic fluid source in the x-direction. In view of this, we considered that, the anisotropic fluid corresponding to the one dimensional cosmic strings should be aligned along the x-axis. However, it should be noted here that, the choice of the direction for the anisotropic pressure due to the anisotropic fluid source will definitely affect the dynamics of the model. In some of our earlier works we have already investigated about this feature (Ray *et al.* 2019; Mishra *et al.* 2019).

Some relevant quantities in the context of discussion of geometrical aspect of the model include

$$\text{Hubble rate:}\quad H=\frac{\dot{\mathcal{R}}}{\mathcal{R}}=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right), \tag{16}$$

$$\text{Expansion scalar:}\quad \theta=u^l_{;l}=\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right), \tag{17}$$

$$\text{Deceleration parameter:}\quad q=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \tag{18}$$

3. Anisotropic nature of the model

In the present work, we have considered a spatially homogeneous and anisotropic BVI_h universe with different expansion rates along different spatial directions. The quantities that measure the departure from spatial isotropy are the Shear scalar σ^2 and the average anisotropy parameter $\mathcal{A}$ defined respectively as

$$\sigma^2=\frac{1}{2}\left(\sum H_i^2-\frac{1}{3}\theta^2\right)=\frac{1}{3}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)^2, \tag{19}$$

$$\mathcal{A}=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\triangle H_i}{H}\right)^2. \tag{20}$$

$\triangle H_i=H_i-H$, where $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$ and $H_3=\frac{\dot{C}}{C}$ are the directional Hubble rates along x, y and z-axes respectively. In view of Eq. (15), we have $H_2=H_3$. For isotropic models these quantities σ^2 and $\mathcal{A}$ identically vanish. From observational perspectives, the anisotropic nature of a model is usually quantified through the estimation of the amplitude of shear $\frac{\sigma}{H}$ at the present epoch. Using the data from differential microwave radiometers aboard the Cosmic Background Explorer (COBE), Bunn *et al.* (1996) have placed an upper limit to this quantity as $\left(\frac{\sigma}{H}\right)_0<3\times10^{-9}$. For the best case with the total density parameter $\Omega_0=1$, they have obtained $\left(\frac{\sigma}{H}\right)_{pl}\simeq 10^{-3}$–$10^{-4}$. They have also concluded that primordial anisotropy should have been fine-tuned to be less than 10^{-3} of its natural value in the Planck era. Saadeh *et al.* (2016a, b) used cosmic microwave background temperature and polarisation data from Planck and obtained a tighter limit to the anisotropic expansion as $\left(\frac{\sigma}{H}\right)_0<4.7\times10^{-11}$. In view of these recent observational limits on cosmic shear and anisotropic expansion rates, we have adopted a simple approach in the present work and have assumed a proportional relationship between the amplitude of shear σ and Hubble rate. This assumption leads to an anisotropic relation among the directional Hubble rates $H_1=kH_2$. The parameter k takes care of the anisotropic feature of the model. Obviously, $k\neq1$ provides an anisotropic model.

For the BVI_h metric, we obtain the amplitude of shear expansion in the present epoch as

$$\left(\frac{\sigma}{H}\right)_0=\sqrt{3}\left(\frac{k-1}{k+2}\right). \tag{21}$$

Even though tighter constraints on cosmic anisotropy are available in literature and evidences against the departure from global isotropy are being gathered (Deng & Wei 2018), these observational analysis need to be fine tuned as the analysis are prior dependent (Saadeh *et al.* 2016a). It is worth to mention here that, the cosmological principles assuming a homogeneous and isotropic universe may be a good approximation to the present universe. However, it has not yet been well proven in the scales ≥ 1 Gpc (Caldwell & Stebbins 2008). In view of this, in the present work, we wish to construct some accelerating anisotropic models keeping enough room for any amount of cosmic anisotropy. However, we can set some constraints on the parameter k basing upon the observationally found upper bound on $\left(\frac{\sigma}{H}\right)_0$. While the bounds of Bunn *et al.* (1996) constrain k as $k=1.000000008$, that of Saadeh *et al.* (2016a, b) disfavours any classical finite departure from $k = 1$. However, in the present work, we consider $k = 1.0000814$ that provides $\left(\frac{\sigma}{H}\right)_0 = 4.7 \times 10^{-5}$.

Within the formalism discussed here it is easy to show that $\mathcal{A} = \frac{2}{9}\left(\frac{\sigma}{H}\right)^2$. Consequently, the average anisotropic parameter in the present epoch can be calculated as $\mathcal{A}_0 = 4.91 \times 10^{-10}$. Since the large scale structure of the universe may show a departure from isotropy, the cosmic anisotropy can be estimated from Hemispherical asymmetries in the Hubble expansion. In a recent work, Kalus *et al.* (2011) estimated the Hubble anisotropy of supernova type Ia Hubble diagrams at low redshifts ($z < 0.2$) as $\frac{\Delta H}{H} < 0.038$. Using the value of the anisotropy parameter k at the present epoch, we obtain the expansion asymmetry as $\frac{\Delta H}{H} = 0.814 \times 10^{-4}$. One can note that, the predicted anisotropy from our model is well within the observationally set up bounds (Campanelli *et al.* 2011).

4. Equation of state parameter and energy conditions

The presumed anisotropic relation among the directional Hubble rates has a simplified structure and within this formalism it can provide us a simple approach to study the cosmic dynamics. For a given anisotropic parameter k, the directional Hubble rates become $H_1 = \left(\frac{3k}{k+2}\right)H$ and $H_2 = H_3 = \left(\frac{3}{k+2}\right)H$. Obviously for $k = 1$, the directional Hubble rates become equal to the Hubble parameter H. In our formalism, the presumption of the proportionality relation between the shear scalar and scalar expansion leads to the calculation of the equation of state (EoS) parameter in terms of the Hubble rate. Also, the energy conditions for the present model will depend on the Hubble rate.

