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Bursts of gravitational waves due to crustquake from pulsars

BISWANATH LAYEK* and PRADEEPKUMAR YADAV

Birla Institute of Technology and Science, Pilani Campus, Pilani, Jhunjhunu 333 031, India.
E-mail: layek@pilani.bits-pilani.ac.in

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Abstract. We revisit here a possibility of generation of gravitational wave (GW) bursts due to a very quick change in the quadrupole moment (QM) of a deformed spheroidal pulsar as a result of crustquake. Since it was originally proposed as a possible explanation for sudden spin-up (glitch) of pulsars, the occurrence of crustquake and its various consequences have been studied and discussed quite often in the literature. Encouraged by recent development in gravitational wave (GW) astronomy, we re-investigate the role of crustquake in the emission of GWs. Assuming exponential decay of excitation caused by crustquake, we have performed a Fourier analysis to estimate the GW strain amplitude $h(t)$, characteristic signal amplitude $h_c(f)$ and signal-to-noise ratio (SNR) of the burst for the Crab pulsar. For exotic quark stars, multifold enhancement of these quantities are expected, which might make quark star a potential source of gravitational waves. The absence of such bursts may put several constraints on pulsars and such hypothetical stars.

Keywords. Neutron star—pulsar—crustquake—gravitational wave.

1. Introduction

The remarkable first-ever direct detection of GWs in 2015 by LIGO (Abbott *et al.* 2016) opened a new era in gravitational astronomy. Observations supplemented with numerical simulations identified a black hole merger as the source for ripples' in the space-time. Since then there have been quite a few significant detections of GWs. The peak strain amplitude (h) for all these detections have been in the range 10^{-21}–10^{-22}. In view of these detections, we should be hopeful that the gravitational waves produced by compact isolated astrophysical objects like isolated pulsars can be measurable in the near future by more sensitive upcoming ground-based advanced detectors, namely, aLIGO, VIRGO, the third generation Einstein Telescope (ET), etc. Prior to the GWs detections as mentioned above, there have been a few attempts for GWs searches from isolated pulsars as well. The attempt for GWs search associated with the timing glitch in the Vela pulsar in August 2006 (Abadie *et al.* 2011) was one among those which are worth mentioning. The motivation of the above searches were based on suggestions made by several authors (see Abadie *et al.* 2011 and the references therein) through their works in this area. As per those suggestions, several sources namely the flaring activity, the formation of hypermassive NS following coalescence of binary neutron stars, etc. are capable of exciting quasinormal modes of a pulsar and hence may emit GWs. The timing glitch can be one of these sources, which has the potential to excite quasinormal modes in the parent pulsar. Although the searches for GWs during August 2006 Vela pulsar (Abadie *et al.* 2011) timing glitch produced no detectable GWs, with the improving sensitivity of advanced detectors, continuous attempts in this direction may produce more conclusive results in near future.

In the literature, there have been a few other theoretical works that advocate gravitational wave astronomy in the context of isolated neutron stars. There have been discussions on the emission of GWs from deformed neutron stars (Zimmermann & Szedenits 1979), crustal mountains in radio pulsars or in low mass X-ray binaries (Haskell *et al.* 2015), continuous emission of GWs from a triaxially symmetric rotating neutron star due to permanent ellipticity

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(Jones 2002), etc. Further details on the above-mentioned sources of GWs from isolated pulsars can be found in a detailed review by Lasky (2015). In the context of bursts of GWs from an isolated NS, there has also been an interesting suggestion by Bagchi *et al.* (2015), where the authors have proposed a unified model for glitches, antiglitches, and generation of GW from isolated pulsars due to phase transitions inside the core of a pulsar.

In view of these theoretical proposals, we explore here another possible source of GWs from isolated pulsars, which may arise due to a very fast-changing elastic deformation (ϵ) and hence the change in quadrupole moment (Q) of a pulsar as a result of crustquake. Nearby Crab pulsar exhibiting small size glitches of order 10^{-8} that can be explained successfully using the crustquake model will be the most likely candidate to test our proposal. In this context, we should mention the work by Keer and Jones (2015) on crustquake initiated excitation of various oscillation modes in a pulsar and hence the emission of possible GWs from such oscillations. In their work, the authors had assumed that excitations decay after executing several oscillations and they made an order of magnitude estimate for the strain amplitude of GWs arising due to the excitation of f-mode. According to their picture, the estimated strain amplitude depends on various factors such as amplitude of oscillations, rotational frequency of the star, etc. in contrast to our picture where h turns out to be independent of these quantities.

