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A computational study of the properties of the quasi-perpendicular fast forward shock event during solar maximum

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Abstract. MHD shock waves and discontinuities are often observed in the solar wind. We studied a strong shock wave event on 7 June 2014, during a solar maximum. The properties of forward shock are investigated and all physical parameters of interplanetary shock are analyzed. The upstream parameters, such as Alfven velocity and sound speed are calculated, and the angle (θ) between the upstream magnetic field direction and the shock normal is estimated. To determine the propagation speed of the shock, a full numerical solution of shock adiabatic equation is carried out. The thermal pressure for electrons and ions are calculated and also the pressure ratio and the entropy change across the shock are estimated. The results show that the shock strength is about 3.88 with $\theta \approx 72°$ (a quasi-perpendicular case), and shock propagation velocity is about 678 km/s. The increase in the specific entropy is also evaluated.

Keywords. Shock waves—entropy—solar wind.

1. Introduction

Shock processes occur naturally in various astrophysical and space situations, the most common being supernova explosions and solar winds in the interplanetary medium. From a theoretical point of view, the magnetohydrodynamics model applies to plasma magnetic flows and the propagation of shock waves in such a medium (De Hoffmann & Teller 1950; Sen 1956) is of great interest to many researchers in various fields such as astrophysics, space plasma physics, and geophysics. Shock waves in the solar wind are often referred to as interplanetary shocks. Interplanetary shocks and discontinuities (Colburn & Sonett 1966) have been observed for several decades. Most of the interplanetary shocks have been seen in the solar wind near the Earth, especially during solar activity (Watari *et al.* 2001; Echer *et al.* 2003; Mullan & Smith 2006; Kilpua *et al.* 2015). They are important as accelerators of energetic particles, and as generators of plasma waves. Usually, interplanetary shocks are observed as abrupt changes in all physical parameters of solar wind, such as flow speed, density, pressure, magnetic field and temperature (Burlaga 1995; Kivelson & Russell 1995), but entropy change from the upstream to downstream side of the shock ramp is most important. Also, interplanetary structures can be effective, on the Earth's magnetic field thereby causing magnetic storms (Gonzalez *et al.* 1999).

A number of statistical studies about fast and slow shock waves were performed, and often the shock was assumed to be spherically symmetric with the flow perpendicular to the shock surface. The following relation (Kivelson & Russell 1995) was used in order to estimate shock speed:

$$v_{\text{shock}} = \frac{\rho_{\text{md}} u_{\text{u}} - \rho_{\text{mu}} u_{\text{d}}}{\rho_{\text{md}} - \rho_{\text{mu}}}, \quad (1)$$

where ρ_{mu}, ρ_{md}, u_{u} and u_{d} are the mass density and flow velocity in the upstream and downstream regions respectively. In applying (1) to the shocks in space plasmas, there are some limitations. First, the magnetic field is neglected. Second, the shock is assumed to be spherically symmetric with the flow perpendicular to the shock surface and only gives the radial component of the shock speed (Kallenrode 2004). It, therefore, can be used as a lower limit only. In this work, we study the interplanetary shock parameters during high solar activity conditions. We studied a strong shock wave event on 7 June 2014, during a solar maximum for analysis. Plasma and magnetic field parameter variations through the shocks were calculated, and derived quantities for

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shock and shock speed. The upstream parameters, such as Alfven velocity and sound speed are calculated, and the angle (θ) between the upstream magnetic field direction and the shock normal is estimated. To determine the propagation speed of the shock, a full numerical solution of shock adiabatic equation is carried out. The thermal pressure for electrons and ions are calculated and also pressure ratio and the entropy change across the shock are estimated.

2. Data

Interplanetary plasma high resolution data used in this study were obtained by sensors onboard the WIND spacecraft (Vogl *et al.* 2001) and the ACE spacecraft in solar maximum. We used the five-minutes averaged solar wind parameters (density, speed, temperature and magnetic field), compiled in the OMNI database. The typical observation of the fast forward shock analyzed in this study is found during the solar maximum on 7 June 2014, as shown in the next section.

