J. Astrophys. Astr. (2020)41:21
https://doi.org/10.1007/s12036-020-09638-7

Approximate analytical solution for the propagation of shock waves in self-gravitating perfect gas via power series method: isothermal flow

G. NATH

Department of Mathematics, Motilal Nehru National Institute of Technology, Allahabad 211 004, India.
*Corresponding author. E-mail: gnath@mnnit.ac.in; gn_chaurasia_univgkp@yahoo.in;

MS received 11 December 2019; accepted 7 July 2020

Abstract. For the propagation of a shock (blast) in a self-gravitating perfect gas in case of spherical and cylindrical symmetry, an approximate analytical solution is investigated. The shock wave is considered to be a strong one, with the ratio $\left(\frac{C}{V_S}\right)^2$ to be a small quantity, where c is the sound speed in an undisturbed medium and V_S is the shock wave velocity. The initial density in the undisturbed medium is taken to be varying according to a power law. To obtain the approximate closed-form similarity solution, the flow variables are expanded in a power series of $\left(\frac{C}{V_S}\right)^2$. The first- and second-order approximations are discussed with the help of power series expansion. The analytical solutions are constructed for the first-order approximation. The distribution of the flow variables for first-order approximation in the flow field region behind the shock wave is shown in graphs for both the cylindrical and spherical geometries. The effect of flow parameters, namely, ambient density variation index α, adiabatic exponent $\gamma$and gravitational parameter G_0, are studied on the flow variables and on the total energy of disturbance in the case of the first approximation to the solutions. It is shown that the total energy of the disturbance in the flow field region behind the shock wave decreases with an increase in initial density variation index or adiabatic exponent, i.e. shock strength increases with increase in the value of adiabatic exponent or initial density variation index. A comparison is also made between the solutions obtained for non-gravitating and self-gravitating gases.

Keywords. Shock and blast waves—self-gravitating perfect gas—perturbation method—similarity solution—isothermal flows.

1. Introduction

Shock wave problems are of immense significance from the standpoint of both fundamental research and practical applications. Due to the deposition of a huge amount of energy in a very small region over short intervals, as in the case of spark discharges in air or in an explosion, the shock wave phenomena arise. The thorough study of shocks within stars began by Ro and Matzner (2017), who analyzed the birth and early phase of shock front propagating radially. Dessart *et al.* (2010) pointed out that shocks may be responsible for many types of astrophysical phenomenon such as outbursts and they occur naturally when energy is released over a period shorter than the dynamical time. Barring reflection and dissipation by other means, all acoustic waves steepen into shocks in limited time (Landau & Lifshitz 1959). In a one-dimensional case, the point-like strong explosions problems have been investigated in detail since the important work by Sedov 1946. Also, the formation of a blast wave formed by a very powerful explosion was studied by Taylor (1950a, b). Sakurai (1953, 1954) generalized the work of Taylor (1950a, b) to the case of plane and cylindrical shock waves and he obtained the first and second approximation to the solution using the power series method in Sakurai (1953) and Sakurai (1954), respectively. Sedov's problem of explosion is still in use for uncomplicated modeling of supernova remnants (Allen *et al.* 2015; Leahy &

Published online: 07 August 2020

Ranasinghe 2016; Lerche & Vasyliunas 1976; Solinger *et al.* 1975; Vink 2012) in their early evolutionary states (Vink 2012; Woltjer 1972).

The generation and propagation of a shock wave are key to understanding successful, core-collapse supernovae; in the limit that the energy associated with the neutron star bounce far outweighs the gravitational energy of the star, the shock generated from the bounce propagates through and unbinds the stellar envelope. When the density profile of the progenitor can be approximated as a power law, the well-known, self-similar, Sedov–Taylor blast wave (Taylor 1950a, b; Sedov 1959) describes the propagation of the shock and the temporal evolution of the post-shock fluid; as long as the density of the ambient medium does not fall off steeper than $\rho \propto r^{-3}$, the shock decelerates during the blast wave phase. When the density falls off more steeply than $\rho \propto r^{-3}$, a different phase of self-similar propagation is realized owing to the divergence of the energy of the Sedov–Taylor solution and the existence of a critical point in the flow (Koo & McKee 1990; Waxman & Shvarts 1993; Coughlin *et al.* 2002).

The X-ray observation of supernova remnants (SNRs) shows that these are less than 1000 years old and likely to be in the 'early' phase (Crab Nebula, Tycho, Cas A) (Gorenstein *et al.* 1974). In this phase, the effects of swept-up matter are relatively unimportant and the development of the SNRs depends on the nature of explosion, including perhaps dynamical effects of a residual pulsar (Woltjer 1972). The remaining SNRs are usually presumed to be in what is generally referred to as the 'adiabatic phase' (Gorenstein *et al.* 1974; Winkler & Clark 1974). The dynamics of this phase are dominated by swept-up cold interstellar material, and the fluid elements within the spherically expanding shock front are generally assumed to evolve adiabatically (Woltjer 1972).

Several important supernova remnant parameters are derived from X-ray measurements with the aid of a specific theoretical model. Solinger *et al.* 1975 have raised the issues of the reliability (i.e. model dependence) of these derived parameters. They have examined the validity of the 'adiabatic' assumption and explore the alternative extreme of an 'isothermal' blast wave. Because of thermal conductivity, the large temperature gradients predicted by the adiabatic model probably are not maintained in nature, and the effect of thermal conduction must be considered. Thus, the heat conduction is a factor that clearly should be considered in the evolution of SNRs. In the limiting case of infinite conductivity, the SNR would be isothermal. This is the simplest case to treat and illustrates the main effects of the conduction currents. The thermal conduction can alter the evolution of a blast wave in two ways: in a homogeneous medium, it reduces the temperature gradients and eliminates the temperature singularity that occurs at the origin in the Sedov–Taylor solution; in an inhomogeneous medium, it leads to evaporation of the embedded clouds. The simplest way to treat conduction in a homogeneous medium is to assume that it is so efficient that it renders the temperature constant throughout the remnant. Such isothermal blast waves were first analyzed by Korobeinikov (1956) and Korobeinikov *et al.* (1962). The isothermal blast wave is amenable to a self-similar solution and has been treated for the case of a uniform medium by Korobeinikov (1956) in a non-astrophysical context. In order to further understanding of the properties of the self-similar solution, Solinger *et al.* (1975) have treated the general case of an isothermal blast wave in a medium in which ambient density ρ depends on a power of the central distance: $\rho = \rho^* r^{-\alpha}$, where ρ^* and α are constants. The approach adopted in Solinger *et al.* (1975) is: (I) The conductivity is taken to be infinite at all points within the supernova shell, i.e., $\frac{\partial T}{\partial r} \equiv 0$. (II) The hydrodynamic equations are studied to determine whether this somewhat extreme assumption leads to any significant changes in the evolutionary picture.

The implication of isothermal blast wave for SNRs is also considered by Solinger *et al.* (1975). The blast wave radius differs only slightly from the Sedov value, but the density jump at the shock differs significantly from the adiabatic value due to conduction of energy to the shock front from the hot interior (for a blast wave in a steep density gradient, the heat flow is reversed). Also, Naidu *et al.* (1983) have developed approximate analytical solutions using the technique of Laumbach and Probstein (1969), and they have generalized the treatment to allow for energy injection. Although the assumption of isothermality is convenient and approximately accurate, it is physically inconsistent when the conduction is due to electrons in a fully ionized plasma (Lerch & Vasyliunas 1976): if the electrons are hot enough to transport heat efficiently across the blast wave, they are too hot to maintain equipartition between the electron and ion temperatures. Numerical solutions (Cowie 1977), including the saturation of the heat flux, which occurs when the temperature gradient length is comparable to or larger than the blast wave radius, show that the electrons are nearly isothermal and ions nearly adiabatic at early times; at intermediate times, thermal conductions and Coulomb collisions between the electrons and ions result in a structure somewhat similar to the idealized isothermal blast wave (Ostriker 1988).

For isothermal blast waves, the assumption of a vanishing temperature gradient ahead of the blast wave (Korobeinikov 1956, 1976). This concept leads to an adiabatic flow through the discontinuity and the principle of energy conversation therefore holds. Together with boundary conditions given by adiabatic Rankine-Hugoniot conditions (Rankine 1870; Hugoniot 1887), well-defined solutions exist. Nevertheless, the above assumptions describe a mixed system of adiabaticity and isothermality and hence this is not a genuinely isothermal system. Deschner *et al.* (2018) have studied the self-similar solution to isothermal shock problems. They presented the solutions for both the genuinely isothermal implosion problem and explosion problem with one governing set of equations. In the investigated cases, the fluid will cool down immediately on both sides of the discontinuity to ensure a constant temperature throughout the system. This violates the principle of energy conservation but avoids the additional assumption of a vanishing temperature gradient ahead of the shock.

For the internal motion in stars, the explanation and analysis are one of the fundamental in astrophysical problems. The unsteady motion of huge mass of the gas followed by the swift release of energy results flare-ups in novae and supernovae is supported by the observational data. Qualitative performance of the gaseous mass may be discussed using the equations of motion and equilibrium taking into consideration the gravitational forces. For the self-similar problem, the numerical solution was for the first time obtained by Taylor (1950a, 1946) and Sedov (1959) separately. Also, the numerical solutions for self-similar adiabatic flows in self-gravitating gas were obtained by Sedov (1959) and Carrus *et al.* (1951), independently. The homothermal flows in the case of a spherical shock wave in a self-gravitating gas using the similarity method was discussed by Purohit (1974) and Singh and Vishwakarma (1983). A new code for the numerical solution of three-dimensional self-gravitating hydrodynamical problems was described by Truelove *et al.* (1998). Other important works on self-gravitating gas using the similarity method are (Nath 2013, 2014a, 2016; Nath *et al.* 2013, 2018; Nath & Vishwakarma 2016).

Self-gravitational inflows (contractions or accretions or collapses) and outflows (expansions or winds) of an isothermal fluid with spherical symmetry have been investigated over recent decades in various astrophysical context (star formation, supernova explosions, formation and evolution of galaxy clusters, etc.). Under isothermal condition and spherical symmetry, Larson (1969a, b) and Penston (1969a, b) independently found a self-similar solution (Bian & Lou 2005). Shu (1977) obtained another class of similarity solutions containing the so-called 'expansion wave collapse solution', which describes an expanding region of core collapse from a surrounding molecular cloud. They have shown that the adiabatic cooling never dominates over photo-ionization heating and the isothermal assumption for the gas holds. Further, in the case when the shock speed is large, the assumption of an isothermal shock will remain valid for the scales relevant in molecular clouds. Thus, the photo-ionized (H II) regions are essentially isothermal and are modeled as isothermal blast waves that do not conserve energy because their temperature is fixed. Nath and Sinha (2011) have obtained the similarity solution for shock wave in a self-gravitating perfect gas with variable density and azimuthal magnetic field in the case of isothermal flow and the numerical solution was obtained by them. Shock waves through a variable density medium have been treated by Sedov (1959), Sakurai (1956), Rogers (1957), Rosenau and Frankenthal (1976), Vishwakarma and Yadav (2003), Nath (2011, 2020) and others. Their results are further appropriate to the shock formed in the deep interior of stars.