In this work, we assume that the decay of crustquake initiated excitation behaves like a critically damped oscillator and estimate the characteristics of GW bursts. We will take pulsar of spheroidal (oblate) shape, the ellipticity/oblateness (ϵ) of which can be defined through, $I_{zz} \neq I_{xx} = I_{yy}$ and $\epsilon = \frac{I_{zz} - I_{xx}}{I_0}$. Here, $I_{zz} = \frac{2}{5}Mc^2, I_{xx} = \frac{1}{5}M(a^2 + c^2)$ are the MI of the star about the (symmetric) z-axis and x-axis, respectively ($c < a$). I_0 is the MI of the spherical star. The quadrupole moment of the spheroidal star can be taken as $Q_{zz} = \frac{2}{5}(c^2 - a^2) \equiv Q$ which is related to the ellipticity through, $Q = -2I_0\epsilon$. The negative value of Q arises solely due to the oblate shape of the star. Obviously, a spheroidal star does not radiate continuous gravitational waves due to its spherical symmetry. However, the crustquake can initiate excitations to the star and the star de-excites itself to achieve a new equilibrium (more oblate to less oblate) through several oscillations (like an underdamped oscillator) as discussed by Keer and Jones (2015) or it can decay like a critical/overcritical damped oscillator (as we propose in this paper). This causes a change in the pulsar's oblateness and hence it should emit gravitational radiation. Note that the detailed mechanism of excitation or de-excitations are still unknown, neither the consequences of these are established experimentally. In this work, we assume the evolution of ellipticity behaves like $\epsilon(t) \propto \exp(-\frac{t}{\tau})$. where τ characterizes the relaxation time scale during which major changes of $\Delta\epsilon$ or equivalently, ΔQ of QM occurs following crustquake. Such an assumption can be justified from the fact that the oscillations caused by excitation is not expected to have a well-defined single (angular) frequency (ω). Rather, it is more natural to expect that it consists of an infinitely large number of normal mode frequencies. The study of de-excitation can then be achieved through Fourier analysis by modeling the evolution of ellipticity like critical/overcritical damped oscillators. Also, as the strain amplitude $h(t)$ depends on $\ddot{Q}^2(t)$, generation of GWs with a significant strain amplitude is expected only if Q changes in a very short duration (i.e., smaller τ) which can be achieved only if the star behaves like critical/overcritical damped oscillators. It will be interesting to study the properties of matter in detail to check if at all a star behaves like such a damped oscillator.

The motivation for theoretical studies on possible emission of GWs from isolated pulsars can be manifold. Firstly, the event rate of (number of events which can produce significant GWs in a galaxy per year) GWs emission from pulsars is expected to be quite high in comparison to the event rate from various other sources of gravitational radiation, namely mergers of neutron stars (NS) and/or black holes (BH). For example, it has been mentioned in Riles (2013) that event rates lie in the range 2×10^{-4}–0.2 per year for initial LIGO detection of a NS–NS coalescence, and 7×10^{-5}–0.1 per year for a NS–BH coalescence, etc. In contrast, there is a huge catalogue of pulsars and more than 450 glitch events (crustquake can account for small size 10^{-8}–10^{-12} glitches) have been recorded[1] and analyzed (Ezpinoza *et al.* 2011) quite extensively. Thus crustquake can be quite a frequent phenomenon when one considers a very large number of pulsars in our galaxy. The other purpose of the GWs study from pulsar might be to understand a few features of the pulsar itself. For example, in this proposal, the strain amplitude of GWs crucially depends on the relaxation time scale (τ) towards the new equilibrium value of oblateness (hence, QM). Thus gravitational wave studies from pulsars can shed light

[1] http://www.jb.man.ac.uk/pulsar/glitches/gTable.html.

on crustquake, possible mechanism of strain relaxation, etc. To the best of our knowledge, so far no serious attempt has been made to determine strain relaxation time scale. This is because of the fact that in the context of crustquake model for glitches, τ is not relevant in determining glitch size and other features of glitches. However, as we show below, τ is crucial to have a significant value of h in our model. Similarly, if a few small size glitches are caused by crustquake, then the glitches will be followed by little bursts of GWs which can be a falsifiable prediction of a crustquake.

The paper is organized in the following manner. In Section 2, we briefly review the basic relevant features of crustquake model as pictured by Baym and Pines (1971). We shall also discuss here the maximum possible values of ellipticity parameter (ϵ) of a neutron star crust which can sustain the crustal stress. The methodology is presented in Section 3. Here, the expressions of GW strain amplitude $h(t)$, characteristic time scale τ, characteristic signal amplitude $h_c(f)$ and the signal-to-noise ratio (SNR) are presented. The results are presented in Section 4, followed by the concluding remarks in Section 5.