4.1 *EoS parameter*

The physical properties of the model such as pressure, energy density and string tension density are obtained from the field equations (11)–(15) as

$$p = \left(\frac{1}{\alpha^2 - \beta^2}\right) \times [(S_1(H) - S_2(H) + S_3(H))\beta - S_2(H)\alpha] + \frac{\Lambda_0}{\alpha + \beta}, \tag{22}$$

$$\rho = \left(\frac{1}{\alpha^2 - \beta^2}\right) \times [S_3(H)\alpha - S_1(H)\beta - (\alpha - \beta)\Lambda_0], \tag{23}$$

$$\xi = \frac{S_1(H) - S_2(H)}{\alpha - \beta}, \tag{24}$$

where

$$S_1(H) = \frac{1}{(k+2)^2} \times [6(k+2)\dot{H} + 27H^2 + (k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}], \tag{25}$$

$$S_2(H) = \frac{1}{(k+2)^2} \times [3(k^2+3k+2)\dot{H} + 9(k^2+k+1)H^2] - \mathcal{R}^{-\left(\frac{6k}{k+2}\right)}, \tag{26}$$

$$S_3(H) = \frac{1}{(k+2)^2} \times [9(2k+1)H^2 - (k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}]. \tag{27}$$

Algebraic simplification of the above expressions yield

$$p = -\left(\frac{1}{\alpha^2 - \beta^2}\right)[\phi_1(k,\beta)\dot{H} + \phi_2(k,\beta)H^2] + \frac{\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}}{\alpha - \beta} + \frac{\Lambda_0}{\alpha + \beta}, \tag{28}$$

$$\rho = \left(\frac{1}{\alpha^2 - \beta^2}\right)[\phi_3(k,\beta)\dot{H} + \phi_4(k,\beta)H^2] - \frac{\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}}{\alpha - \beta} - \frac{\Lambda_0}{\alpha + \beta}, \tag{29}$$

$$\xi = \left(\frac{1}{\alpha - \beta}\right)[\phi_5(k)\left(\dot{H} + 3H^2\right) + 2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}], \tag{30}$$

where

$$\phi_1(k,\beta) = \frac{3}{k+2}[(k+1)\alpha + (k-1)\beta], \tag{31}$$

$$\phi_2(k,\beta) = \left(\frac{3}{k+2}\right)^2[(k^2+k+1)\alpha + (k^2-k-3)\beta], \tag{32}$$

$$\phi_3(k,\beta) = -\frac{6\beta}{k+2}, \tag{33}$$

$$\phi_4(k,\beta) = \left(\frac{3}{k+2}\right)^2[(2k+1)\alpha - 3\beta], \tag{34}$$

$$\phi_5(k) = \frac{3(1-k)}{k+2}. \tag{35}$$

It is interesting to note that for $\alpha+\beta=0$, i.e. for $\beta=-2\pi$, we have

$$\phi_1(k,\beta) = \phi_3(k,\beta) \quad \text{and} \quad \phi_2(k,\beta) = \phi_4(k,\beta) \tag{36}$$

and consequently in the limit $\beta \to -2\pi$,

$$p = -\rho. \tag{37}$$

In other words, within the scope of the present formalism, ΛCDM model with $p=-\rho$ can be recovered from the model for $\alpha+\beta=0$. Of course, overlapping of the present model with that of ΛCDM requires a negative coupling constant.

The Equation of State parameter (EoS) ω is defined as the pressure to energy density ratio, $\omega = \frac{p}{\rho}$. For $\alpha \neq \pm\beta$, it is straightforward to obtain ω as

$$\omega = -1 + (\alpha+\beta)\frac{S_2(H) - S_3(H)}{S_1(H)\beta - S_3(H)\alpha + (\alpha-\beta)\Lambda_0}. \tag{38}$$

As is obvious from the above expression, the dynamical behaviour of the EoS parameter depends on the parameters of the Hubble rate H and the coupling constant β. For any realistic cosmological model, the Hubble parameter is a decreasing function of time and therefore at late phase of cosmic evolution, we expect that, the functionals $S_2(H)$ and $S_3(H)$ will behave alike thereby cancelling each other at late times. Therefore, for any value of $\beta(\neq -2\pi, \neq -4\pi)$, the EoS parameter behaves as a cosmological constant $(\omega=-1)$ at late epoch. However, at an early epoch, the Hubble rate assumes a very high value thereby pushes ω to a larger value.

In the limit $\beta \to 0$, the model reduces to that of GR and the EoS parameter becomes

$$\omega = -1 + \frac{S_2(H) - S_3(H)}{\Lambda_0 - S_3(H)} \tag{39}$$

which becomes $\omega = -\frac{S_2(H)}{S_3(H)}$ in the absence of a cosmological constant Λ_0 term in the field equations. One can note that, similar conclusion on the dynamical evolution of ω as above may be derived for $\beta \to 0$. In other words, the dynamical behaviour of the EoS parameter will not be sensitive to the choice of the coupling constant at late times. All the trajectories of ω will behave alike at late phase of cosmic evolution. However, at an early epoch, the model will pass through different trajectories which may be β dependent.

Another dynamical parameter is the effective cosmological constant Λ_{eff} that appears in the equivalent Einstein field equation for the extended gravity theory. Unlike the dynamical cosmological constant in GR, this effective cosmological constant depends on the matter field content such as the pressure and energy density. We can obtain Λ_{eff} as

$$\Lambda_{\text{eff}} = \left(\frac{\beta}{\alpha+\beta}\right)[(S_1(H) + S_3(H)) - 2\Lambda_0] + \Lambda_0. \tag{40}$$

In terms of the Hubble parameter, we may express Λ_{eff} as

$$\Lambda_{\text{eff}} = \frac{\beta}{\alpha+\beta}\left[\frac{6}{k+2}(\dot{H} + 3H^2) - 2\Lambda_0\right] + \Lambda_0. \tag{41}$$

For $\alpha+\beta \neq 0$, the magnitude of the effective cosmological constant decreases with the growth of cosmic time. The sign of this quantity will depend on the sign of β. At late times, the behaviour of the effective cosmological constant depends on the contribution coming from $\dot{H} + 3H^2$. In many models, this term either vanishes or have a negligible contribution. For such models, Λ_{eff} reduces to $\left(\frac{\alpha-\beta}{\alpha+\beta}\right)\Lambda_0$. Obviously as mentioned earlier, for a vanishing coupling constant β, it reduces to the usual time independent cosmological constant Λ_0.