3. Observations

Figure 1 shows the data coverage of the solar wind observation for the OMNI database, which shows an example of a fast forward interplanetary shock on 7 June 2014, during solar maximum. Figures 1(a)–(d) are proton number density (cm^{-3}), total magnetic field, B (nT), flow speed V_{p} (km/s) and proton temperature T_{p}(K). The shock is indicated by the letter "S" and discontinuity is indicated by the blue dashed lines. For a fast forward shock, the values of the four parameters mentioned above are increasing. In Figure 1, the upstream and downstream sides of the shock are indicated by the letters "U" and "D" respectively, and the average parameters were calculated for these interval limited time. As shown in Figure 2, about two hours after shock, AE-index abruptly increased which indicates the influence of shock on the magnetosphere and the current in the magnetopause.

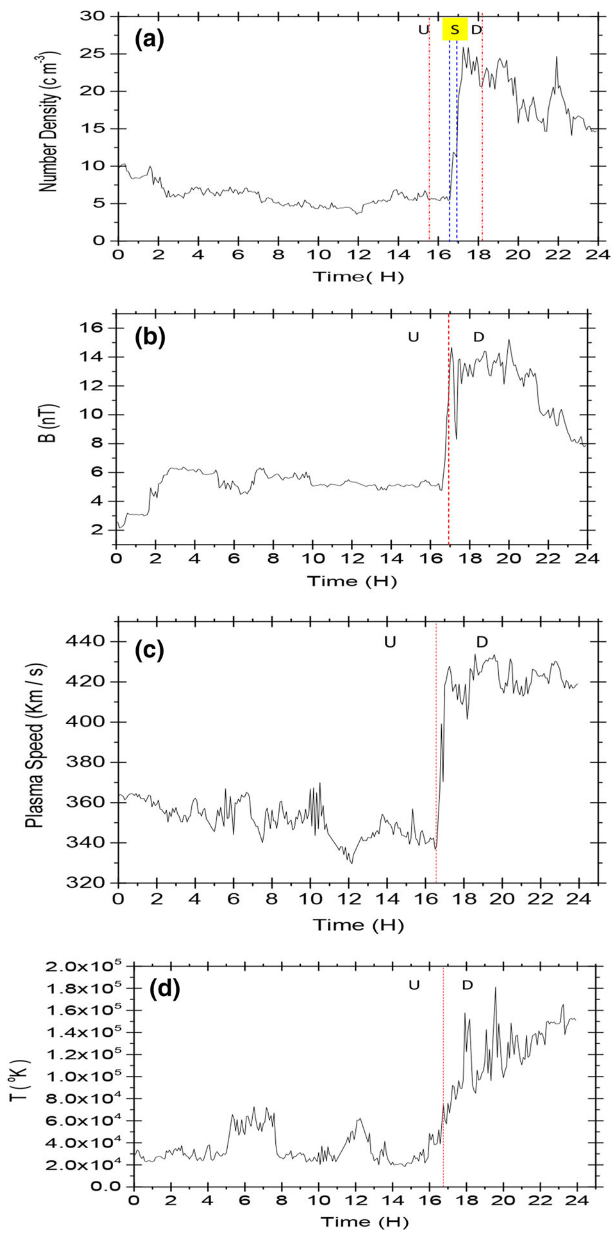

Figure 1. A fast forward interplanetary shock on 7 June 2014, during solar maximum. **(a)** Proton number density, **(b)** total magnetic field, **(c)** speed of flow and **(d)** proton temperature. The shock is indicated by the letter "S" and discontinuity is indicated by the blue dashed lines.