2. Crustquake: The basic relevant features

Crustquake was originally proposed by Ruderman (1968) as a possible explanation for the sudden spin-up (glitches) of otherwise highly periodic pulsar PSR 0833-45 (Vela). Currently, the superfluid vortex model (Anderson & Itoh 1975) is the leading model to explain glitches. However, in the context of glitches or otherwise, crustquake model has been revisited repeatedly. There have been discussions in the literature suggesting the involvement of crustquake in NS physics, such as an explanation for the giant magnetic flare activities observed in magnetars (Thompson & Duncan 1995; Lander *et al.* 2015), a trigger mechanism for angular momentum transfer by vortices (Eichler & Shaisultanov 2010; Melatos *et al.* 2008; Warszawski & Melatos 2008) or to explain the change in spin down rate that continues to exist after a glitch event (Alpar *et al.* 1994). In view of these discussions, crustquake seems to be an inevitable event for a large number of NS. This work will assume the occurrence of crustquake in a NS and study one of its many possible consequences, namely, the generation of short duration (*bursts*) GWs due to crustquake.

The crustquake in a neutron star is caused due to the existence of a solid elastic (Ruderman 1969; Smoluchowski & Welch 1970) deformed crust of thickness about 10^5 cm. The deformation parameter of the crust can be characterized by its ellipticity, $\epsilon = \frac{I_{zz} - I_{xx}}{I_0}$ as defined earlier in Section 1. Here, we briefly discuss the basic features of crustquake model (Baym & Pines 1971). At an early stage of formation, the crust solidified with initial oblateness ϵ_o (unstrained value or reference value) at a much higher rotational frequency. As the star slows down, the ellipticity $\epsilon(t)$ decreases, leading to the development of strain in the crust due to the inherent crustal rigidity. Finally, once the breaking stress is reached, the crust cracks and releases its (partial) energy. The total energy of the pulsar can be written as (Baym & Pines 1971)

$$E = E_0 + \frac{L^2}{2I} + A\epsilon^2 + B(\epsilon - \epsilon_0)^2, \tag{1}$$

where A and B are two coefficients, the values of which are available (Baym & Pines 1971). The equilibrium value of ϵ can be obtained by minimizing E while keeping angular momentum ($L = I\Omega$) fixed. The value thus obtained is given by

$$\epsilon = \frac{\Omega^2}{4(A+B)} \frac{\delta I}{\delta \epsilon} + \frac{B}{A+B}\epsilon_0 \simeq \frac{B}{A+B}\epsilon_0. \tag{2}$$

The crustquake causes the shift in the reference value of ϵ_0 by $\Delta\epsilon_0$, which in turns changes the equilibrium value ϵ by $\Delta\epsilon$, where

$$\Delta\epsilon = \frac{B}{A+B}\Delta\epsilon_0. \tag{3}$$

For normal neutron star of 10 km radius and 1 km crust thickness, we have $A \simeq 6 \times 10^{52}$ erg and $B \simeq 6 \times 10^{47}$ erg (Baym & Pines 1971) which in turn provides the value of ϵ as

$$\epsilon = 10^{-5}\epsilon_0. \tag{4}$$

The upper limit of ellipticity can be constrained by noting that the crust has a critical strain, say, $\Delta_{\rm cr}$ such that

$$|\epsilon - \epsilon_0| = \frac{A}{B}\epsilon < \Delta_{\rm cr}. \tag{5}$$

The theoretical value of $\Delta_{\rm cr}$ (10^{-2}–10^{-4}) had been estimated earlier by Jones (2002). However, in a recent work (Horowitz & Kadau 2009) on crustal breaking strain of NS, the authors have done detailed molecular dynamics simulations by modeling the crust by a single pure crystal and obtained the value, $\Delta_{\rm cr} = 0.1$. As per their work, the value of $\Delta_{\rm cr}$ remains around 0.1 even in the presence of impurities, defects, etc. Note that this value is at least an order of magnitude higher than the maximum value quoted in Jones

(2002). Here, for the estimate of maximum GWs strain amplitude, we take $\Delta_{\rm cr} = 0.1$ and we obtain the upper limit of ϵ as

$$\epsilon < \frac{B}{A}\Delta_{\rm cr} = 10^{-6}. \tag{6}$$

At the onset of crustquake, the above value will be taken as an initial oblateness, $\epsilon_i = 10^{-6}$ for the calculation of GW strain amplitudes. We will assume the change of ellipticity $\Delta\epsilon = \eta\epsilon$ (where $0<\eta<1$ is a numerical factor characterizing the fraction of strain released due to crustquake), to estimate strain relaxation time τ. The above estimate has been provided for normal NS. It has been proposed that more exotic quark star (see Keer & Jones (2015) and references therein) may exist having a very large solid core which can sustain a large ellipticity. For such a star, $A \simeq 8\times 10^{52}$ erg and $B \simeq 8\times 10^{50}$ erg and hence ϵ_i can be as large as 10^{-3} (for the same value of $\Delta_{\rm cr} = 0.1$ as above) as obtained from Equation (6). Thus, the quark star can also be a potential source of gravitational wave burst as a result of crustquake.