To model a variety of physical problems occurring around us the nonlinear models may be used. The study of nonlinear growth equations has great significance as these equations play an important role in a variety of physical problems modeling in different fields of nonlinear science, such as fluid dynamics, nuclear physics, chemical physics, plasma physics, optical fiber, physics of solid-state and geochemistry. In the nonlinear model studies, we need to identify the fundamental procedure and the importance of the fretful parameters which cannot be understood at a swift look on the model. The analytical solution determination of nonlinear equations is very difficult and only in particular cases can we obtain the solution in closed form (Yan & Zhang 2001; Daghan & Donmez 2016). The exact similarity solution for shock wave propagation in a non-ideal gas with radiation heat flux, magnetic field, radiation energy, radiation pressure and in the influence of gravitational field for spherical geometry is obtained by Nath *et al.* (2019). Nath and Singh (2019) have obtained the approximate analytical solution for unsteady isothermal flow behind the shock wave in the rotating medium in the influence of the magnetic field.

To the best of our knowledge, in the studies related to the shock or blast wave propagation, no one has obtained the analytical solution for the propagation of shock wave in a self-gravitating gas in the case of isothermal flow. We know that for an extremely high temperature radiating flows, such as occurs at once after an intense explosion, the radiative heat flux is so large that the temperature profile is essentially flat and, consequently, the temperature gradient to approach zero i.e. $\frac{\partial T}{\partial r} \rightarrow 0$. Therefore, the dependent temperature in the flow field depends only on time t and not on the distance from the centre of the explosion, i.e. $T = T(t)$, and the flow is known as isothermal flow (Solinger *et al.* 1975; Laumbach & Probstein 1970; Sachdev & Ashraf 1971; Ashraf & Ahmad 1975; Korobeinikov 1976; Zhuravskaya & Levin 1996; Vishwakarma & Nath 2006, 2007, 2009, 2012; Nath 2010, 2011, 2012a, b, 2013, 2014a, b, 2015, 2020; Gretler & Regenfelder 2005; and many others). Because of an intense explosion, a strong shock is generated and then because of high temperature in the flow field behind the shock, intense radiation heat transfer takes place. For this type of flows, the assumption of adiabaticity may not be valid. Thus, the supposition that such flows are isothermal may be an excellent approximation. In the present work, we attempted to find self-similar approximate analytical solutions for the unsteady isothermal flow behind the shock wave in a self-gravitating perfect gas in the case of cylindrical and spherical geometry.

The stability of blast waves has been a long-standing problem for physicists and astrophysicists equally. A question that comes readily to mind is whether the Sedov solutions are stable, particularly when initial density ρ_1 is allowed to vary with position. The stability of the Primakoff–Sedov blast wave and its generalizations have been studied by Bernstein and Book (1980). They have discussed the stability by writing the self-similar solutions of the ideal fluid equations behind a strong spherical or cylindrical shock propagating into a medium with density varying as $\rho_1 = \rho^* r_s^{\alpha}$ in terms of functions of the similarity variable. It is not a trivial exercise to demonstrate the stability even of plane shocks, a problem which has been treated by Erpenbeck (1962) in ideal gas-dynamic systems and by Gardner and Kruskal (1964) for magnetohydrodynamic shocks. All of these calculations found that shock waves propagating in a uniform medium are stable. The physical mechanism is easy to describe. A small ripple in the shock front gives rise to divergence (convergence) in the curved portion ahead (behind) the main front. These regions become weaker (stronger) than the unperturbed shock, and hence propagate slower (faster) than average, thus reducing the amplitude of the ripple. Evidently the longer the perturbation wavelength, the weaker the stabilizing effect of this mechanism. When this interpretation is applied to spherical shock waves, it becomes obscure and must be regarded as no better than a plausibility argument. If the density ρ_1 drops sufficiently rapidly with increasing radius, it is possible that the ripples ahead of the shock run away, while those behind fall farther behind, leading to instability (Bernstein and Book 1980). Lerche and Vasyliunas (1976) and Isenberg (1977) have claimed that for at least some decreasing power-law density distributions, Sedov shocks are unstable for any value of the adiabatic index γ. Their conclusions, obtained after very sophisticated analysis, were outstanding in predicting instability at short wavelengths, contradicting the intuitive argument appropriate to the planar case. Newman (1979) has disputed the results, on the ground that Lerche and Vasyliunas (1976) and Isenberg (1977) improperly treated the boundary condition at the shock front.

A more physically pertinent calculation by Bernstein and Book (1978) has shown that in the earliest stages of supernova blast wave evolution when the blast wave can be modeled as a hot gas accelerating the swept-up interstellar gas, the interface is Rayleigh-Taylor unstable. A complete theoretical analysis of the perturbations of the Sedov solution for arbitrary values of γ has not yet been done. However, it is usually believed, based on a good deal of experimental confirmation (Newman 1980), that it is stable for any of the standard values of the adiabatic index γ. The stability of a shock wave in the later stages of its evolution, when adiabaticity no longer holds, is not well established either experimentally or theoretically. Calculations by Elmegreen and Elmegreen (1978) of the gravitational stability of a pressure-confined slab suggests that cold shock waves can become unstable, but that the time scale for the instability can be quite long. It has been shown by several authors (McCray *et al.* 1975; Mufson 1975; Chevalier and Imamura 1982) that there can be thermal instabilities within the shock front, although the long-term effects of such instabilities are probably small. Also, Giuliani (1979) has proposed that ionization-shock fronts have a rapidly growing oscillatory instability.

The physics of converging shock waves and particularly initiation of the instability of the converging shock waves has been studied experimentally and numerically by Watanable and Takayama (1991). In the experiments Watanable and Takayama (1991) have found that the initial shock wave configuration looked cylindrical, it was gradually deformed with propagation towards the centre and finally showed model-four instability. However, the stability of diverging cylindrical shock or blast waves is one of the unsolved problems of shock wave dynamics associated with shock wave generated due to explosion. Also, the stability of a shock or blast wave for isothermal flow is not well established either experimentally or theoretically in both the spherical and cylindrical geometry.

The cylindrical shock or blast waves may be produced by the explosion of a long thin wire, or by a long intense electrical discharge in air such as the thunderbolt. Strong cylindrical shock waves are connected with the luminous trails of meteors. Strong cylindrical shock waves studies are not only associated with the explosion of a long thin wire but also to certain axially symmetrical hypersonic flow problems, such as the shock envelope behind a fast meteor, or missile. As an example, when a meteor, or a hypersonic missile, is shooting through the atmosphere at great speed, the shock envelope at distances sufficiently far behind the flying object can be considered as locally one-dimensional (Shao-Chi 1954). Also, the cylindrical shock or blast waves are created by an electrical sliding discharge which is produced between electrodes in a coaxial configuration. This discharge simulates a line explosion, differing from it only by the finite amount of time during which the energy is deposited (Arad *et al.* 1987). The problem of line explosion with time-dependent energy release in a self-gravitating gas with varying density is taken because of its relevance to the formation of a cylindrical spark channel from exploding wires. Due to these important applications of cylindrical shock or blast waves both the cylindrical and spherical geometry is considered in the present study.

Our problem is the generalization of the problem studied by Sakurai (1953, 1954) by taking into account the effect of gravitation in the case of isothermal flow. The density in the ambient medium is taken to be varying and obeying power law and the fluid velocity in the ambient medium is assumed to be zero. Following Taylor (1950a, b), Sakurai (1953, 1954) and Nath and Singh (2019), the flow variables are expanded in the power series of $\left(\frac{C}{V_S}\right)^2$, where c denotes the speed of sound and V_S is the shock propagation velocity. Our solution corresponds to the approximation valid for the case when the velocity of the shock is so large that the ratio $\left(\frac{C}{V_S}\right)^2$ can be neglected. Thus in this work, the solution in the form of power series in $\left(\frac{C}{V_S}\right)^2$ is constructed. The first-order approximate solutions are produced in analytical form. The ODEs for the second approximation in the non-dimensional form to the flow variables and boundary conditions are also presented. The effects of adiabatic exponent, gravitational parameter, initial density variation index on the flow variables and total energy of the disturbance are discussed via tables and figures. Also, a comparison is made between the solution obtained for non-gravitating and self-gravitating gases.

2. Basic equations and boundary conditions

The fundamental equations of motion for one-dimensional unsteady flow of perfect gas, under the assumption that the flow to be isothermal may be written as (Carrus *et al.* 1951; Purohit 1974; Singh & Vishwakarma 1983; Truelove *et al.* 1998; Nath & Sinha 2011; Nath & Singh 2019; Vishwakarma & Nath 2006; 2007; 2009; 2012; Nath 2010, 2011, 2012a, b, 2013, 2014a, b, 2015)

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial v}{\partial r} + \frac{jv}{r}\right) = 0, \tag{1}$$

$$\frac{Dv}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{Gm}{r^j} = 0, \tag{2}$$

$$\frac{\partial T}{\partial r} = 0, \tag{3}$$

$$\frac{\partial m}{\partial r} - 2\pi j r^j \rho = 0, \tag{4}$$

where v is the fluid velocity, ρ is the density, p is the pressure, r is the space and t is the time coordinates, m is the mass contained in a unit sphere or in a cylinder of unit length, $j = 1, 2$ denote cylindrical and spherical symmetries respectively, T is the temperature and G is the gravitational constant. The expression $\frac{D}{Dt}$ is defined by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial r}. \tag{5}$$

If r_s as a function of time t be the position of the shock front, then $V_s = \frac{dr_s}{dt}$ is the propagation velocity of the shock front. For self-similar solution (Sedov 1959; Nath *et al.* 2019; Nath and Singh 2019), the shock velocity is assumed to vary as

$$V_s^2 = A^2 r_s^{-q}, \tag{6}$$

where A and q are constants.

Immediately ahead of the shock front, the flow variables are characterized by

$$v = v_1 = 0,\ \rho = \rho_1 = \rho^* r_s^{\alpha}, \tag{7}$$

where ρ^* and α are constants and the conditions immediately ahead of the shock front is denoted by the subscript 1. For the ambient medium to be hydrostatic the density of the ambient medium does not fall off steeper than $\rho \propto r^{-3}$($i.e.\ \alpha \geq -3$), the shock decelerates during the blast wave phase. From Equations (2), (4) and (5), we obtain

$$m_1 = \frac{2\pi j \rho^* r_s^{\alpha+j+1}}{(\alpha+j+1)} \text{ and}$$

$$p_1 = -\frac{\pi j \rho^{*2}}{(\alpha+j+1)(\alpha+1)} r_s^{2(\alpha+1)} + p_1(0), \tag{8}$$

where α satisfy the condition $-(j+1) < \alpha < -1$ for p_1 to be positive. Equation (8) shows that the pressure p_1 ahead of the shock front is non-constant, but this is taken as a small positive constant in the case of a strong shock to obtain the solution such that z is non-constant (i.e. μ is non-zero; Equation (29) and (31)). Also, for μ to be non-zero with p_1 a small constant, we have the relation $-\alpha + q \neq 0$. As we want to describe the evolution of a blast wave under the assumption that it is self-similar. Since the inclusion of pressure at the centre usually destroys the self-similarity, Therefore, we set $p_1(0) = 0$, here in Equation (8).