4. Results and discussion

Rankine–Hugoniot jump conditions, describe the fundamental physical relationship between the states on both sides (upstream and downstream) of a shock wave. These include four flux conservation laws: the mass flux, the two components of momentum flux, magnetic flux and the energy flux. Rankine–Hugoniot jump conditions can be expressed as

$$\rho_{\mathrm{mu}} U_{\mathrm{nu}} = \rho_{\mathrm{md}} U_{\mathrm{nd}}, \tag{2}$$

$$\rho_{\mathrm{mu}} U_{\mathrm{nu}}^2 + \rho_{\mathrm{mu}} \frac{V_{\mathrm{su}}^2}{\gamma} + \frac{B_{\mathrm{tu}}^2}{2\mu_0}$$

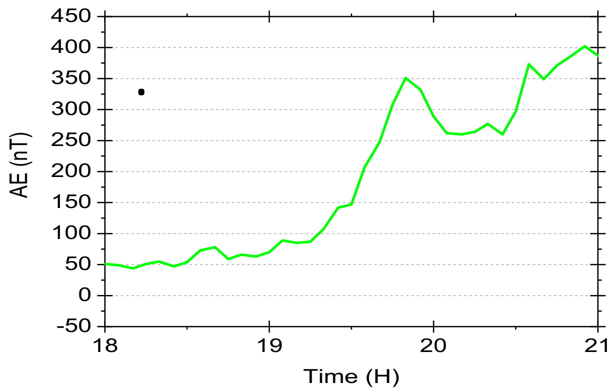

Figure 2. The variations of geomagnetic indices that AE-index abruptly increased, corresponding to the Earth's magnetosphere compression by the shock and the intensification of the current in the magnetopause.

$$= \rho_{md} U_{nd}^2 + \rho_{md} \frac{V_{sd}^2}{\gamma} + \frac{B_{td}^2}{2\mu_0}, \tag{3a}$$

$$\rho_{mu} U_{nu} U_{tu} - B_{nu} \frac{B_{tu}}{\mu_0} = \rho_{md} U_{nd} U_{td} - B_{nd} \frac{B_{td}}{\mu_0}, \tag{3b}$$

$$\rho_{mu} U_{nu} \left(\frac{V_{su}^2}{\gamma - 1} + \frac{U_{tu}^2 + U_{nu}^2}{2} \right) + \frac{B_{tu}}{\mu_0} (B_{tu} U_{nu} - B_{nu} U_{tu}) = \rho_{md} U_{nd} \left(\frac{V_{sd}^2}{\gamma - 1} + \frac{U_{td}^2 + U_{nd}^2}{2} \right) + \frac{B_{td}}{\mu_0} (B_{tu} U_{nd} - B_{nd} U_{td}), \tag{4}$$

$$U_{nu} B_{tu} - U_{tu} B_{nu} = U_{nd} B_{td} - U_{td} B_{nd}, \tag{5a}$$

$$B_{nu} = B_{nd}. \tag{5b}$$

In the above equation, $U_{nu}, U_{nd}, V_{su}, V_{sd}, B_{nu}, B_{nd}$ are the normal propagation of the shock speed, the sound speed and the normal of magnetic field in the upstream and downstream regions, respectively. $U_{tu}, U_{td}, B_{tu}, B_{td}$ are the tangential propagation of shock speed, and the normal of the magnetic field in the upstream and downstream regions. γ is ratio of heat capacity at constant pressure to heat capacity at constant volume.

By using the Rankine–Hugoniot equations, we can obtain an equation, the so-called Shock adiabatic equation (Anderson 1963). To solve the above system of equations, the procedure is that all of the upstream parameters are specified except U and then solve for U_{nu} as a function of the upstream parameters (Gurnett & Bhattacharjee 2017).

Since, there are twelve unknowns (ρ_{mu}, ρ_{md}, U_{nu}, U_{nd}, U_{tu}, U_{td}, V_{su}, V_{sd}, B_{nu}, B_{nd}, B_{tu}, B_{td}) of which four upstream parameters (ρ_{mu}, V_{su}, B_{tu}, B_{nu}) are specified, the jump conditions provide seven equations for eight unknown quantities. Therefore, one more quantity should be specified in order to solve the system of equations. This indeterminacy is due to the fact that no information has been provided about the strength of the shock. To make the problem tractable, using of a new parameter is useful, $r = \rho_{md}/\rho_{mu}$, which is called the shock strength. With this parameter specified, enough equations are now available to determine completely all of the unknown parameters. Also, to provide some further simplification, the following three dimensionless quantities are used:

(1) the Alfven Mach number, M_A that is the ratio of the normal component of the flow velocity to the normal component of the Alfven velocity,
(2) the sonic Mach number, M_s that is the ratio of the normal component of the flow velocity to the normal component of the sound speed,
(3) the angle between the magnetic field and the shock normal, θ.