3. Gravitational wave burst caused by crustquake: Methodology

As per the discussion in Section 1, the strain amplitude due to change of ellipticity following starquake will be determined by assuming that the star behaves like a critically damped oscillator (from now onwards, we will do the analysis for critically damped oscillator; however, the methodology will be equally applicable for overcritical damped oscillator). We will take the evolution of ellipticity as

$$\epsilon(t) = \epsilon_i {\rm e}^{-t/\tau} = \epsilon_i {\rm e}^{-at}; \quad (1/\tau \equiv a). \tag{7}$$

Here, ϵ_i is the initial value of ellipticity of the neutron star at the onset of crustquake. As we are interested to study the evolution of $h(t)$ within characteristic time τ, we neglect the ${\rm e}^{-at}t$ term in the above expression. Expanding $\epsilon(t)$ in a Fourier series,

$$\epsilon(t) = \frac{1}{\sqrt{\pi}}\int_0^\infty {\rm d}\omega\epsilon(\omega){\rm e}^{-i\omega t}. \tag{8}$$

Here, $\epsilon(\omega)$ is the corresponding (complex) amplitude in frequency domain and is given by

$$\epsilon(\omega) = \frac{1}{\sqrt{\pi}}\int_0^\infty {\rm d}t\epsilon(t){\rm e}^{i\omega t} = \frac{\epsilon_i}{\sqrt{\pi}}\left(\frac{a+i\omega}{a^2+\omega^2}\right). \tag{9}$$

Hence, the quadrupole moment can be written as

$$Q(t) = -2I_0\epsilon(t) = -\frac{2I_0\epsilon_i}{\pi}\int_0^\infty \frac{a+i\omega}{a^2+\omega^2}{\rm e}^{-i\omega t}{\rm d}\omega. \tag{10}$$

Expansion of $\epsilon(t)$ and/or $Q(t)$ in a Fourier series in frequency domain allows us to apply linear perturbation theory for the estimate of strain amplitude from a source oscillating with infinitely large number of normal modes. The resultant strain amplitudes $h_{ij}(t)$ can be obtained by summing up the contribution from each frequency. The strain amplitude (i.e., one of the components say, $h_{zz}(t) = h(t)$) in a particular frequency interval) can then be written as (Riles 2013)

$$h(t) = \frac{2G}{c^4 d}\ddot{Q} = \frac{4GI_0}{\pi c^4 d}\int_{\omega_{\min}}^{\omega_{\max}} {\rm d}\omega\epsilon(\omega)\omega^2{\rm e}^{-i\omega t}, \tag{11}$$

where d is the distance of the pulsar from the Earth and we have taken a frequency interval between $\omega_{\min}$ and $\omega_{\max}$ which are the minimum and maximum angular frequencies respectively. The upper cut-off frequency is a requirement to validate the slow motion approximation. In such an approximation, the wavelength λ of gravitational radiation must be much larger than the size of the source. Thus a frequency of about 1 kHz implies the wavelength of radiation as about $\lambda = 300$ km, which is quite large compared to the size of the source (~ 10 km). We will set 1 kHz frequency as an upper limit and slow motion approximation will be assumed up to this frequency. Error arising due to small deviation from this approximation for a one kHz frequency will be neglected. There is no such requirement on minimum limit of frequency, however, we will estimate h in the frequency range 100 Hz–1 kHz in accordance with the sensitivity of existing interferometers. Thus the strain amplitude (the real part of Equation (11)) can be written as

$$h(t) = \frac{4GI_0\epsilon_i}{\pi c^4 d}\int_{\omega_{\min}}^{\omega_{\max}}\left[\frac{a\cos(\omega t)+\omega\sin(\omega t)}{a^2+\omega^2}\right]\omega^2{\rm d}\omega. \tag{12}$$