4.2 *Energy conditions*

Since energy conditions put some additional constraints on the viability of the models we wish to calculate the different energy conditions for the constructed model in the modified gravity theory. In our formalism, the energy conditions are obtained as

$$\text{NEC}: \quad \rho + p = \frac{S_3 - S_2}{\alpha - \beta} \geq 0,$$

$$\text{WEC}: \quad \rho = \frac{S_3\alpha - S_1\beta - (\alpha - \beta)\Lambda_0}{\alpha^2 - \beta^2} \geq 0,$$

$$\text{SEC}: \quad \rho + 3p = \frac{(S_3 - 3S_2)\alpha + (2S_1 - 3S_2 + 3S_3)\beta}{\alpha^2 - \beta^2} + \frac{2\Lambda_0}{\alpha + \beta} \geq 0,$$

$$\text{DEC}: \quad \rho - p = \frac{(S_2 + S_3)\alpha - (2S_1 - S_2 + S_3)\beta}{\alpha^2 - \beta^2} - \frac{2\Lambda_0}{\alpha + \beta} \geq 0,$$

where NEC, WEC, SEC and DEC respectively denote Null energy condition, Weak energy condition, Strong energy condition and Dominant energy condition. These energy conditions are expressed in terms of the Hubble parameter as

$$\text{NEC}: \quad \rho + p = \frac{-1}{\alpha + \beta} \left[\frac{k(k-1)}{(k+2)^2} 9H^2 + \frac{k+1}{(k+2)} 3\dot{H}\right] \geq 0, \tag{42}$$

$$\text{WEC}: \quad \rho \geq 0, \tag{43}$$

$$\text{SEC}: \quad -(\alpha^2 - \beta^2)(\rho + 3p) = \left(\frac{3k^2 + k + 2}{(k+2)^2} 9H^2 + \frac{k+1}{(k+2)} 9\dot{H} - \frac{2}{A^2}\right)\alpha + \left(\frac{k^2 - k - 2}{(k+2)^2} 27H^2 + \frac{3k-1}{(k+2)} 3\dot{H} - \frac{2}{A^2}\right)\beta - 2(\alpha - \beta)\Lambda_0 \geq 0, \tag{44}$$

$$\text{DEC}: \quad (\alpha^2 - \beta^2)(\rho - p) = \left(\frac{k^2 + 3k + 2}{(k+2)^2} 9H^2 + \frac{k+1}{(k+2)} 3\dot{H} - \frac{2}{A^2}\right)\alpha + \left(\frac{k^2 - k - 6}{(k+2)^2} 9H^2 + \frac{k-3}{(k+2)} 3\dot{H} - \frac{2}{A^2}\right)\beta - 2(\alpha - \beta)\Lambda_0 \geq 0. \tag{45}$$

We wish to compel our model in such a manner that the WEC be satisfied through out the cosmic evolution. In order to achieve this, one has to take a balance between the parameters of the Hubble rates and the choice of the coupling constant β. Since at late times, our model overlaps with ΛCDM model, the NEC and DEC are satisfied at least at late phase of cosmic evolution. On the other hand, the SEC condition is violated at late times even though there occurs some possibility that SEC be satisfied at an early epoch. In fact, a detailed analysis on these energy condition may be possible once the cosmic dynamics is fixed up from an assumed or derived Hubble rate.

5. Scalar field reconstruction

In GR, the late time cosmic acceleration phenomena is modelled usually through a scalar field ϕ which may either be quintessence like or phantom like with the EoS parameter being $\omega \geq -1$ or $\omega \leq -1$ respectively. The action for such cases is given by

$$S_\phi = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi} + \frac{\epsilon}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right], \tag{46}$$

where $\epsilon = +1$ for quintessence field and $\epsilon = -1$ for phantom field. $V(\phi)$ is the self interacting potential of the scalar field. The scalar field dynamically rolls down the potential and thereby mediating for cosmic acceleration. In this work, we wish to draw a correspondence between the geometrically modified gravity theory discussed above with that of the scalar field cosmology and also wish to reconstruct the scalar field along with the scalar potential. In a flat Friedman background, the energy density and pressure are expressed by

$$\rho_\phi = \frac{\epsilon}{2}\dot{\phi}^2 + V(\phi), \tag{47}$$

$$p_\phi = \frac{\epsilon}{2}\dot{\phi}^2 - V(\phi). \tag{48}$$

A direct correspondence of our model with the scalar field yields

$$\dot{\phi}^2 = \frac{\epsilon}{\alpha - \beta}[S_3 - S_2], \tag{49}$$

$$V(\phi) = \frac{\{S_2 + S_3\}\alpha - \{2S_1 - S_2 + S_3\}\beta}{\alpha^2 - \beta^2} - \frac{2\Lambda_0}{\alpha + \beta}. \tag{50}$$

Since the factor $S_3(H) - S_2(H)$ decreases with the cosmic evolution, we expect the magnitude of $\dot{\phi}$ to decrease with cosmic time. It is worth to mention here that the exact behaviour of the scalar field will be model dependent and can be investigated with some specific evolutionary behaviour of the Hubble rate.