In the next steps, all quantities on the downstream of the jump conditions are eliminated, except the density ρ_{md}, which will be combined as a ratio with ρ_{mu} to introduce the shock strength r. However, the result, after some lengthy algebra (substitutions and simplifying), is

$$\left(M_{Au}^2 - 1\right)^2 \left[M_{su}^2 - \frac{2r}{r+1-\gamma(r-1)}\right] - r \tan^2\theta_u M_{su}^2 \left[\frac{2r-\gamma(r-1)}{r+1-\gamma(r-1)} M_{Au}^2 - r\right] = 0. \tag{6}$$

By using the relations $M_{Au} = U_{nu}/(V_{Au} \cos\theta_u)$ and $M_{su} = U_{nu}/V_{su}$ and after making these substitutions and simplifying, the above equation becomes

$$\left(U_{nu}^2 - r V_{Au}^2 \cos^2\theta_u\right)^2 \left[U_{nu}^2 - \frac{2r V_{su}^2}{r+1-\gamma(r-1)}\right] - r \sin^2\theta_u U_{nu}^2 V_{Au}^2 \left[\frac{2r-\gamma(r-1)}{r+1-\gamma(r-1)} U_{nu}^2 - r V_{Au}^2 \cos^2\theta_u\right] = 0. \tag{7}$$

This is the so-called shock adiabatic equation. By using the Rankine–Hugoniot equations, we can obtain the so-called shock adiabatic equation (Anderson 1963) as follows:

$$\left(U_{\rm nu}^2 - rV_{\rm Au}^2\cos^2\theta_{\rm u}\right)^2\left[U_{\rm nu}^2 - \frac{2rV_{\rm su}^2}{r+1-\gamma(r-1)}\right] - r\sin^2\theta_{\rm u}U_{\rm nu}^2V_{\rm Au}^2\left[\frac{2r-\gamma(r-1)}{r+1-\gamma(r-1)}U_{\rm nu}^2 - rV_{\rm Au}^2\cos^2\theta_{\rm u}\right] = 0. \quad (8)$$

It gives the propagation as a function speed of the shock strength and the upstream parameters (Gurnett & Bhattacharjee 2017). In the above equation, $U_{\rm nu}$, $V_{\rm Au}$ and $V_{\rm su}$ are the normal propagation of shock speed, Alfven velocity and the sound speed in the upstream region, respectively. γ and r are the ratio of heat capacity at constant pressure to heat capacity at constant volume (we considered $\gamma = 5/3$ for computation) and the shock strength (which is the ratio of downstream to upstream plasma densities), respectively. θ_u is the angle between the magnetic field and the shock normal in the upstream region. To calculate the shock speed, in addition to knowing the shock strength, we need to know Alfven velocity, the sound speed and the angle between the magnetic field and the shock normal. The Alfven speed ($V_{\rm A}$) for the solar wind and the sound speed ($V_{\rm s}$) can be calculated using the following formulas:

$$V_{\rm A} = \frac{B}{(\mu_0\rho_{\rm m})^{1/2}}, \quad (9)$$

$$V_{\rm s} = \left(\frac{\gamma P_{\rm T}}{\rho_{\rm m}}\right)^{1/2}, \quad (10)$$

where $\rho_{\rm m}$ is the mass density ($\rho_{\rm m} = \sum_{\rm s} n_{\rm s}m_{\rm s}$, subscript 's' indicates the kind of particle) and $P_{\rm T}$ is the thermal pressure that can calculated by