The internal energy per unit mass of the perfect gas and state equation are (Nath 2010; 2014b, Nath and Sinha 2011)

$$e_m = C_v T = \frac{p}{(\gamma-1)\rho};\, p = \rho R T, \tag{9}$$

where the gas constant is R, the specific heat at constant volume is $C_v = \frac{R}{(\gamma-1)}$, and the ratio of the specific heats of the gas is γ and the internal energy per unit mass is e_m.

From Equations (3) and (9), we obtain

$$\frac{p}{p_2} = \frac{\rho}{\rho_2}. \tag{10}$$

A strong cylindrical or spherical shock (blast) waves is supposed to be propagating in the undisturbed medium. The Rankine–Hugonoit jump conditions at the shock front are given by the principle of conservation of momentum, energy and mass (Nath and Sinha 2011; Vishwakarma & Nath 2006, 2007, 2012; Nath 2010, 2020), namely,

$$\rho_1(v_1 - V_s) = \rho_2(v_2 - V_s), \tag{11}$$

$$\rho_1(v_1 - V_s)^2 + p_1 = \rho_2(v_2 - V_s)^2 + p_2, \tag{12}$$

$$\frac{1}{2}(v_1 - V_s)^2 + \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} - \frac{\theta_1}{\rho_{1V_s}} = \frac{1}{2}(v_2 - V_s)^2 + \frac{\gamma}{(\gamma-1)}\frac{p_2}{\rho_2} - \frac{\theta_2}{\rho_{1V_s}}, \tag{13}$$

$$m_1 = m_2, \tag{14}$$

where the conditions immediately behind the shock front are denoted by the subscript 2 and θ is the radiation heat flux, i.e. the flux of energy that enforces isothermality within the blast wave interior.

We have $v_1 = 0$, $p_1 = \text{constant}$, $v_2 = (v)_{(r=r_s)}$, $\rho_2 = (\rho)_{(r=r_s)}$, $p_2 = (p)_{(r=r_s)}$ and $m_2 = m_{(r=r_s)}$ at the shock front $r = r_s(t)$. Substituting these values in Equations (11) to (14), we obtain the Rankine-Hugoniot jump conditions across the shock front as

$$\left(V - (v)_{(r=r_s)}\right) = \beta V_s, \tag{15}$$

$$(p)_{(r=r_s)} = (1-\beta)\rho_1 V_s + p_1, \tag{16}$$

$$(\rho)_{(r=r_s)} = \frac{\rho_1}{\beta}, \tag{17}$$

$$(m)_{(r=r_s)} = m_1, \tag{18}$$

where β is density ratio across the shock front. The expression for $\beta(0<\beta<1)$ is obtained as:

$$\frac{1}{2}(1+\beta) + \frac{1}{(\gamma-1)}\left(\frac{C}{V_s}\right)^2 - \frac{\gamma\beta}{(\gamma-1)} = \frac{(\theta_1-\theta_2)\left[\frac{1}{\gamma}\left(\frac{C}{V_s}\right)^2 + (1-\beta)\right]}{p_{r=r_s} V_s (1-\beta)}. \tag{19}$$

In Equation (19), $(\theta_1 - \theta_2)$ is assumed to be negligible in comparison to the product of V_s and $p_{r=r_s}$, and $C = \left(\frac{\gamma p_1}{\rho_1}\right)^{\frac{1}{2}}$ is the speed of sound in the undisturbed medium. Therefore, Equation (19) reduces to the following form

$$\frac{1}{2}(1+\beta)+\frac{1}{(\gamma-1)}\left(\frac{C}{V_s}\right)^2-\frac{\gamma\beta}{(\gamma-1)}=0. \quad (20)$$

To obtain the closed-form solution using power series, we need to obtain the boundary conditions in terms of $\left(\frac{C}{V_s}\right)^2$. Thus, from Equation (20), we obtain the expression for β in terms of $\left(\frac{C}{V_s}\right)^2$ as

$$\beta = \frac{(\gamma-1)}{(\gamma+1)}\left[1+\frac{2}{(\gamma-1)}\left(\frac{C}{V_s}\right)^2\right]. \quad (21)$$

Substituting the value of β in Equations (15) to (17), we obtain the Rankine–Hugoniot jump conditions across the shock front as

$$(v)_{(r=r_s)}=\frac{2V_s}{\gamma+1}\left[1-\left(\frac{C}{V_s}\right)^2\right], \quad (22)$$

$$(p)_{(r=r_s)}=\frac{2\rho_1 V_s^2}{(\gamma+1)}\left[1-\frac{(\gamma-1)}{2\gamma}\left(\frac{C}{V_s}\right)^2\right], \quad (23)$$

$$(\rho)_{(r=r_s)}=\frac{(\gamma+1)}{(\gamma-1)}\frac{\rho_1}{\left[\frac{2}{\gamma-1}\left(\frac{C}{V_s}\right)^2+1\right]}, \quad (24)$$

$$(m)_{(r=r_s)}=m_1. \quad (25)$$

The energy supplied by the explosive is equal to the energy carried out by the shock wave and thus remains invariable (Sakurai 1953 ; Nath and Singh 2019). Thus, we obtain

$$E_i=\int_0^{r_s}\left\{\frac{1}{2}v^2+\frac{1}{\gamma-1}\left(\frac{p}{\rho}-\frac{p_1}{\rho_1}\right)-\frac{Gm}{r^{j-1}}\right\}\rho r^j dr, \quad (26)$$

where E_i is the explosion energy per unit area or length in the case of spherical and cylindrical geometry of the surface of the blast wave respectively when r_s equals unity. Taking into consideration the equation

$$\int_0^{r_s}\frac{\rho}{\rho_1}r^j dr=\frac{r_s^{j+1}}{(j+1)}, \quad (27)$$

obtained from the Lagrangian equation of continuity and using Equation (27) in Equation (26), we obtain

$$E_i=\int_0^{r_s}\left\{\frac{1}{2}\rho v^2+\frac{1}{\gamma-1}p-\frac{Gm\rho}{r^{j-1}}\right\}r^j dr - \frac{p_1}{(\gamma-1)}\frac{r_s^{j+1}}{(j+1)}. \quad (28)$$

To obtain v, p, ρ and m as functions of r and t, and r_s as functions of t only, we shall use the system of five Equations (1), (2), (4), (9) and Equation (28), and system of boundary conditions (22) to (25).

3. Similarity transformations of fundamental equations

Introducing new independent variables x, z in place of r and t defined by (Sakurai 1953, 1954; Nath & Singh 2019)

$$\frac{r}{r_s}=x,\ \left(\frac{C}{V_s}\right)^2=z. \quad (29)$$

Expressing the quantities v, p, ρ and m in the following form

$$v=V_sX(x,z), p=p_1\left(\frac{V_s}{C}\right)^2P(x,z)=\frac{p_1P(x,z)}{z},$$
$$\rho=\rho_1h(x,z),\ m=m_1M(x,z), \quad (30)$$

where X, p, h and M are functions of non-dimensional new variables x and z, and using Equations (29) and (30), we obtain

$$\frac{\partial}{\partial r}=\frac{1}{r_s}\frac{\partial}{\partial x},\frac{D}{Dt}=\frac{V_s}{r_s}\left\{(X-x)\frac{\partial}{\partial x}+\mu z\frac{\partial}{\partial z}\right\}, \quad (31)$$

where $\mu=r_s\left(\frac{dz}{dr_s}\Big/z\right)$ and μ is a function of z only.

By using Equations (29)–(31) in Equations (1), (2), (4) and (10), we obtain

$$(X-x)\frac{\partial X}{\partial x}+\mu z\frac{\partial X}{\partial z}-\frac{1}{2}\mu X = -\frac{1}{\gamma h}\frac{\partial P}{\partial x}+\frac{G_0 jM}{(\alpha+j+1)x^j}, \quad (32)$$

$$\mu z\frac{\partial h}{\partial z}+(X-x)\frac{\partial h}{\partial x}=-h\left(\frac{jX}{x}+\frac{\partial X}{\partial x}\right)+\alpha h, \quad (33)$$

$$\frac{\partial N}{\partial x}=(\alpha+1+j)hx^j, \quad (34)$$

$$P=\frac{h}{(\gamma+1)^2}\left[2\gamma(\gamma-1)+z\left(4\gamma-(\gamma-1)^2\right)-2z^2(\gamma-1)\right], \quad (35)$$

where $G_0=\frac{2G\pi\rho^*}{A^2}$ is taken as the gravitational parameter and $\alpha+q+2=0$ was necessary to obtain the similarity solution. The conditions $-(j+1)<\alpha<-1$ (Equation (8)) and $\alpha+q+2=0$ arises because of the introduction of self-gravity, i.e. due to the consideration of self-gravitating gas. If the gas is non-gravitating, then we have an energy-conserving blast wave obeying Sedov's scaling. Thus, the similarity criterion is only relevant when gravity is a significant dynamical force.

From Equation (35), we have after the partial differentiation

$$\frac{\partial P}{\partial x}=\frac{1}{(\gamma+1)^2}\left[2\gamma(\gamma-1)+z\left(4\gamma-(\gamma-1)^2\right)-2z^2(\gamma-1)\right]\frac{\partial h}{\partial x}. \quad (36)$$

Using Equation (36) in Equation (32), we obtain

$$h\left\{(X-x)\frac{\partial X}{\partial x}+\mu z\frac{\partial X}{\partial z}-\frac{1}{2}\mu X\right\}+\frac{1}{\gamma(\gamma+1)^2}\left[2\gamma(\gamma-1)+z\left(4\gamma-(\gamma-1)^2\right)-2z^2(\gamma-1)\right]\frac{\partial h}{\partial x}+\frac{G_0 jMh}{(\alpha+j+1)x^j}=0, \quad (37)$$

Using Equations (29) to (31) in energy Equation (28), we obtain

$$z\left(\frac{r_{s_0}}{r_s}\right)^{j+1}=\int_0^1\left[\frac{\gamma hX^2}{2}+\frac{h}{(\gamma-1)(\gamma+1)^2}\left\{2\gamma(\gamma-1)+z\left(4\gamma-(\gamma-1)^2\right)-2z^2(\gamma-1)\right\}-\frac{G_0\gamma jMh}{(\alpha+j+1)x^{j-1}}\right]x^j dx-\frac{z}{(j+1)(\gamma-1)}, \quad (38)$$

where $r_{s_0}=\left(\frac{E_i}{p_1}\right)^{\frac{1}{(j+1)}}$. By using transformations (30), the boundary conditions (22) to (25) change into

$$X(1,z)=\frac{2(1-z)}{(\gamma+1)}, h(1,z)=\frac{(\gamma+1)}{(\gamma-1)}\left[1+\frac{2}{\gamma-1}z\right]^{-1},$$
$$P(1,z)=\frac{2\gamma}{(\gamma+1)}\left[1-\frac{(\gamma-1)}{2\gamma}z\right], M(1,z)=1. \quad (39)$$