6. Model with a hybrid scale factor

The formalism developed in this work can be used to investigate certain aspects of cosmic dynamics. One can note that all the dynamical properties are expressed in terms of the Hubble rate H. Therefore, for a given dynamics, if the Hubble rate is known, then it becomes easy to track the evolution history. In view of this, we employ a hybrid scale factor (HSF) $\mathcal{R} = e^{at}t^{b}$ in the formalism. Here a and b are the model parameters and are constrained from different observational and physical basis. The reason behind the choice of such a scale factor is that it simulates a transition from a decelerated universe in recent past to an accelerated one. Moreover, the dynamical behaviour of HSF as predicted remains intermediate to that of the power law expansion and exponential expansion. Transit redshift z_t is an important cosmological parameter which has been recently constrained from an analysis of type Ia Supernova observation and Hubble parameter measurements as $z_t = 0.806$ (Jesus *et al.* 2018; Farooq *et al.* 2017). Tripathy *et al.* (2020a, b) have used the $H(z)$ data to construct four different HSF models that provide a good description of the cosmic transit phenomena. In those works (Tripathy *et al.* 2020a, b), the parameters of the HSF are constrained as (0.585, 0.3), (0.65, 0.2), (0.47, 0.3) and (0.51, 0.2). It has been claimed that these constructed HSF models are compatible to the observational $H(z)$ data. The cosmic transit redshift z_t as obtained for all the HSF models are close to either 0.5 or 0.8. In a recent work, we have constrained the parameters of HSF as $a = 0.695$ and $b = 0.085$ so as to obtain a transition redshift $z_t = 0.806$ (Mishra *et al.* 2018a). Eventhough it is more logical to use SN Ia data, to constrain the model parameters and to apply the formalism developed in the present work to test the viability, in the present work, we have used the HSF parameters as constrained in Mishra *et al.* (2018a) to construct a cosmological model under the modified gravity and the formalism described in previous sections. The viability of the constructed model will be tested in Section 7 through different diagnostic approaches. The Hubble parameter for the HSF is given by $H = a + \frac{b}{t}$ so that the directional Hubble rates become $H_1 = \frac{3k}{k+2}\left(a + \frac{b}{t}\right)$ and $H_2 = H_3 = \frac{3}{k+2}\left(a + \frac{b}{t}\right)$. The deceleration parameter for HSF is $q = -1 + \frac{b}{(at+b)^2}$. In Fig. 1(a), we have shown the deceleration parameter q which displays the signature flipping behaviour at a suitable transit redshift. The deceleration parameter decreases from $q = -1 + \frac{1}{b} \simeq 10.765$ at an early time to $q \simeq -1$ at late time. At the present epoch, this models predicts the deceleration parameter to be $q = -0.86$. Recently, analysis from a host of Hubble parameter measurements and type Ia supernova observational data casts a doubt that, the universe has already reached the peak of its acceleration and may be we are currently witnessing a possible slowing down (Shefieloo *et al.* 2009; Jhang & Xia 2018). Such a feature is investigated through the reconstruction of the slope of the deceleration parameter from observations. In order to check whether the HSF can predict such a feature we have plotted the function $q'(z) = \frac{dq}{dz}$ as a function of redshift $z = \frac{\mathcal{R}_0}{\mathcal{R}} - 1$ in Fig. 1(b). Here $\mathcal{R}_0$ is the scale factor at present epoch. The figure shows that there is no slowing down in cosmic acceleration at late phase of cosmic time. However, we find an interesting feature where $q'(z)$ peaks up at around $z = 1.5$. In order to have a quantitative idea about the deceleration parameter, we have listed some of its values at different epochs in Table 1.

In Fig. 2, the dynamical aspect of the model is assessed through the plot of the EoS parameter as function of redshift. In the figure ω is shown for a fixed anisotropic parameter $k = 1.0000814$ and for four different values of the coupling constant β. In general, the EoS parameter decreases from an initial positive value to behave like a cosmological constant at late phase. The initial positive value depends on the choice of β. Higher is the value of β, lower is the initial value. As expected, at late times, the model is insensitive to the choice of the coupling constant β. However at an early epoch, the EoS parameter evolves through different trajectories for different choices of β. Lower is the value of β, higher is the slope of the EoS curve at late times. In other words, at an early epoch, the ω trajectory for low values of β remains in the top of others. In the figure, for a comparison, we have also shown the trajectories for two well known ω parametrizations such as CPL (Chevallier & Polarski 2001; Linder 2003) and BA (Barboza & Alcaniz 2012) given by $\omega(z) = \omega_a + \omega_0 \frac{z}{1+z}$ and $\omega(z) = \omega_0 + \omega_a \frac{z(1+z)}{1+z^2}$ respectively. At the present epoch, the values of the EoS parameter from the HSF read as $\omega(\beta = -0.5) = -0.892$, $\omega(\beta = 0) = -0.888$, $\omega(\beta = 0.5) = -0.883$ and $\omega(\beta = 10.65) = -0.832$. For the CPL and BA models the EoS at the present epoch respectively becomes -0.9 and -1.1 corresponding to the density parameter $\Omega_m = 0.3$ at the present epoch.

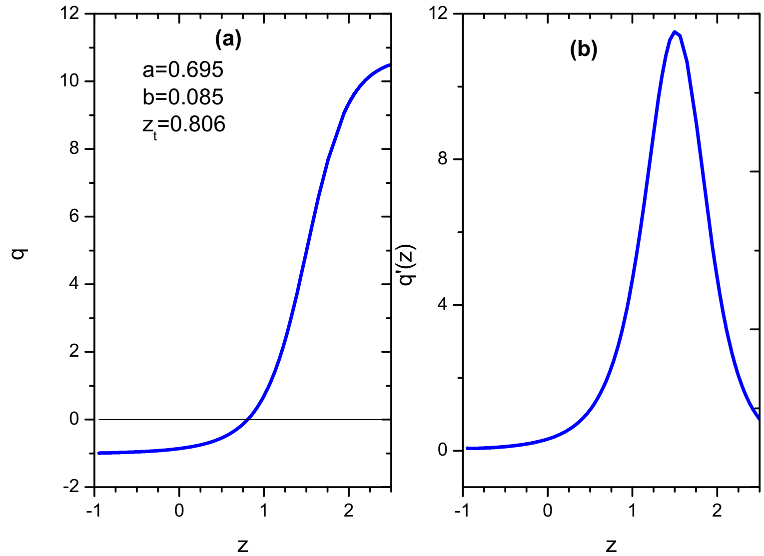

Figure 1. (**a**) Deceleration parameter for HSF showing the transition redshift, (**b**) $q'(z)$ as a function of redshift. The model does not favour a slowing down at late phase.