$$P_{\rm T} = \sum_{\rm s} Kn_{\rm s}T_{\rm s}, \quad (11)$$

where K, $n_{\rm s}$ and $T_{\rm s}$ are the Boltzmann constant, density and temperature of each piece (proton, alfa and electron), respectively. The difficulty arises when we estimate the angle between the magnetic field and the shock normal. To estimate $\theta_{\rm u}$, we should use another physical relationship. By definition of $U_{\rm n}$ in terms of Alfven Mach number which is the ratio of the normal component of the Alfven velocity, and sonic Mach number which is the ratio of the normal component of the flow speed to the sound speed, we obtain an equation for $\theta_{\rm u}$ as

$$\theta_{\rm u} = \cos^{-1}\left(\frac{V_{\rm Au}\cos\theta_{\rm u}}{V_{\rm su}}\right)\left(\frac{\beta\gamma}{2}\right)^{1/2}, \quad (12)$$

where β is the plasma beta (which is the ratio of the plasma pressure to the magnetic field pressure) that can be calculated by

$$\beta = \frac{2\mu_0 P_{\rm T}}{B^2}. \quad (13)$$

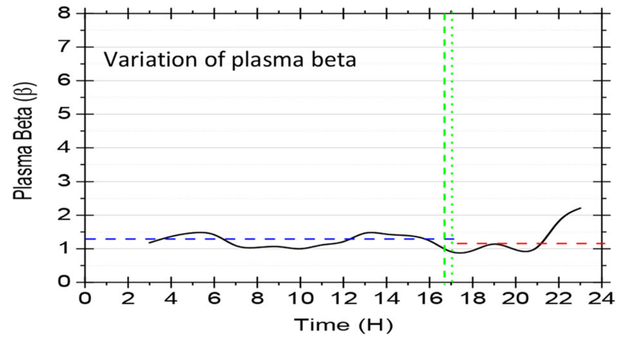

Figure 3. Plot of the variations of plasma beta (hourly average) on 7 June 2014. The dashed lines indicate a mean value of plasma beta in the upstream and downstream regions.

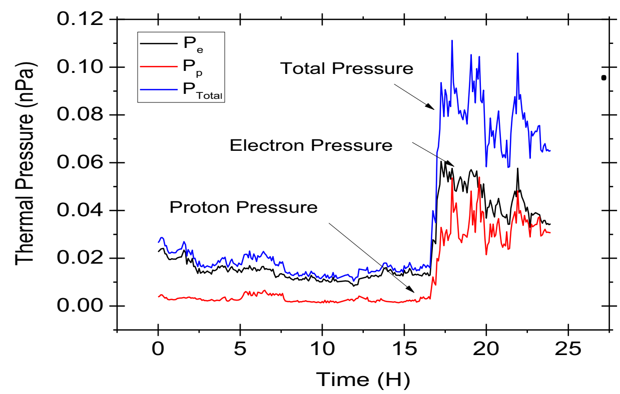

Figure 4. Plot of the variations of thermal pressure (np).

Table 1. The value of shock strength and the average of the calculating parameters corresponding to the upstream time window.

Shock strength	Plasma beta	Alfven velocity (km/s)	Sound speed (km/s)	$\theta_{\rm u}$
3.88	1.41	43.0	47.45	71.8

Figure 3 shows the variation of plasma beta (hourly average) on 7 June 2014. Since Equation (12) is a function of the cosine of theta, this computation must be done numerically. We estimated the shock strength, Alfven velocity and the sound speed from $\rho_{\rm md}/\rho_{\rm mu}$, $B_{\rm u}/(\mu_\circ\rho_{\rm mu})^{1/2}$ and $(\gamma P_{\rm Tu}/\rho_{\rm mu})^{1/2}$, respectively, where $\rho_{\rm mu}$ and $\rho_{\rm md}$ are the mean value of mass densities corresponding to the upstream and downstream sides of shock, and $P_{\rm Tu}$ is the thermal pressure. Figure 4 shows the electron thermal pressure, proton