Expressing in terms of dimensionless quantity $\omega' = \frac{\omega}{a} \equiv \omega\tau$, the above expression can be re-written as

$$\begin{aligned} h(t) =& \frac{4GI_0\epsilon_i}{\pi c^4 d\tau^2}\int_{\omega'_{\min}}^{\omega'_{\max}}\left[\frac{\cos\left(\frac{\omega' t}{\tau}\right)+\omega'\sin\left(\frac{\omega' t}{\tau}\right)}{1+\omega'^2}\right]\omega'^2{\rm d}\omega' \\ =& 3.5\times 10^{-24}\left(\frac{1\ {\rm kpc}}{d}\right)\left(\frac{\epsilon_i}{10^{-6}}\right)\left(\frac{10^{-4}\ {\rm s}}{\tau}\right)^2 K(t). \end{aligned} \tag{13}$$

The numerical prefactor is calculated by taking MI of the star I_0 as 10^{45} gm-cm^2. The function $K(t)$ (for a fixed set of values of $\omega'_{\rm min}$ and $\omega'_{\rm max}$) is given by

$$K(t) = \int_{\omega'_{\rm min}}^{\omega'_{\rm max}} \left[\frac{\cos\left(\frac{\omega' t}{\tau}\right) + \omega' \sin\left(\frac{\omega' t}{\tau}\right)}{1+\omega'^2} \right] \omega'^2 {\rm d}\omega'. \quad (14)$$

The frequency-dependent information in the burst can also be inferred from the frequency density distribution, $h_{\omega'}(t) = \frac{{\rm d}h(t)}{{\rm d}\omega'}$ and is given by (using Equation (13))

$$h_{\omega'}(t) = N \left[\frac{\cos\left(\frac{\omega' t}{\tau}\right) + \omega' \sin\left(\frac{\omega' t}{\tau}\right)}{1+\omega'^2} \right] \omega'^2. \quad (15)$$

Note that $h_{\omega'}(t)$ is a dimensionless quantity, to be multiplied by τ to get the proper frequency distribution $h_\omega(t)$. The prefactor (N) in the above equation is of order 10^{-24} (to be discussed in the next section). Now, as the strain amplitude depends on the relaxation time τ, we will provide a rough estimate for τ as follows. Here we should mention that, so far no serious attempt has been made to determine the precise value of τ (which is a typical glitch time). It is understood, since in the context of crustquake model for glitches, this is not necessary as the size of glitches and other relevant features of glitches are not sensitive to the value of τ. If crustquake causes glitches, then uncertainty of such time scale may be resolved through pulsar timing by narrowing down to the characteristic time-scale for spin up events/glitches. At this stage, we may take it as a parameter which can be fixed by observation in future by studying the impact of τ on various consequences. However, we will provide a rough estimate of τ by assuming that the strain energy goes into gravitational radiation and the emission is dominated within the time interval τ. Thus the estimate provided below may set a lower limit on τ. With this assumption, we can now write the rate of energy loss due to GW radiation as

$$\frac{{\rm d}E}{{\rm d}t} = -\frac{3G}{10c^5} \dddot{Q}2 = -\frac{6G}{5c^5} I_0^2 \epsilon_i^2 a^6 {\rm e}^{-2at}. \quad (16)$$

Here we have used $Q_{zz} = -2Q_{xx} = -2Q_{yy}$ relevant for spheroidal pulsar and quadrupole moment tensor Q_{ij} which is traceless, i.e., $Q_{zz} + Q_{xx} + Q_{yy} = 0$. The net energy loss ΔE in a typical time τ now can be obtained from Equation (16) as

$$\Delta E = -\frac{6G}{5c^5} I_0^2 \epsilon_i^2 a^6 \int_0^{1/a} {\rm d}t {\rm e}^{-2at} \simeq \frac{3G}{5c^5} \frac{I_0^2 \epsilon_i^2}{\tau^5}. \quad (17)$$

We can estimate the value of the relaxation time scale τ by (approximately) equating the above energy loss with the strain energy released by the star using Equation (1) as

$$\Delta E = B\Delta\epsilon \simeq \frac{3G}{5c^5} \frac{I_0^2 \epsilon_i^2}{\tau^5}, \quad (18)$$

i.e.,

$$\tau = 5 \times 10^{-5} \left(\frac{\epsilon_i}{10^{-6}} \right)^{2/5} \left(\frac{10^{39} {\rm erg}}{B\Delta\epsilon} \right)^{1/5} {\rm s}. \quad (19)$$

We will use Equations (13), (14) and (19) to determine the GW strain amplitude h by taking the appropriate values of ϵ_i and $\Delta\epsilon$. Here, we should mention that while calculating rate of energy loss using Equation (16), we have essentially taken contribution of all frequencies. However, for consistency, one should apply Equation (10) and carry out the integration over frequency up to a maximum limit (say, 1 kHz) to validate slow motion approximation. However, the rate of energy loss being $\frac{1}{\tau^5}$ dependent, the order of magnitude of τ will not be affected significantly due to the above factor.