Table 1. Deceleration parameter at different epochs.

Epoch	z	q
Late phase	−0.9	−0.99
Present	0	−0.86
At transit	0.8	0
Early phase	1.5	4.93

Our model with $\beta = 10.65$ predicts the density parameter at the present epoch as $\Omega_m = 0.3$. In view of this, we may compare our model for $\beta = 10.65$ with those of CPL and BA. It is clear from the comparison that, in a time zone in the range $-0.25 \leq z \leq 0.6$, the EoS from HSF is closer to that of other models.

The effect of the anisotropic parameter k on the EoS is investigated in Fig. 3. In the figure we have shown

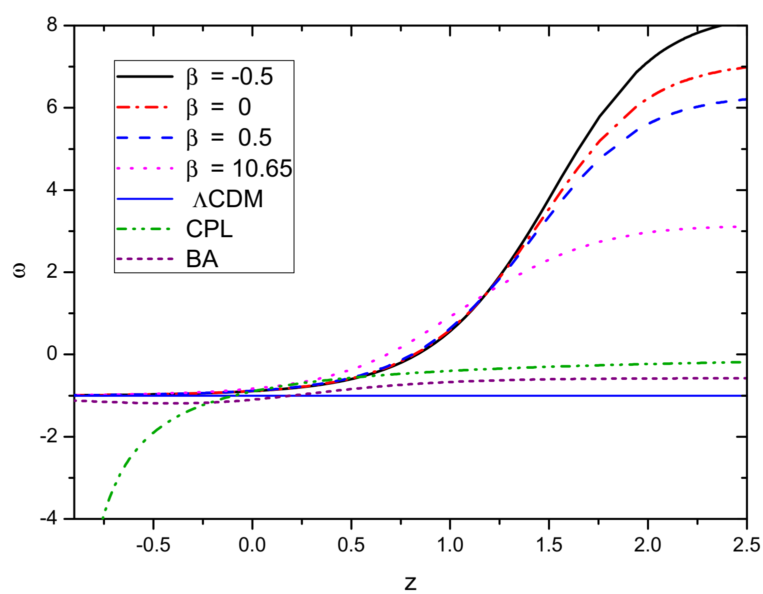

Figure 2. Equation of state parameter for four representative values of the coupling constant β. $k = 1.0000814$. At the present epoch, the values of the EoS parameter from the HSF read as $\omega(\beta = -0.5) = -0.892$, $\omega(\beta = 0) = -0.888$, $\omega(\beta = 0.5) = -0.883$ and $\omega(\beta = 10.65) = -0.832$. For the CPL and BA models the EoS at the present epoch respectively becomes -0.9 and -1.1 corresponding to the density parameter at the present epoch $\Omega_m = 0.3$.

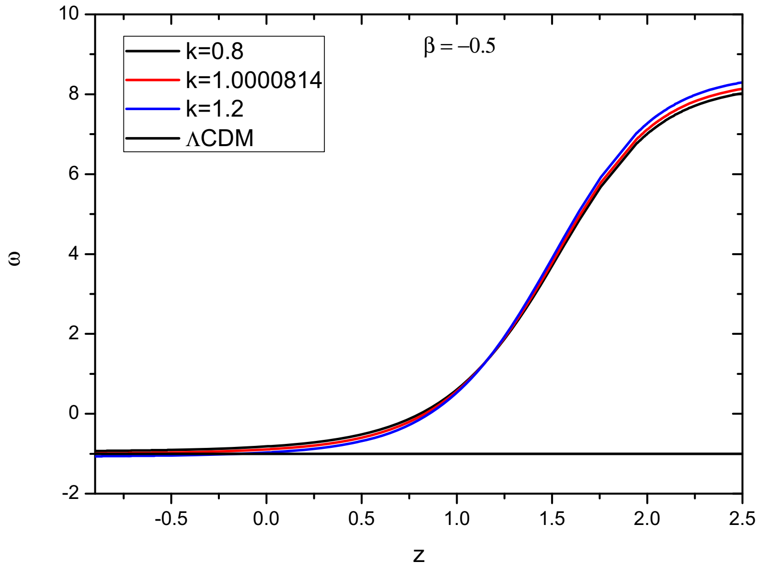

Figure 3. Equation of state parameter for three representative values of the anisotropy parameter k. $\beta = -0.5$.

Table 2. Variation of EoS parameter $\omega(z)$ with anisotropy parameter.

Epoch	z	$\omega(k=0.8)$	$\omega(k=1.0000814)$	$\omega(k=1.2)$
Late phase	−0.9	−0.932	−0.994	−1.064
Present	0	−0.819	−0.892	−0.971
At transit	0.8	−0.012	−0.08	−0.152
Early phase	1.5	3.68	3.76	3.86

the evolution of ω for a given coupling constant $\beta = -0.5$ for three representative values of k namely $k = 0.8, 1.0000814$ and 1.2. We note here that, we have considered a specific shear expansion to Hubble rate ratio in the present epoch within the observational limits and have constrained k to be 1.0000814. This value of k shows a very little departure from its isotropic value. Unlike that of the coupling constant, cosmic expansion anisotropy affects the cosmic dynamics both at an early epoch and at late times. The effect of cosmic anisotropy is almost symmetrical about $z = 1.25$. At epochs $z < 1.25$, higher the value of k, lower is the value of ω and at epochs $z \geq 1.25$, the EoS shows an opposite behaviour i.e. higher the value of k, higher is the value of ω. A quantitative idea on the effect of the cosmic expansion anisotropy on the EoS can be obtained from the values listed in Table 2.