After differentiating Equation (38) with respect to z, the expression for μ is obtained as

$$\mu=\frac{\left[(j+1)L-\frac{z}{(\gamma-1)}\right]\left[1+\frac{\alpha}{(j+1)}\right]}{\left[L-z\frac{dL}{dz}\right]}, \quad (40)$$

where

$$L=\int_0^1\left[\frac{\gamma hX^2}{2}+\frac{h}{(\gamma-1)(\gamma+1)^2}\left\{2\gamma(\gamma-1)+z\left(4\gamma-(\gamma-1)^2\right)-2z^2(\gamma-1)\right\}-\frac{G_0\gamma jMh}{(\alpha+j+1)x^{j-1}}\right]x^j dx. \quad (41)$$

4. Construction of closed-form solution in power series in z

As the shock wave is a strong one, the shock velocity V_s is large compared with sound velocity C. Thus $z=\left(\frac{C}{V_s}\right)^2$ is considered to be small there, so that the quantities X, P, h and M can be expanded in power series in powers of z such as follows:

$$X=X^{(0)}+zX^{(1)}+z^2X^{(2)}+z^3X^{(3)}+\cdots+z^lX^{(l)}+\cdots, \quad (42)$$

$$h = h^{(0)} + zh^{(1)} + z^2h^{(2)} + z^3h^{(3)} + \cdots + z^l h^{(l)} + \cdots, \tag{43}$$

$$P = P^{(0)} + zP^{(1)} + z^2P^{(2)} + z^3P^{(3)} + \cdots + z^l P^{(l)} + \cdots, \tag{44}$$

$$M = M^{(0)} + zM^{(1)} + z^2M^{(2)} + z^3M^{(3)} + \cdots + z^l M^{(l)} + \cdots, \tag{45}$$

where $X^{(l)}$, $h^{(l)}$, $p^{(l)}$ and $M^{(l)}$ $(l = 0, 1, 2.\ldots..)$ are all functions of x only. Using these Equations (42) to (45) in the expression for L, i.e. in Equation (41), we obtain

$$L = L_0\left(1 + z\sigma_1 + z^2\sigma_2 + z^3\sigma_3 +\right), \tag{46}$$

where

$$L_0 = \int_0^1 \left[\frac{\gamma h^{(0)}}{2}\left(X^{(0)}\right)^2 + \frac{2\gamma}{(\gamma+1)^2}h^{(0)} - \frac{G_0 j M^{(0)} h^{(0)}}{(\alpha+j+1)x^{j-1}}\right]x^j dx, \tag{47}$$

$$\sigma_1 L_0 = \int_0^1 \left[\frac{\gamma}{2}\left\{\left(X^{(0)}\right)^2 h^{(1)} + 2X^{(0)}h^{(0)}X^{(1)}\right\} + \frac{h^{(0)}\left(4\gamma - (\gamma-1)^2\right)}{(\gamma-1)(\gamma+1)^2} + \frac{2\gamma}{(\gamma+1)^2}h^{(1)} - \frac{G_0 j\left(M^{(0)}h^{(1)} + M^{(1)}h^{(0)}\right)}{(\alpha+j+1)x^{j-1}}\right]x^j dx, \tag{48}$$

$$\sigma_2 L_0 = \int_0^1 \left[\frac{\gamma}{2}\left\{\left(X^{(1)}\right)^2 h^{(0)} + \left(X^{(0)}\right)^2 h^{(2)} + 2X^{(0)}X^{(1)}h^{(1)} + 2X^{(0)}X^{(2)}h^{(0)}\right\} + \frac{h^{(1)}\left(4\gamma - (\gamma-1)^2\right)}{(\gamma-1)(\gamma+1)^2} - \frac{2}{(\gamma+1)^2}h^{(0)} + \frac{2\gamma}{(\gamma+1)^2}h^{(2)} - \frac{G_0 j\gamma\left(M^{(0)}h^{(2)} + M^{(1)}h^{(1)} + M^{(2)}h^{(0)}\right)}{(\alpha+j+1)x^{j-1}}\right]x^j dx, \tag{49}$$

and so on.

Using Equations (41) and (46) in Equation (38), we obtain

$$z\left(\frac{r_{s_0}}{r_s}\right)^{(j+1)} = L_0\left[1 + \left\{\sigma_1 - \frac{1}{L_0(\gamma-1)(j+1)}\right\}z + \sigma_2 z^2 + \sigma_3 z^3 + \cdots\right], \tag{50}$$

Or in view of Equation (29),

$$\left(\frac{C}{V_s}\right)^2\left(\frac{r_{s_0}}{r_s}\right)^{j+1} = L_0\Big[1 + \left\{\sigma_1 - \frac{1}{L_0(\gamma-1)(j+1)}\right\}\left(\frac{C}{V_s}\right)^2 + \sigma_2\left(\frac{C}{V_s}\right)^4 + \sigma_2\left(\frac{C}{V_s}\right)^6 + \cdots\Big]. \tag{51}$$

The above Equation (51) is in the form of power series in $\left(\frac{C}{V_s}\right)$, which provide the relation between the propagation velocity $V_s$and the position of the shock front r_s.

By using Equation (46) in Equation (40), μ may be expressed as

$$\mu = (\alpha + j + 1)\left[1 + \mu_1 z + 2\mu_2 z^2 + \cdots\right], \tag{52}$$

where

$$\mu_1 = \sigma_1 - \frac{2}{(j+1)L_0(\gamma-1)},\ \mu_2 = \sigma_2,$$

$$\mu_3 = \sigma_3 \text{ and so on.} \tag{53}$$

Substituting the values from Equations (42) to (45) and value of μ from Equation (52) in Equations (33) to (35) and (37) and comparing the like power of z on both sides, we get the following system of equations:

For the zeroth power of z:

$$\left(X^{(0)} - x\right)h_x^{(0)} + h^{(0)}X_x^{(0)} + \alpha h^{(0)} + \frac{jh^{(0)}X^{(0)}}{x} = 0, \tag{54}$$

$$\left(X^{(0)} - x\right)h^{(0)}X_x^{(0)} - \frac{(\alpha+j+1)}{2}h^{(0)}X^{(0)} + \frac{2(\gamma-1)}{(\gamma+1)^2}h_x^{(0)} + \frac{G_o j}{(\alpha+j+1)x^j}h^{(0)}M^{(0)} = 0, \tag{55}$$

$$M_x^{(0)} - (\alpha + j + 1)x^j h^{(0)} = 0, \tag{56}$$

$$P^{(0)} - \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} h^{(0)} = 0, \tag{57}$$

For the first power of z:

$$\left(X^{(0)} - x\right) h_x^{(1)} + X^{(1)} h_x^{(0)} + (\alpha + j + 1) h^{(1)} + h^{(0)}\left(X_x^{(1)} + \frac{jX^{(1)}}{x}\right) + h^{(1)}\left(X_x^{(0)} + \frac{jX^{(0)}}{x}\right) + \alpha h^{(1)} = 0, \tag{58}$$

$$-\frac{(\alpha+j+1)}{2} h^{(0)}\left\{X^{(1)} + \mu_1 X^{(0)}\right\} -\frac{(\alpha+j+1)}{2} h^{(1)} X^{(0)} + h^{(0)}\left\{\left(X^{(0)} - x\right) X_x^{(1)} + X_x^{(0)} X^{(1)}\right\} + h^{(1)}\left(X^{(0)} - x\right) X_x^{(0)} + (\alpha+j+1) h^{(0)} X^{(1)} + \frac{1}{\gamma(\gamma+1)^2} \left\{h_x^{(0)}\left(4\gamma - (\gamma-1)^2\right) + 2\gamma(\gamma-1) h_x^{(1)}\right\} + \frac{G_o j}{(\alpha+j+1)x^j}\left\{h^{(1)} M^{(0)} + h^{(0)} M^{(1)}\right\} = 0, \tag{59}$$

$$M_x^{(1)} = (\alpha + j + 1)x^j h^{(1)}, \tag{60}$$

$$P^{(1)} = \frac{1}{(\gamma + 1)^2}\left[\left(4\gamma - (\gamma - 1)^2\right) h^{(0)} + 2\gamma(\gamma - 1) h^{(1)}\right] \tag{61}$$

For the second power of z:

$$X^{(2)} h_x^{(0)} + X^{(1)} h_x^{(1)} + X^{(0)} h_x^{(2)} - x h_x^{(2)} + (\alpha + j + 1)\left[\mu_1 h^{(1)} + 2h^{(2)}\right] + \left[h^{(0)} X_x^{(2)} + h^{(1)} X_x^{(1)} + h^{(2)} X_x^{(0)} + \frac{j}{x}\left\{h^{(0)} X^{(2)} + h^{(1)} X^{(1)} + h^{(2)} X^{(0)}\right\}\right] + \alpha h^{(2)} = 0, \tag{62}$$

$$h^{(0)}\left(X^{(0)} - x\right) X_x^{(2)} + h^{(1)}\left(X^{(0)} - x\right) h_x^{(1)} + h^{(2)}\left(X^{(0)} - x\right) X_x^{(0)} + h^{(0)} X^{(1)} X_x^{(1)} + h^{(1)} X^{(1)} X_x^{(0)} + h^{(0)} X^{(2)} X_x^{(0)} + (\alpha + j + 1)\left\{\mu_1 h^{(0)} X^{(1)} + 2h^{(0)} X^{(2)} + h^1 X^{(1)}\right\} - \frac{(\alpha + j + 1)}{2}\left\{2\mu_2 h^{(0)} X^{(0)} + \mu_1 h^{(1)} X^{(0)} + h^{(1)} X^{(1)} + \mu_1 h^{(0)} X^{(1)} + h^{(2)} X^{(0)} + h^{(0)} X^{(2)}\right\} + \frac{1}{\gamma(\gamma + 1)^2}\{h_x^{(1)}\left(4\gamma - (\gamma - 1)^2\right) - 2(\gamma - 1) h_x^{(0)} + 2\gamma(\gamma - 1) h_x^{(2)}\} + \frac{G_o j}{(\alpha + j + 1)x^j}\left\{h^{(2)} M^{(0)} + h^{(1)} M^{(1)} + h^{(0)} M^{(2)}\right\} = 0, \tag{63}$$

$$M_x^{(2)} = (\alpha + j + 1)x^j h^{(2)}, \tag{64}$$

$$P^{(2)} = \frac{1}{(\gamma + 1)^2}\left[\left(4\gamma - (\gamma - 1)^2\right) h^{(1)} + 2\gamma(\gamma - 1) h^{(2)} - 2(\gamma - 1) h^{(0)}\right]. \tag{65}$$

By putting the values from Equations (42) to (45) in Equation (39) and on both sides comparing the equal power of z, we obtain the boundary conditions as

For the zeroth power of z, we have the shock jump conditions as:

$$X^{(0)}(1) = \frac{2}{\gamma + 1}, P^{(0)}(1) = \frac{2\gamma}{(\gamma + 1)}, h^{(0)}(1) = \frac{\gamma + 1}{\gamma - 1}, M^{(0)}(1) = 1. \tag{66}$$