4. Results

The strain amplitude in Equation (13) of gravitational waves in a given frequency interval in our scenario now depends on ϵ_i and on the time scale τ of strain relaxation. Following the arguments provided in Section 2, for a normal NS the maximum possible value of ϵ_i is in the order of 10^{-6} and the value of $\Delta\epsilon$ (required to determine τ) will be determined by the prefactor η $(0<\eta<1)$. The value of pre-factor η and hence, $\Delta\epsilon$ can be constrained through the crustquake model for glitches. For small size glitches in Crab pulsar, $\Delta\epsilon = 10^{-8}$ (Ezpinoza *et al.* 2011), which corresponds to $\eta = 0.01$ (i.e., 1% release of strain). For consistency, we should mention that for a Crab pulsar, the above value of $\Delta\epsilon$ corresponds to only few years waiting time for the next glitch to occur, which is much less than the spin-down time scale ($\approx 10^3$ years) of Crab pulsar and this fact is fairly consistent with the observations. For a quark star, the value of ϵ_i can be of the order 10^{-3}, an enhancement of order 10^3 times compared to a normal neutron star.

We will take the value of τ as obtained from Equation (19). Assuming $B = 6 \times 10^{47}$ erg, $\epsilon_i = 10^{-6}$ and $\Delta\epsilon = 10^{-8}$, we obtain the value of $\tau \simeq 10^{-4}$ s. For exotic quark stars, τ being proportional to $(\frac{\epsilon_i^2}{B\Delta\epsilon})^{1/5}$ is of the same order as for a normal NS. The strain amplitude $h(\tau)$ for Crab as obtained from Equations (13)–(14) is provided in Table 1. The time evolution of $h(t)$ is shown in Figure 1(a). Here, $h(t)$ is the frequency integrated (over the frequency range 100 Hz–1 kHz) strain amplitude. It is obvious that shorter the time scale τ for spin up events, larger the strain amplitude one expects. As we see from the table, the value of strain amplitude for Crab is expected to be of the order 10^{-25}. For a same distance quark star h ($=10^{-22}$) is enhanced by order 10^3 compared to Crab. The frequency density distribution of the strain amplitude $\frac{dh}{d\omega'}/h(\tau)$ (dividing by $h(\tau)$ is just for convenience) evaluated at $t=\tau$ as obtained from Equation (15) is shown in Figure 1(b). The plot shows that the proper frequency density $\frac{dh}{d\omega}/h(\tau)$ (using $\frac{dh}{d\omega}=\tau\frac{dh}{d\omega'}$) is of order 10^{-3}. Thus, the strain amplitude $h(t,\omega)$ is almost frequency-independent in the frequency range 100 Hz–1 kHz.

Now whether such bursts of GWs have the potential to be detectable against signal noises depend on various factors such as sensitivity of interferometer, proper 'template' to analyze burst, etc. There are several ways to characterize the detectability of signal strength from a source against the background noise. One such commonly used method (Sathyaprakash & Schutz 2009; Moore *et al.* 2014) is the comparison of square root of power spectral density (PSD) for the source, $\sqrt{S_h(f)}=\frac{h_c(f)}{\sqrt{f}}$ and for the noise, $\sqrt{S_n(f)}=\frac{h_n(f)}{\sqrt{f}}$. Here, $h_c(f)$ is the *characteristic signal amplitude* in the frequency domain and $h_n(f)$ is the *effective noise* that characterizes the sensitivity of a detector. Note that $S_h(f)$ and $S_n(f)$ both have a dimension of $\mathrm{Hz}^{-1/2}$ (see Sathyaprakash & Schutz 2009; Moore *et al.* 2014 for details), whereas $h_c(f)$ and $h_n(f)$ are dimensionless. The characteristic signal amplitude $h_c(f)$ is defined as $h_c(f)=f|\tilde{h}_c(f)|$, where