In order to assess a simultaneous effect of the coupling constant β and the cosmological constant Λ_0 on the EoS parameter, we have shown the variation of ω at the present epoch with respect to β for four different values of Λ_0 and a given cosmic anisotropy $k = 1.0000814$ in Fig. 4. The representative values of Λ_0 are considered as multiples of the present value of energy density ρ_0 as calculated from the present model. ω increases with the increase in the coupling constant for a given value of Λ_0. Also for a given β, it increases with the increase in Λ_0. However, the net variation of ω with respect to β decreases with an increase in Λ_0. To get a quantitative view, in Table 3, the values of the EoS parameter at the present epoch are given for some representative values of the cosmological constant and the coupling constant. In Fig. 5, we have plotted the density parameter Ω_m as a function of redshift for different values of the model parameter β and a given value of the anisotropy parameter $k = 1.0000814$. The density parameter Ω_m at the present epoch respectively becomes 0.902, 0.829 and 0.767 corresponding to the β values -0.5, 0 and 0.5. One should note that, the present value of the density parameter is $\Omega_m = 0.3$. In our HSF model, the same value of Ω_m is reproduced for $\beta = 10.65$.

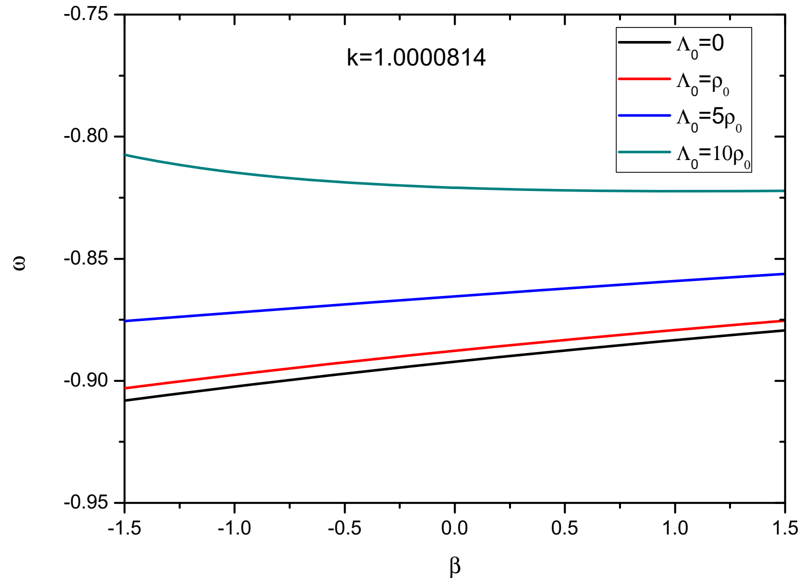

Figure 4. Equation of state parameter as function of coupling constant β for three representative values of the cosmological constant $k = 1.0000814$.

Table 3. EoS parameter at present epoch for different values of cosmological constant Λ_0 and the coupling parameter β.

Λ_0	$\omega_0(\beta = -1)$	$\omega_0(\beta = -0.5)$	$\omega_0(\beta = 0)$	$\omega_0(\beta = 0.5)$	$\omega_0(\beta = 1)$
0	−0.902	−0.897	−0.892	−0.888	−0.883
ρ_0	−0.898	−0.892	−0.888	−0.883	−0.879
$5\rho_0$	−0.872	−0.869	−0.866	−0.862	−0.859
$10\rho_0$	−0.815	−0.819	−0.821	−0.822	−0.822

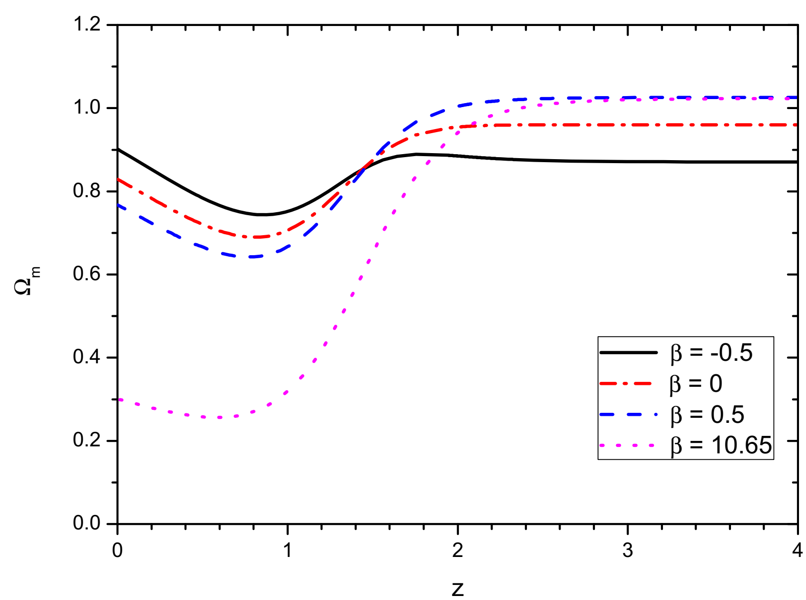

Figure 5. The density parameter Ω_m as a function of redshift for the anisotropic parameter $k = 1.0000814$ and four representative values of β. At the present epoch Ω_m respectively becomes 0.902, 0.829, 0.767 and 0.3 corresponding to the β values −0.5, 0, 0.5 and 10.65.