For the first power of z, we obtain the shock jump conditions as:

$$X^{(1)}(1) = \frac{-2}{\gamma + 1}, P^{(1)}(1) = \frac{(1 - \gamma)}{(\gamma + 1)}, h^{(1)}(1) = -\frac{2(\gamma + 1)}{(\gamma - 1)^2}, M^{(1)}(1) = 0. \tag{67}$$

For the second power of z, we obtain the shock jump conditions as:

$$X^{(2)}(1) = 0, P^{(2)}(1) = 0, h^{(2)}(1) = \frac{4(\gamma + 1)}{(\gamma - 1)^3}, M^{(2)}(1) = 0. \tag{68}$$

For the solution of the problem, first, we need to solve the system of ODE's Equations (54) to (56) with the boundary condition (66) for $X^{(0)}, h^{(0)}, M^{(0)}$ and substituting this solution to Equation (57), we obtain $P^{(0)}$. Then, from Equations (30), (47) and (51), we obtain the first approximation to the structure and velocity of the shock (blast) wave in the form

$$v = V_s X^{(0)}(x), p = p_1 \left(\frac{V_s}{C}\right)^2 P^{(0)}(x), \rho = \rho_1 h^{(0)}(x),$$

$$m = m_1 h^{(0)}(x), \left(\frac{C}{V_s}\right)^2 \left(\frac{r_{s_0}}{r_s}\right)^2 = L_0. \tag{69}$$

In the same way, to obtain the second approximation to the solution, we require to substitute the first approximations $X^{(0)}, P^{(0)}, h^{(0)}\, and\, M^{(0)}$ in Equations (58)–(61) to get a system of ordinary differential equations in second approximation to the solution $X^{(1)}, p^{(1)}, h^{(1)}$and $M^{(1)}$ which contain an undetermined parameter μ_1.

5. The first approximation to solution

The Equations (54) to (57) are written in the form:

$$\frac{h_x^{(0)}}{h^{(0)}} = \frac{\left\{X_x^{(0)} + \frac{jX^{(0)}}{x} + \alpha\right\}}{(x - X^{(0)})}, \tag{70}$$

$$\left(X^{(0)} - x\right)h^{(0)}X_x^{(0)} + \frac{2(\gamma-1)}{(\gamma+1)^2}h_x^{(0)} + \frac{G_o j}{(\alpha+j+1)x^j}h^{(0)}M^{(0)} = \frac{(\alpha+j+1)}{2}h^{(0)}X^{(0)}, \tag{71}$$

$$M_x^{(0)} = (\alpha+j+1)x^j h^{(0)}, \tag{72}$$

$$P^{(0)} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2}h^{(0)}. \tag{73}$$

Using (70) Equation (71) transform into

$$X_x^{(0)} = \frac{\frac{(\alpha+j+1)}{2}\left(x - X^{(0)}\right)X^{(0)} - \frac{G_o j\left(x - X^{(0)}\right)}{(\alpha+j+1)x^j}M^{(0)} - \frac{2(\gamma-1)}{(\gamma+1)^2}\frac{jX^{(0)}}{x} - -\frac{2(\gamma-1)}{(\gamma+1)^2}\alpha}{-\frac{2(\gamma-1)}{(\gamma+1)^2} - \left(X^{(0)} - x\right)^2}. \tag{74}$$

Using Equation (66) the above Equation (74) becomes

$$X_x^{(0)}(1) = \frac{(j+1-\alpha) - \frac{G_o j(\gamma+1))}{(\alpha+j+1)} - \frac{4j}{(\gamma+1)}}{(3-\gamma)}. \tag{75}$$

Following Taylor (1950a, b), Sakurai (1953, 1954) and Nath and Singh (2019), the approximate solution of $X^{(0)}$ of X is supposed to be of the form

$$X^{(0)}(x) = \frac{x}{\gamma} + Bx^n, \tag{76}$$

where the constants B and n are determined by the values of $X^{(0)}$and $X_x^{(0)}$ at $x = 1$, which are given by the boundary conditions (66) and the Equation (75). Thus, the constants B and n are given by

$$B = \frac{(\gamma-1)}{\gamma(\gamma+1)}, \tag{77}$$

and

$$n = \frac{\gamma(\gamma+1)}{(\gamma-1)}\left[\frac{(j+1-\alpha) - \frac{G_o j(\gamma+1))}{(\alpha+j+1)} - \frac{4j}{(\gamma+1)}}{(3-\gamma)} - \frac{1}{\gamma}\right]. \tag{78}$$

Further, using Equation (76) with the value of constants B and n from Equations (77) and (78) into Equation (70), we get, after integration,

$$h^{(0)}(x) = \left(\frac{\gamma+1}{\gamma-1}\right)(x)^{\left[\frac{(j+1+\alpha\gamma)}{\gamma-1}\right]} \left[\frac{\gamma}{(\gamma+1) - x^{n-1}}\right]^{\left[\frac{(j+1+\alpha\gamma)}{(n-1)(\gamma-1)} + \frac{(n+1)}{(n-1)}\right]}, \tag{79}$$

To determine the integration constants, the boundary conditions (66) are used.

From Equation (73), after integration with respect to x, we obtain

$$M^{(0)} = \int_0^x \left[\alpha + j + 1)x^j\right]x^j h^{(0)}(x)dx. \tag{80}$$

From Equations (80) and (74), we obtain

$$P^{(0)} = \frac{2\gamma}{(\gamma+1)}(x)^{\left[\frac{(j+1+\alpha\gamma)}{\gamma-1}\right]}\left[\frac{\gamma}{(\gamma+1) - x^{n-1}}\right]^{\left[\frac{(j+1+\alpha\gamma)}{(n-1)(\gamma-1)} + \frac{(n+1)}{(n-1)}\right]}. \tag{81}$$

Equations (76) to (81) give the first approximate analytical solution of the considered problem.

6. The second approximation to solution

In this section, we have to derive the system of ordinary differential equations for the second approximation $X^{(1)}, P^{(1)}, h^{(1)}$ and $M^{(1)}$ to the flow variables X, P, h and M respectively. We obtain the system of Equations (58) to (61) and boundary conditions (67). Splitting $X^{(1)}, P^{(1)}, h^{(1)}$and $M^{(1)}$ as

$$X^{(1)} = X_1^1 + \mu_1 X_2^1,\ P^{(1)} = P_1^1 + \mu_1 P_2^1,$$
$$h^{(1)} = h_1^1 + \mu_1 h_2^1 \text{ and } M^{(1)} = M_1^1 + \mu_1 M_2^1. \tag{82}$$

Using Equation (82) in Equations (58) to (61), we obtain two sets of equations for $X_1^{(1)}$, $P_1^{(1)}$, $h_1^{(1)}$, $M_1^{(1)}$ and $X_2^{(1)}$,$P_2^{(1)}$, $h_2^{(1)}$, $M_2^{(1)}$ independent of μ_1.

For the zeroth power of μ_1, the subsequent set of equations are given as:

$$\left(X^{(0)} - x\right)h_{1x}^{(1)} + (\alpha + j + 1)h_1^{(1)} + h^{(0)}\left(X_{1x}^{(1)} + \frac{jX_1^{(1)}}{x}\right) + h_1^{(1)}\left(X_x^{(0)} + \frac{jX^{(0)}}{x}\right) + X_1^{(1)}h_x^{(0)} + \alpha h_1^{(1)} = 0, \tag{83}$$

$$h^{(0)}\left\{\left(X^{(0)} - x\right)X_{1x}^{(1)} + X_1^{(1)}X_x^{(0)}\right\} - \frac{(\alpha + j + 1)}{2}\left\{X_1^{(1)}h^{(0)} + h_1^{(1)}X^{(0)}\right\} + h_1^{(1)}\left\{\left(X^{(0)} - x\right)X_x^{(0)} + (\alpha + j + 1)X_1^{(1)}h^{(0)} + \frac{1}{\gamma(\gamma+1)^2}\left\{h_x^{(0)}\left(4\gamma - \left(\gamma - 1\right)^2\right) + 2\gamma(\gamma - 1)h_{1x}^{(1)}\right\} + \frac{G_o j}{(\alpha + j + 1)x^j}\left\{h^{(0)}M_1^{(1)} + h_1^{(1)}M^{(0)}\right\} = 0, \tag{84}$$

$$M_{1x}^{(1)} = (\alpha + j + 1)x^j h_1^{(1)}, \tag{85}$$

$$p_1^{(1)} = \frac{1}{(\gamma+1)^2}\left[\left(4\gamma - (\gamma - 1)^2\right)h^{(0)} + 2\gamma(\gamma - 1)h_1^{(1)}\right]. \tag{86}$$

For the first power of μ_1, the subsequent set of equations are given as:

$$\left(X^{(0)} - x\right)h_{2x}^{(1)} + (\alpha + j + 1)h_2^{(1)} + h^{(0)}\left(X_{2x}^{(1)} + \frac{jX_2^{(1)}}{x}\right) + h_2^{(1)}\left(X_x^{(0)} + \frac{jX^{(0)}}{x}\right) + X_2^{(1)}h_x^{(0)} + \alpha h_2^{(1)} = 0, \tag{87}$$

$$h^{(0)}\left\{\left(X^{(0)} - x\right)X_{2x}^{(1)} + X_2^{(1)}X_x^{(0)}\right\} - \frac{(\alpha + j + 1)}{2}\left\{X_2^{(1)}h^{(0)} + h^{(0)}X^{(0)} + h_2^{(1)}X^{(0)}\right\} + h_2^{(1)}\left\{\left(X^{(0)} - x\right)X_x^{(0)} + (\alpha + j + 1)X_2^{(1)}h^{(0)} + \frac{1}{\gamma(\gamma+1)^2}\left\{2\gamma(\gamma - 1)h_{2x}^{(1)}\right\} + \frac{G_o j\left\{h^{(0)}M_2^{(1)} + h_2^{(1)}M^{(0)}\right\}}{(\alpha + j + 1)x^j} = 0, \tag{88}$$

$$M_{2x}^{(1)} = (\alpha + j + 1)x^j h_2^{(1)}, \tag{89}$$

$$p_2^{(1)} = \frac{1}{(\gamma+1)^2}\left[2\gamma(\gamma - 1)h_2^{(1)}\right]. \tag{90}$$

Putting the values from Equation (82) in Equation (67) and equating to zero the coefficient of zeroth and first power of μ_1, we obtain the following:

For the first power of μ_1:

$$X_1^{(1)}(1) = \frac{-2}{\gamma+1}, h_1^{(1)}(1) = -\frac{2(\gamma+1)}{(\gamma-1)^2}, P_1^{(1)}(1) = -\frac{(\gamma - 1)}{(\gamma + 1)}, M_1^{(1)}(1) = 0, \tag{91}$$

and for the second power of μ_1:

$$X_2^{(1)}(1) = 0, h_2^{(1)}(1) = 0, P_2^{(1)}(1) = 0, M_2^{(1)}(1) = 0. \tag{92}$$