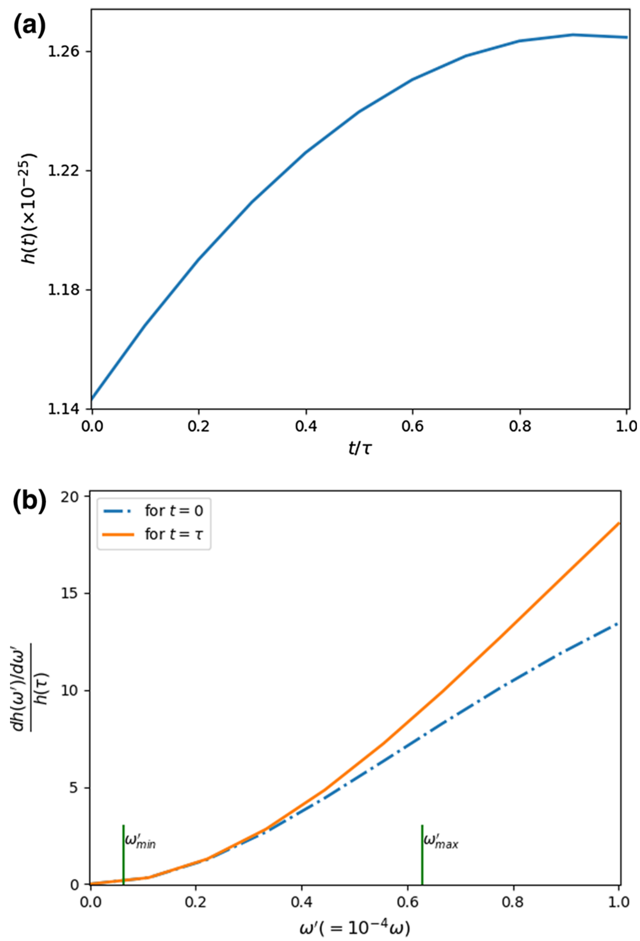

Figure 1. (**a**) Plot shows the time evolution of strain amplitude $h(t)$ within characteristic time, $\tau=10^{-4}$ s. Here, $h(t)$ is the frequency integrated (over the frequency range 100 Hz–1 kHz) amplitude. (**b**) Plot shows the frequency density distribution of the strain amplitude $\frac{dh(\omega')}{d\omega'}$ as a function of ω' ($=\omega\tau$). $\frac{dh(\omega')}{d\omega'}$ has been calculated at time $t=\tau$. Note, the above distribution is dimensionless and it has to be multiplied by τ to get the proper frequency density distribution $\frac{dh(\omega)}{d\omega}$.

Table 1. Typical order of magnitude for strain amplitude, h and characteristic signal amplitude, $h_c(f)$ for the bursts. The values of h are provided for Crab and a near-by hypothetical quark star at $t=\tau$ for $\tau=10^{-4}$ s. The values of ϵ_i are chosen as per the argument provided in the text. Characteristic signal amplitude $h_c(f)$ is quoted for a frequency of 1 kHz.

Star	ϵ_i	$h(t=\tau)$	$h_c(f=1\ \mathrm{kHz})$
Crab ($d=2$ kpc)	10^{-6}	10^{-25}	10^{-26}
Quark star ($d=2$ kpc)	10^{-3}	10^{-22}	10^{-23}

$\tilde{h}_c(f)$ is the Fourier transform of the signal amplitude $h(t)$. In our case, $\tilde{h}_c(f)$ can be estimated (Sathyaprakash & Schutz 2009) as

$$\tilde{h}_c(f)\propto\int_0^{\infty}\mathrm{d}t\,h(t)\mathrm{e}^{i2\pi ft}\simeq h\left(\frac{a+i2\pi f}{a^2+4\pi^2f^2}\right). \tag{20}$$

In the above equation, we have used the fact that the strain amplitude is approximately constant during the time interval $\Delta t=\tau$ ($\frac{\Delta h}{h}\simeq0.1$ as shown in Figure 1(a)). The characteristic signal amplitude is then given by

$$h_c(f) = |f\tilde{h}_c(f)| = h\frac{f}{\sqrt{a^2 + 4\pi^2 f^2}}. \quad (21)$$

For $\tau = 10^{-4}$ s and for a frequency range 100 Hz–1 kHz, the characteristic signal strain lies in the range $h_c(f) = (0.01\text{–}0.1)$ h. Hence, the signal-to-noise ratio, $\mathrm{SNR} = \frac{h_c(f)}{h_n(f)}$ can be estimated using the data provided for the noise amplitude $h_n(f)$. For $h_n(f)$, we utilize the data available in the literature (Sathyaprakash & Schutz 2009; Hild & Abernathy 2011) for the Einstein Telescope[2] and the values lie between 10^{-23}–10^{-22} for the frequency range 100 Hz–1 kHz. For these values of $h_n(f)$, SNR (at $f = 1$ kHz) in our case turns out be of order 10^{-4} for Crab and 0.1 for a same distance quark star.