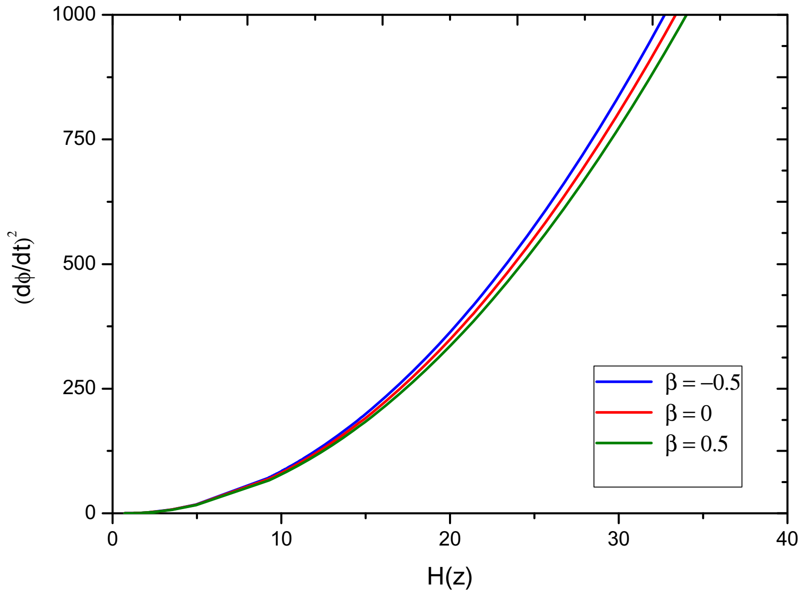

Figure 6. Squared slope of reconstructed scalar field as a function of Hubble rate.

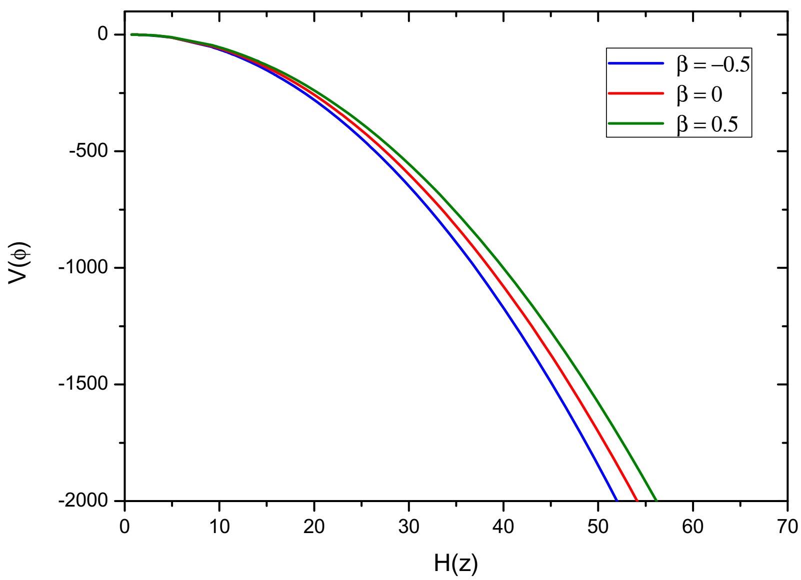

Figure 7. Scalar potential as a function of Hubble rate.

In Fig. 6, the quintessence like scalar fields are reconstructed from our model for three representative values of the coupling constant β. The anisotropy parameter k and the time independent cosmological constant Λ_0 are chosen to be 1.0000814 and ρ_0 respectively. As per expectation, the scalar field is found to decrease with the cosmic expansion. In Fig. 7, the evolution of the self interacting potential for the quintessence like scalar field is plotted. The self interacting potential increases with cosmic expansion. The choice of the coupling constant β does not affect the general evolutionary behaviour of these two quantities. However with an increase in β value at a given epoch, the scalar field decreases and the potential increases.

7. Diagnostic approach

There are two important diagnostic approaches used in literature. They are the determination of the state finder pair $\{j, s\}$ in the $j - s$ plane and the $Om(z)$ diagnostics. These geometrical diagnostic approaches are useful tools to distinguish different Dark Energy models. While the state finder pair involve third derivatives of the scale factor, the $Om(z)$ parameter involve only the first derivative of the scale factor appearing through the Hubble rate $H(z)$.

7.1 *Statefinder pair*

State finder pairs provide an useful tool to distinguish Dark Energy models since they involve the third derivative of the scale factor. They are defined as

$$j = \frac{\dddot{\mathcal{R}}}{\mathcal{R}H^3} = \frac{\ddot{H}}{H^3} - (2+3q), \tag{51}$$

$$s = \frac{j-1}{3(q-0.5)}. \tag{52}$$

In our formalism, the deceleration parameter is a time varying quantity and therefore the state finder pair evolve with time. In Fig. 8, the $j-s$ trajectory in the $j-s$ plane is shown for the HSF considered in this work. The plot can be divided into two regions: quintessence region ($j<1, s>0$) and the Chaplygin gas (CG) region ($j>1, s<0$). Both the regions meet at the ΛCDM point with $j=1, s=0$. The model evolves from a Chapligyn gas region into the quintessence region and then recurves back to overlap with the ΛCDM model. The small black square in the figure represent the present values of the statefinder pair ($j=0.939, s=0.0149$) as predicted by the model. It should be noted here that, our model lies in the quintessence region in the present epoch and its behaviour is in accordance with the recent Planck results (Ade *et al.* 2016).

7.2 *Om(z) diagnostic*

Another geometric diagnostic methods is the $Om(z)$ diagnostic that involves first derivative of the scale factor and therefore becomes easier to apply to distinguish between different Dark Energy models (Sahni *et al.* 2008). The $Om(z)$ parameter is defined by

$$Om(z) = \frac{E^2(z)-1}{(1+z)^3-1}, \tag{53}$$

where $E(z)=\frac{H(z)}{H_0}$ is the dimensionless Hubble parameter. Here H_0 is the Hubble rate at the present epoch. If $Om(z)$ becomes a constant quantity, the DE model is considered to be a cosmological constant model with $\omega=-1$. If this parameter increases with z with a positive slope, the model can be a phantom model with $\omega<-1$. For a decreasing $Om(z)$ with negative slope, quintessence model are obtained ($\omega>-1$). In Fig. 9, the $Om(z)$ parameter for HSF is shown as a function of redshift. It can be observed from the figure that, for a substantial time zone in the recent past ($0\leq z\leq 0.7$), the parameter $Om(z)$ appears to be a constant and therefore the HSF model looks like a cosmological constant model. However, before this period, the model evolves as a quintessence field.