The system of Equations (83) to (86), and the system of Equations (87) to (90), can be integrated numerically or can be solved analytically with the help of boundary condition Equations (91) and (92), to obtain $X_1^{(1)}$, $P_1^{(1)}$, $h_1^{(1)}$, $M_1^{(1)}$ and $X_2^{(1)}$, $P_2^{(1)}$, $h_2^{(1)}$, $M_2^{(1)}$, respectively. After substituting the values obtained above in Equation (48), we have from Equation (53) to determine μ_1 as given below:

$$\mu_1 = \frac{I_1 - \frac{1}{(j+1)(\gamma-1)}}{L_0 - I_2}, \tag{93}$$

where

$$I_1 = \int_0^1 \left[\gamma X^{(0)}h^{(0)}X_1^{(1)} + \frac{\gamma\left(X^{(0)}\right)^2 h_1^{(1)}}{2} + \frac{2\gamma}{(\gamma+1)^2}h_1^{(1)} - \frac{G_o j\gamma}{(\alpha + j + 1)x^{j-1}}\left\{h^{(0)}M_1^{(1)} + h_1^{(1)}M^{(0)}\right\} + \frac{\left(4\gamma - (\gamma - 1)^2\right)}{(\gamma - 1)(\gamma + 1)^2}h^{(0)}\right]x^j dx, \tag{94}$$

and

$$I_2 = \int_0^1 \left[\gamma X^{(0)} h^{(0)} X_2^{(1)} + \frac{\gamma \left(X^{(0)}\right)^2 h_2^{(1)}}{2} + \frac{2\gamma}{(\gamma+1)^2} h_2^{(1)} - \frac{G_o j \gamma \left\{ h^{(0)} M_2^{(1)} + h_2^{(1)} M^{(0)} \right\}}{(\alpha + j + 1) x^{j-1}} \right] x^j dx. \quad (95)$$

With the above values of μ_1 and $X_1^{(1)}, P_1^{(1)}, h_1^{(1)}, M_1^{(1)}$ and $X_2^{(1)}, P_2^{(1)}, h_2^{(1)}, M_2^{(1)}$; we can calculate the second approximation $X^{(1)}, P^{(1)}, h^{(1)}$and $M^{(1)}$ to the flow variables X, P, h and M using Equation (82).

7. Results and discussion

In the case of first approximation to the closed-form solution, the distributions of the flow variables fluid velocity $X^{(0)}(x)$, pressure $P^{(0)}(x)$, density $h^{(0)}(x)$ and mass $M^{(0)}(x)$are drawn using Equations (76) to (81) in Figures 1(a–d), 2(a–d) and 3(a–d). By the numerical integration of Equation (80) using Equation (79), the distribution of mass in the case of the first approximation is obtained. The value of the physical parameters for calculation are taken to be adiabatic exponent $\gamma = \frac{7}{5}, \frac{5}{3}$; initial density variation index $\alpha = -1.1, -1.25, -2, -2.8$; gravitational parameter $G_0 = 0, 0.01, 0.02, 0.03, 0.05, 0.07$; in the case of cylindrical $(j = 1)$ and spherical $(j = 2)$ geometry. The value $G_0 = 0$ (curves 1 and 4 in Figures 1(a–d), 2(a–d) and 3(a–d)) corresponds to non-gravitating case (i.e. similar to the solution given by Sakurai (1953) in adiabatic flow case). To the best of our knowledge, there are, unfortunately, no analytical results that can be used as a benchmark for the presently considered isothermal flow system. In the present problem, we have obtained the solution in the case of isothermal flow with gravitation, while Sakurai (1953) has obtained the solution in the case of adiabatic flow without taking into consideration the effect of gravitation.

The effect of an increase in the value of initial density variation index α and the gravitational parameter G_0 on the flow variables and total energy of disturbance due to the propagation of cylindrical shock wave and spherical shock wave are shown in Figures 1(a–d) and 2(a–d) respectively. Also, the effect of adiabatic index γ and the gravitational parameter G_0 on the flow variables and total energy of disturbance due to the propagation of a cylindrical shock wave is shown in Figure 3(a–d).

Tables 1 and 2 exhibit the values of B,n and first approximation to the values of the total energy of disturbance L_0 for different values of the physical parameters.

From Figures 1(a, d), 2(a, d) and 3(a, d), it is obtained that the flow variables mass and fluid velocity decrease as we move inward from shock wave to the axis of symmetry in the cylindrical case and to the point of symmetry in spherical case, These flow variables have a minimum value at the axis or point of symmetry and maximum value at the shock. From Figures 1(b, c) and 3(b, c), it is obtained that the flow variables pressure and density decrease as we move inward from shock front to the axis of symmetry; whereas these flow variables increase after attaining minimum for $\gamma = \frac{5}{3}$, $\alpha = -1.25$. Also, from Figure 2(b, c) it is shown that the flow variables pressure and density decrease as we move inward from shock front to the axis of symmetry for $\alpha = -2$; whereas these flow variables increase for $\alpha = -2.8$.

7.1 *Effect of an increase in the value of gravitational parameter G_0*

The flow variable fluid velocity $X^{(0)}(x)$ increases (Figures 1(a), 2(a) and 3(a)); whereas the mass$M^{(0)}(x)$ decreases (Figures 1(d), 2(d) and 3(d)) with an increase in the value of G_0. Pressure $P^{(0)}(x)$ and density $h^{(0)}(x)$ increase near shock but decrease near axis or point of symmetry in general (Figures 1(b, c) and 3(b, c)) except in the case of $j = 2$, $\gamma = \frac{7}{5}$ and $\alpha = -2.8$ (Figures 2(b, c)). The total energy of disturbance L_0 decreases in general with an increase in the value of G_0; whereas it increases with an increase in the value of G_0 for $j = 1$, $\gamma = \frac{5}{3}$, $\alpha = -1.25$ or $j = 2$, $\gamma = \frac{7}{5}$, $\alpha = -2.8$ (Tables 1 and 2). Physically, the decrease in the total energy of disturbance means that the shock becomes stronger in self-gravitating gas.

7.2 *Effect of an increase in adiabatic exponent γ value*

The flow variable fluid velocity $X^{(0)}(x)$ increases (see Figure 3(a)); whereas the mass $M^{(0)}(x)$ and pressure $P^{(0)}(x)$ decrease (Figures 3(c, d)) with an increase in γ. Density $h^{(0)}(x)$ increases near shock but decreases near the axis of symmetry (Figure 3(b)). The total energy of disturbance L_0 decreases with an increase in the value of γ (Table 2), i.e. the shock strength

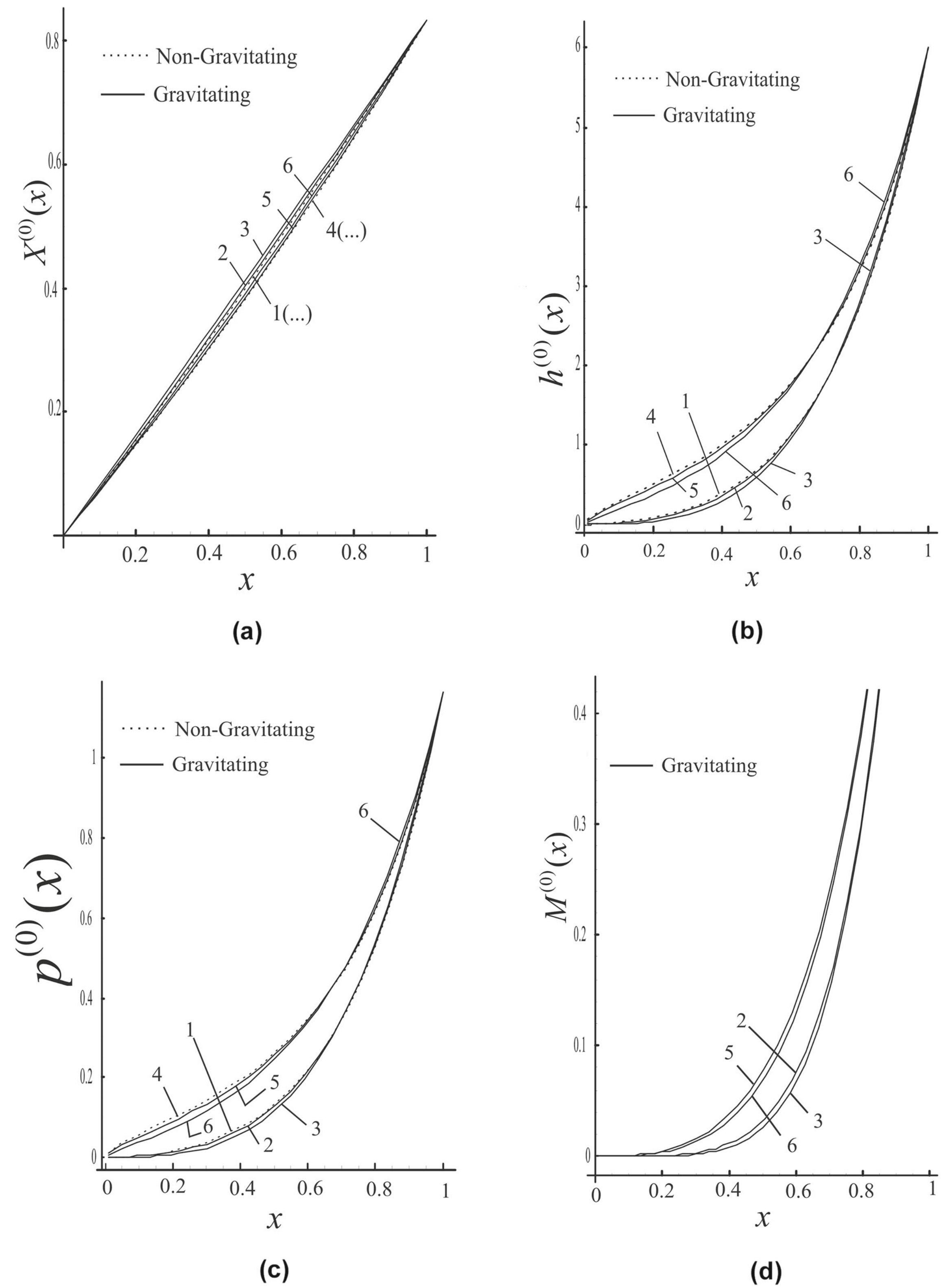

Figure 1. Flow variables distribution in the region behind the cylindrical shock wave $(j = 1)$ with $\gamma = 1.4$: (**a**) fluid velocity$X^{(0)}(x)$, (**b**) density $h^{(0)}(x)$, (**c**) pressure $P^{(0)}(x)$, (**d**) mass $M^{(0)}(x)$: 1.$G_0 = 0$, $\alpha = -1.1$; 2. $G_0 = 0.01$, $\alpha = -1.1$; 3. $G_0 = 0.03$, $\alpha = -1.1$; 4. $G_0 = 0$, $\alpha = -1.25$; 5. $G_0 = 0.01$, $\alpha = -1.25$; 6. $G_0 = 0.03$, $\alpha = -1.25$.