We conclude this section by comparing strain amplitudes as obtained from isolated pulsars (through different mechanism) which were suggested by several authors earlier. For example, as we have already pointed out in Section 1 that Keer and Jones (2015) had estimated the value of strain amplitude resulting from neutron star oscillations initiated due to starquake (through somewhat different model). Their suggested values for a kHz frequency are larger by an order of magnitude, however, there was an assumption that energy transfer to oscillations is instantaneous (i.e., causality is not taken into consideration).

Similarly, GW emission from a triaxial pulsar has been discussed in the literatures (Jones 2002). Although the triaxiality is not a criterion in our scenario, however, even a triaxial star may undergo crustquake. In this case, the continuous emission of GWs will be followed by sudden emission of GW burst. The strain amplitudes from a triaxial Crab pulsar turns out to be of order $(h)_{\mathrm{tr}} \sim 10^{-26}$ for the values of ellipticity, $\epsilon_{\mathrm{tr}} = 10^{-6}$. The above quoted values of $(h)_{\mathrm{tr}}$ is provided for Crab (time period $T =$ 33 ms). As $(h)_{\mathrm{tr}}$ depends on time period of rotation, the magnitude can be even smaller for slowly rotating NS. Our estimate of strain amplitude for the burst of GW as a result of crustquake (refer Table 1), turns out to be of an order of magnitude larger compared to the corresponding value for triaxial stars. We should emphasize that GW bursts in our case is expected even for a spheroidal star for which continuous emission will be completely absent.

Also, there is a proposal by Bagchi *et al.* (2015) that several phase transitions inside the core of a pulsar can generate quadrupole moment which can result in emission of GW bursts. Our estimated strain amplitude turns out to be smaller than the values as obtained in Bagchi *et al.* (2015). This is mainly due to smaller value of characteristic time scale τ (about one microsecond, typical value for a QCD phase transition time). We have also presented our estimate of h for a typical quark star (assumed to be at a same distance as Crab pulsar). The strain amplitude for such exotic star (about 10^{-22}) is increased by about 10^3 order of magnitude compared to a normal star. This happens due to high ellipticity ϵ_i at the onset of crustquake.

[2]refer http://www.et-gw.eu/index.php/etsensitivities#references.

5. Conclusion

We investigated a possibility of generation of gravitational wave burst caused by crustquake due to a very fast change of QM of the star. Modeling the decay of crustquake initiated excitation behaves like a critically damped system, we estimated the possible values of strain amplitudes of the burst. As we have mentioned, such modeling (i.e., exponential decay) can be justified as the excitation expected to have a very large number of frequencies instead of a well-defined single frequency. With this model, GW strain amplitude depends on ellipticity ϵ_i at the onset of crustquake and relaxation time scale τ. We fixed the value of ϵ_i ($=10^{-6}$) as suggested in Horowitz and Kadau (2009) based on detailed molecular dynamics simulations on NS crust. For τ, we have provided an order of magnitude estimate ($\simeq 10^{-4}$) by assuming that the strain energy goes into emission of GWs. For this set of values, $h(\tau)$ (integrated over frequency range 100 Hz–1 kHz) turns out to be of order 10^{-25} for Crab and the value is almost constant within time τ. We also estimated the characteristic signal amplitude $h_c(f)$ and the signal-to-noise ratio (SNR) for the burst. We noticed that there is a multifold increase in the values of these quantities for an exotic quark star. We again point out that the value of $h(t)$ depends on τ, the precise value of which is still unknown. It is therefore important to fix this characteristic time scale (which is a typical spin-up time of glitch) through observation to reduce the uncertainty in estimating strain amplitude in this model. So far, the best resolved time for spin-up has been reported (Dodson *et al.* 2002) to be $\approx$ 40 s for Vela pulsar, which is far away from the requirement. We are hopeful that next-generation telescopes such as MeerKAT, Giant Magellan, Square Kilometer Array, etc. will enhance the chances of direct glitch (sort of *as it happens*) detections and constrain the characteristic spin-up time for pulsar glitches.

As far as detectability of gravitational waves is concerned, it is more challenging to detect short duration bursts than continuous GWs. We hope such challenges will be overcome through proper analysis suitable for burst study and eventually by detecting with more sensitive advanced detectors. If so, the study of gravitational wave astronomy of pulsars can shed light on possible occurrence of the crustquake event itself. Finally, although we have restricted our study to crustquake (for the purpose of fixing various parameters from observation or otherwise), this analysis has the potential to be used for the study of any excitations which might be caused by other mechanisms in the context of pulsars.

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