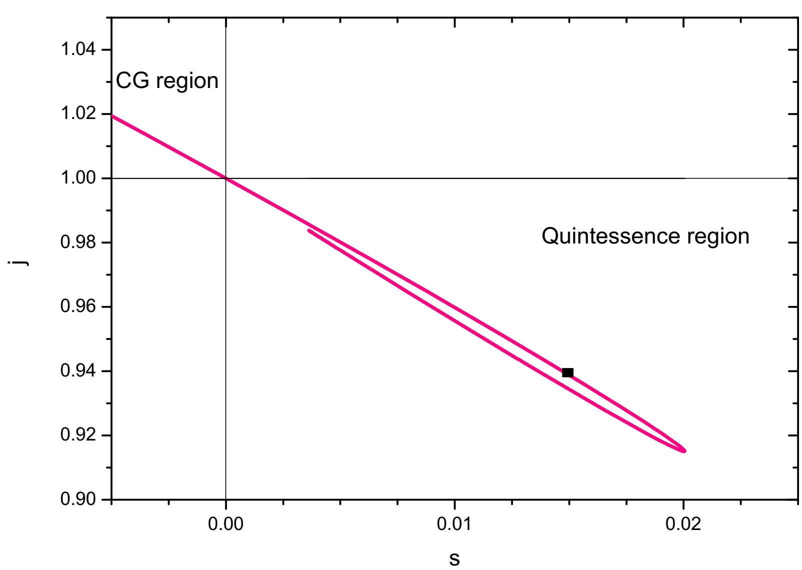

Figure 8. $j-s$ trajectory in the $j-s$ plane. The plot is divided into the Chaplygin gas region and quintessence region. The intersection of the two regions shows the prediction of ΛCDM model. The small black square in the quintessence region shows the prediction from our model in the present epoch.

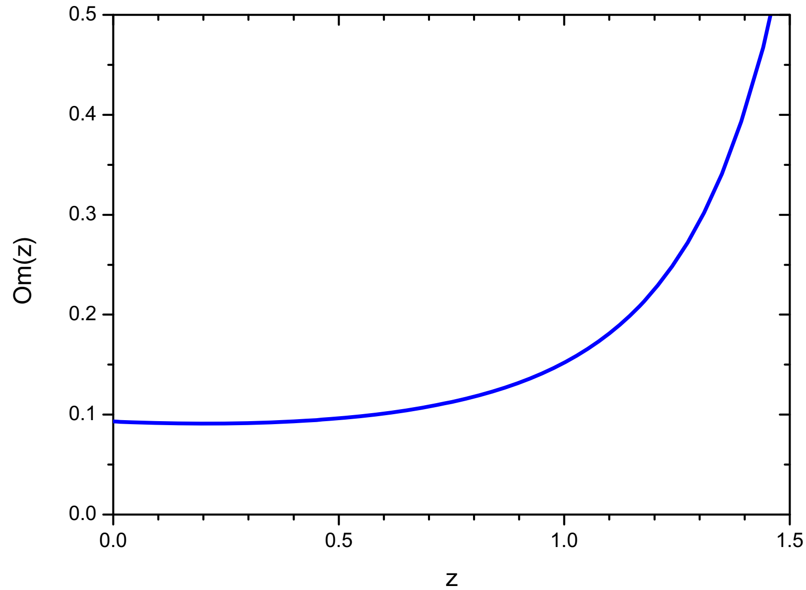

Figure 9. Evolution of the $Om(z)$ parameter for the HSF model. For most of the cosmic time zone in recent past, the model looks like a cosmological constant model.

8. Conclusion

In the present work, we have constructed a cosmological model in an extended theory of gravity by considering the functional $f(R,T) = R + 2\Lambda_0 + \beta T$, where Λ_0 is a constant. This model reduces to the usual GR equations with a cosmological constant in the limit of a vanishing coupling constant β. Investigation of dynamical features of universe in such an extended theory requires an involved calculation. In order to study certain dynamical cosmic aspects, we have adopted an interesting approach in the present work and obtained the expressions in a more general manner. Although the cosmological principle assuming a homogeneous and isotropic universe is a good approximation to the present universe, it is yet to be proven in high energy scales. In view of this, we have considered an anisotropic universe which is more general than the FRW model for our purpose. The anisotropic model we have constructed can be applicable to any amount of cosmic anisotropy. The anisotropic behaviour can be assessed through the value of the anisotropy parameter at the present epoch which has been constrained as $k = 1.0000814$. This value of cosmic anisotropy leads to $\left(\frac{\sigma}{H}\right)_0 = 4.7 \times 10^{-5}$. The expansion asymmetry from our model is obtained to be $\frac{\triangle H}{H} = 0.814 \times 10^{-4}$ which is in conformity with the observations.

A dynamically changing universe with a feature of early deceleration and late time cosmic acceleration is simulated through a hybrid scale factor. The parameters of the HSF are constrained from some physical basis to reproduce the transition redshift as obtained from different observational analysis. This HSF provides a good estimate of the deceleration parameter and the Hubble rate at the present epoch. Recently, there has been a belief that, we are at the peak of the cosmic acceleration and the universe is now slowing down. We have investigated such a feature of the universe employing the HSF and obtained that there is no such slowing down in recent past or recent future.

The dynamical behaviour of the model is assessed through the calculation of the EoS parameter employing the HSF. The EoS parameter decreases from a positive value in an early phase to a value closer to -1 at late times. The behaviour of the EoS parameter is insensitive to the choice of the coupling constant at late times and all the trajectories of EoS parameter for different choices of the coupling parameter behave alike at late phase. However at an early phase, the trajectory splits into different β channels. Trajectory with low value of β lies in the top of all trajectories. Different diagnostic approaches have been adopted to analyse the viability of the present constructed model. At late phase, the model looks like a ΛCDM model for a substantial cosmic time zone in recent past. In the rest phase, it behaves as a quintessence field.

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