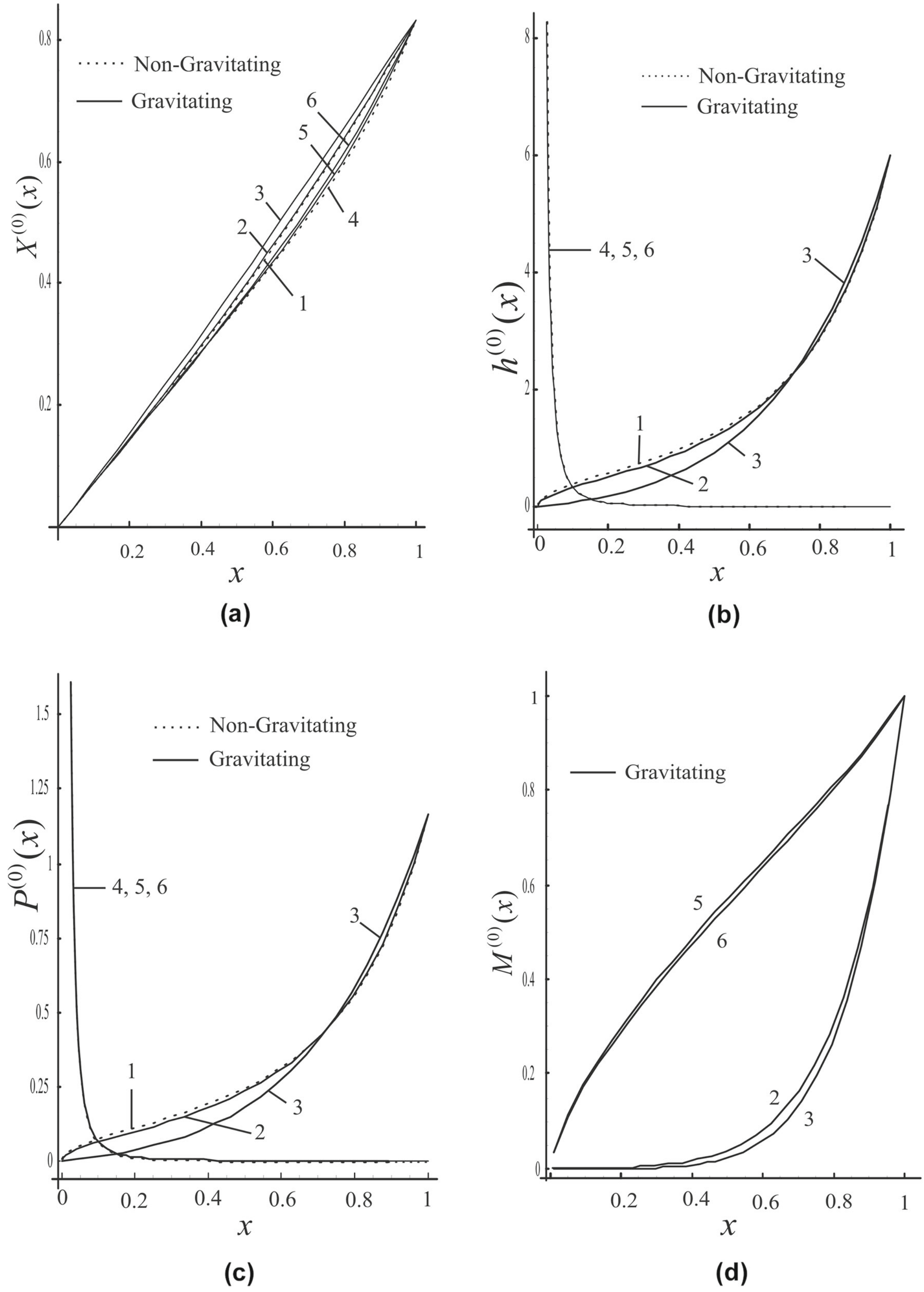

Figure 2. Flow variables distribution in the region behind the spherical shock wave $(j = 2)$ with $\gamma = 1.4$: (**a**) fluid velocity $X^{(0)}(x)$, (**b**) density $h^{(0)}(x)$, (**c**) pressure $P^{(0)}(x)$, (**d**) mass $M^{(0)}(x)$: 1.$G_0 = 0$, $\alpha = -2$; 2. $G_0 = 0.01$, $\alpha = -2$; 3. $G_0 = 0.05$, $\alpha = -2$; 4. $G_0 = 0$, $\alpha = -2.8$; 5. $G_0 = 0.01$, $\alpha = -2.8$; 6. $G_0 = 0.02$, $\alpha = -2.8$.

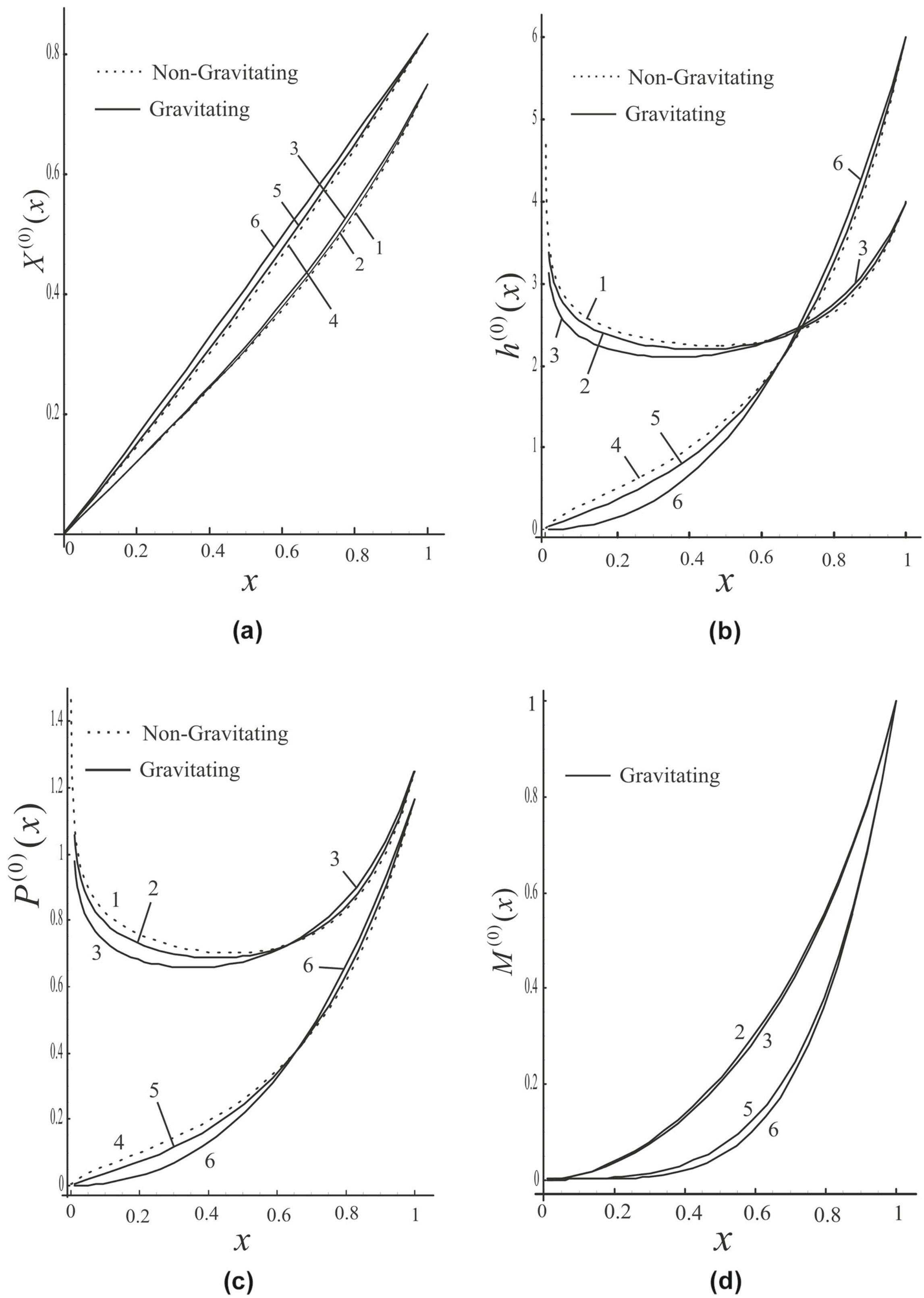

Figure 3. Flow variables distribution in the region behind the cylindrical shock wave $(j = 1)$ with $\alpha = -1.25$: (**a**) fluid velocity $X^{(0)}(x)$, (**b**) density $h^{(0)}(x)$, (**c**) pressure $P^{(0)}(x)$, (**d**) mass $M^{(0)}(x)$: 1.$G_0 = 0$, $\gamma = \frac{7}{5}$; 2. $G_0 = 0.03$, $\gamma = \frac{7}{5}$; 3. $G_0 = 0.07$, $\gamma = \frac{7}{5}$; 4. $G_0 = 0$, $\gamma = \frac{5}{3}$; 5. $G_0 = 0.03$, $\gamma = \frac{5}{3}$; 6. $G_0 = 0.07$, $\gamma = \frac{5}{3}$.

Table 1. The values of n and first approximation to the total energy of disturbance L_0 for different values of α and G_0 with $\gamma = 1.4$.

j	B	α	G_0	n	L_0
1	0.119048	− 1.1	0	1.5250	0.704353
			0.01	1.3850	0.699589
			0.03	1.1050	0.716434
		− 1.25	0	2.3125	0.780479
			0.01	2.1445	0.773008
			0.03	1.8085	0.758899
2	0.119048	− 2	0	2.7500	0.625741
			0.01	2.4980	0.614673
			0.05	1.4900	0.573798
		− 2.8	0	6.9500	1.354300
			0.01	5.6900	1.386550
			0.02	4.4300	1.433370

Table 2. The values of B, n and first approximation to the total energy of disturbance L_0 for different values of γ and G_0 with $j = 1$ and $\alpha = -1.25$.

γ	B	G_0	n	L_0
$\gamma = \frac{7}{5}$	0.1190476	0	2.31250	0.780479
		0.01	2.14450	0.773008
		0.03	1.80850	0.758899
		0.07	1.13650	0.734534
$\gamma = \frac{5}{3}$	0.150026	0	4.75000	0.545831
		0.01	4.57222	0.548484
		0.03	4.21667	0.554203
		0.07	3.50556	0.567629

increases with an increase in the value of adiabatic exponent γ.

7.3 *Effect of an increase in the value of initial density variation index* α

The flow variable fluid velocity $X^{(0)}(x)$ increases (Figures 1(a) and 2(a)). The mass $M^{(0)}(x)$, pressure $P^{(0)}(x)$ and density $h^{(0)}(x)$ decrease with an increase in the value of initial density variation index α (Figures 1(b, c, d) and 2(d)), but in the case of spherical symmetry density $h^{(0)}(x)$ and pressure $P^{(0)}(x)$ decrease near shock and increase near the point of symmetry with an increase in the value of initial density variation index α(Figures 2(b, c)). The total energy of disturbance L_0 decreases with an increase in the value of γ (Table 1), i.e. the shock strength increases with an increase in the value of initial density variation index α.

8. Conclusions

The flow variables of fluid velocity $X^{(0)}(x)$, density $h^{(0)}(x)$, mass $M^{(0)}(x)$ and pressure $P^{(0)}(x)$ together with the value of B and n are given by Equations (76) to (81), which are the approximate analytical solution of the considered problem in the case of the first approximation to the solution. Also, Equation (47) gives the total energy of disturbance L_0 for the first approximation. The present article concerns with the explosion problems an isothermal system in a self-gravitating gas atmosphere. Our work may lead to the applicability of these solutions to early-phase supernova remnants. Shu (1977) has shown that a solution with singular points cannot generally represent the gravitational collapse of isothermal spheres. The obtained approximate analytical solution of the considered problem given by Equations (76) to (81) show that the flow variables vanish in general as x approaches zero. This means that the velocity profile goes monotonically and smoothly (i.e. without a shock) to zero at the origin. As the real core has a finite but small size, the obtained solution should be terminated at some very small value of x by introducing a shock. Thus, our solution corresponds to the gravitational collapse of a gas cloud. A significant difference is obtained in the flow variables' distribution in gravitating and non-gravitating gases. Thus, the shock waves in a self-gravitating perfect gas can be significant for an explanation of shocks in supernova explosions, the central part of starburst galaxies, star formation and shocks in a nuclear explosion, stellar explosion and explosion in the ionosphere. Based on the present study, we may conclude the following:

1. The flow variable fluid velocity $X^{(0)}(x)$ increases, whereas the mass $M^{(0)}(x)$ decreases with an increase in the adiabatic exponent γ or initial density variation index α or due to the assumption of a self-gravitating gas.
2. The density $h^{(0)}(x)$ decreases near the axis of symmetry and increases near the shock in the case of a cylindrical shock with an increase in the value of the gravitational parameter G_0 or adiabatic exponent γ.
3. The pressure $P^{(0)}(x)$ decreases with an increase in the value of the adiabatic exponent γ and initial density variation index α in the case of cylindrical geometry, but in the case of spherical geometry, it decreases near the shock and increases near the point of symmetry.

4. The shock strength increases for $j = 1$ (cylindrical shock) with $\gamma = \frac{7}{5}$; but it decreases for $j = 1$, $\gamma = \frac{5}{3}$, $\alpha = -1.25$ or $j = 2$ (spherical shock) with $\gamma = \frac{7}{5}$, $\alpha = -2.8$ with an increase in the value of the gravitational parameter G_0.
5. An increase in the value of adiabatic exponent γ or initial density variation index α causes a decrease in the total energy of the disturbance, i.e. shock strength increases with an increase in the value of γ or α.

References

Arad B., Gazit Y., Ludmirsk A. 1987, J. Phys. D: Appl. Phys., 20, 360

Allen G. E., Chow K., DeLaney T., Filipovic M. D., Houck J. C., Pannuti T. G., Stage M. D. 2015, ApJ, 798, 82

Ashraf S., Ahmad Z. 1975, Ind. J. P. Appl. Math., 6, 1090

Bernstein I. B., Book D. L. 1978, ApJ, 225, 633

Bernstein I. B., Book D. L. 1980, ApJ, 240, 223

Bian F.-Y., Lou Y.-Q. 2005, Mon. Not. R. Astron. Soc., 363, 1315

Carrus P., Fox P., Haas F., Kopal Z. 1951, Astrophys. J., 113, 496

Chevalier R. A., Imamura J. 1982, ApJ, 261, 543

Coughlin E. R., Quataert E., Ro S. 2002, ApJ, 863, 158

Cowie L. L. 1977, Astrophys. J., 215, 226

Daghan D., Donmez O. 2016, Braz. J. Phys., 46, 321

Dessart L., Livne E., Waldman R. 2010, Mon. Not. Roy. Astron. Soc., 405, 2113

Deschner S. C., Ilenseer T. F., Duschl W. J. 2018, SIAM J. Appl. Math., 78, 80

Elmegreen B. G., Elmegreen D. M. 1978, ApJ, 220, 1051

Erpenbeck J. J. 1962, Phys. Fluids, 5, 604, https://doi.org/10.1063/1.1706664

Gardner C. S., Kruskal M. D. 1964, Phys. Fluids, 7, 700, https://doi.org/10.1063/1.1711271

Giuliani J. L. 1979, ApJ, 223, 280

Gorenstein P., Harden F. R. Jr., Tucker W. H. 1974, ApJ, 192, 661

Gretler W., Regenfelder R. 2005, Eur. J. Mech. B/Fluids, 24, 205

Hugoniot H. 1887, J. Ec. Polytech., 57, 3

Isenberg P. A. 1977, ApJ, 217, 597

Korobeinikov V. P. 1956, Dokl. Akad. Nauk SSSR, 109, 271

Korobeinikov V. P., Melnikova N. S., Ryazanov Y. V. 1962, The Theory of Point Explosion, U.S. Department of Commerce, Washington, D.C., Chap. 7 (English Translation)

Korobeinikov V. P. 1976, Problems in the theory of point explosion in gases, in Proceedings of the Steklov Institute of Mathematics, Volume 119, AMS, Providence, RI

Koo B. C., Mckee C. F. 1990, ApJ, 354, 513

Landau L. D., Lifshitz E. M. 1959, Fluid Mechanics, Pergamon, Oxford

Larson R. B. 1969a, Mon. Not. R. Astron. Soc., 145, 271

Larson R. B. 1969b, Mon. Not. R. Astron. Soc., 145, 405

Laumbach D. D., Probstein R. F. 1969, J. Fluid Mech. Part I, 35, 53

Laumbach D. D., Probstein R. F. 1970, Phys. Fluids, 13, 1178

Lerch I., Vasyliunas V. 1976, Astrophys. J., 210, 85

Leahy D. A., Ranasinghe S. 2016, ApJ, 817, 74

Lerche I., Vasyliunas V. M. 1976, ApJ, 210, 85

McCray R., Stein R. F., Kafatos M. 1975, ApJ, 196, 564

Mufson S. L. 1975, ApJ, 202, 372

Naidu G. N., Rao M. P. R., Yadav H. L. 1983, Astrophys. Space Sci., 89, 77

Nath G. 2010, Res. Astron. Astrophys., 10, 445

Nath G. 2011, Adv. Space Res., 37, 1463

Nath G. 2012a, Meccanica, 47, 1797

Nath G. 2012b, Ain Shams Eng. J., 3, 393

Nath G. 2013, Adv. Space Res., 52, 1304

Nath G. 2014a, J. Theor. Appl. Phys., 8, 331, https://doi.org/10.1007/s40094-014-0131-y

Nath G. 2014b, Shock Waves, 24, 415.

Nath G. 2015, Meccanica, 50, 1701

Nath G., 2016, Astrophys. Space Sci., 361, 31

Nath G. 2020, Indian J. Phys., https://doi.org/10.1007/s12648-02001684-9

Nath G., Vishwakarma J. P., Srivastava V. K., Sinha A. K. (2013) J. Theo. Appl. Phys. 7, 15, https://doi.org/10.1186/2251-7235-7-15

Nath G., Dutta M., Pathak R. P. 2019, Proc. Natl. Acad. Sci., India, Sec. A Phys. Sci., https://doi.org/10.1007/s40010-019-00625-4

Nath G., Pathak R. P., Dutta M. 2018, Acta Astronautica, 142, 152

Nath G., Sinha A. K. 2011, Phys. Res. Inter., 2011, Article ID 782172, 8, https://doi.org/10.1155/2011/782172

Nath G., Singh S. 2019, J. Astrophys. Astron., 40, 50, https://doi.org/10.1007/s12036-019-9616-z

Nath G., Vishwakarma J. P. 2016, Acta Astronatica, 123, 200

Newman W. I. 1979, Astrop. Space Sci., 47, 99

Newman, W. I. (1980), Ap. J., 236, 880-888

Ostriker J. P. 1988, Rev. Mod. Phys., 60, 1

Penston, M. V. 1969a, M.N.R.A.S., 144, 425–448.

Penston, M. V.. 1969b, M.N.R.A.S., 145, 457–485.

Purohit S.C. 1974, J. Phys. Soc. Jpn., 36, 288

Rankine W. J. M. 1870, Roy. Soc. Lond. Philos. Ser. I, 160, 277

Ro S., Matzner C.D. 2017, Astrophys. J., 841, 1

Rogers M. H. 1957, Astrophys. J., 125, 478

Rosenau P., Frankenthal S. 1976, Phys. Fluids, 19, 1889

Sakurai A. 1953, J. Phys. Soc. Jpn., 8, 662

Sakurai A. 1954, J. Phys. Soc. Jpn., 9, 256
Sakurai A. 1956, J. Fluid. Mech., 1, 436
Sachdev P. L., Ashraf S. 1971, J. Appl. Math. Phys., 22, 1095
Sedov L. I. 1946, J. Appl. Maths. Mech., 10, 241
Sedov L. I. 1959, Similarity and Dimensional Methods in Mechanics, Academic Press, New York
Singh J. B., Vishwakarma P. R. 1983, Astrophys. Space Sci., 95, 99
Shao-Chi L. 1954, J. Appl. Phys., 25, 54
Shu F. H. 1977, ApJ, 214, 488
Solinger A., Rappaport S., Buff J. 1975, ApJ, 201, 381
Taylor G. 1946, Proc. Roy. Soc. A, 186, 273
Taylor G. 1950a, Proc. Roy. Soc. Lond. A, 201, 159
Taylor G.I. (1950b), Proc. R. Soc. Lond. A. 2011065 175-186.
Truelove J. K, Klein R. I., McKee C. F., Holliman J. H., Howell L. H., Jeffrey A. G., Wood D. T. 1998, Astrophys. J., 495, 821
Vink J. 2012, Astron. Astrophys. Rev., 20, 49, https://doi.org/10.1007/s00159-011-0049-1
Vishwakarma J.P., Yadav A. K. 2003, Eur. Phys. J. B., 34, 247
Vishwakarma J. P., Nath G. 2006, Phys. Scr., 74, 493
Vishwakarma J. P., Nath G. 2007, Meccanica, 42, 331
Vishwakarma J. P., Nath G. 2009, Meccanica, 44, 239
Vishwakarma J. P., Nath G. 2012, Commun. Nonlinear Sci. Numer. Simul., 17, 2382
Watanable M., Takayama K. 1991 Shock Waves, 1, 149
Waxman E., Shvarts D. 1993, PhFIA, 5, 1035
Winkler P. F., Clark G. W. 1974, ApJ (Lett.), 191, L67
Woltjer L. 1972, Ann. Rev. Astron. Astrophys., 10, 129
Yan Z. Y., Zhang H. Q. 2001, J. Phys. A. Math. Gen., 34, 1785, https://doi.org/10.1088/0305-4470/34/8/320
Zhuravskaya T. A., Levin V. A. 1996, J. Appl. Math. Mech., 60, 745