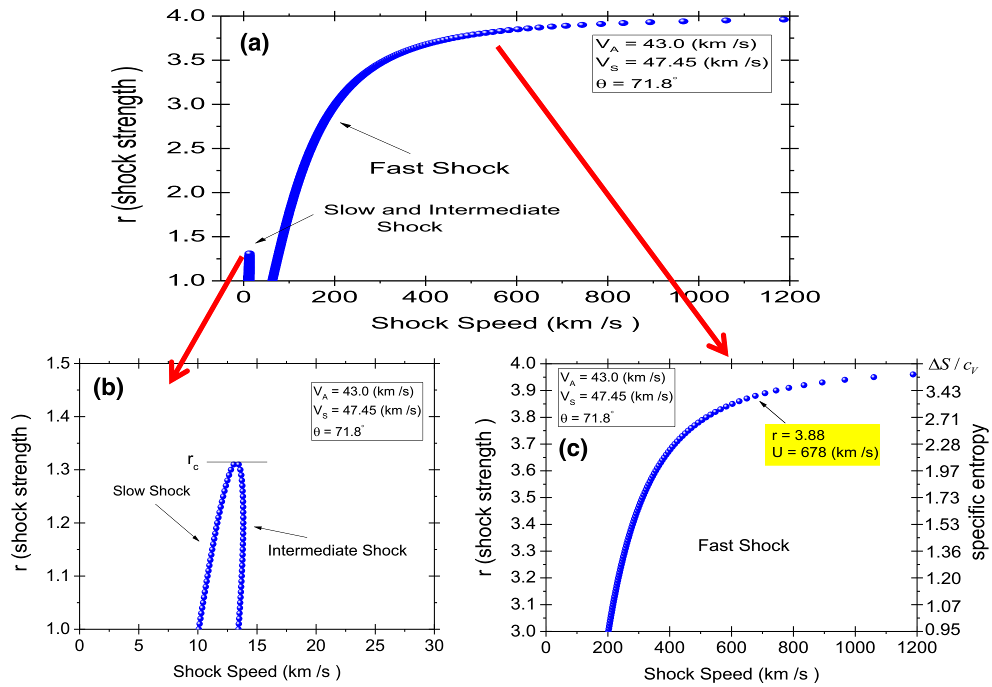

Figure 5. Plot of the shock propagation velocity as a function of the shock strength and for $\theta_u \approx 71.8°$, the angle between the upstream magnetic field direction and the shock normal. With a shock strength equal to 3.88, **(a)** the shock is fast with $U_n = 678$ km/s, **(b)** the slow and intermediate shocks and **(c)** the value of specific entropy change according to shock strength and parameters presented in Table 1.

thermal pressure and total thermal pressure that indicates abrupt changes as the parameter variation through the shock.

Table 1 shows the results of the calculations for the upstream time window. The $\theta_u \approx 71.8°$, indicates that the case is quasi-perpendicular. After determining θ_u, the speed of shock can estimated by Equation (8). From these results, we made a plot of the shock speed as a function of the shock strength and for $\theta_u \approx 71.8°$ as shown in Figure 5. At $r = 3.88$ (an estimation of the shock strength for this case), the shock is fast with $U_n = 678$ km/s. U and B may change direction across the shock, by using the jump conditions such as $B_{nu} = B_{nd}$. The propagation angle in downstream region is estimated to be about 82°.

Figure 6 shows the total magnetic field and the normal component of the magnetic field. The entropy change from the upstream to the downstream side of the shock ramp is one of the most important physical parameters, and it was studied from various viewpoints, for example, Fahr and Siewert (2015) studied entropy generation at multi magnetohydrodynamic shock. They calculated fluid-specific entropy productions from the individual fluid pressures (including the proton fluid and the electron fluid); their results showed that electron is important for the whole entropy production. Vogl *et al.* (2001) considered a model with the pressure anisotropy in the solar wind and solved the jump equation for an anisotropic magnetoplasma. Their results showed that the total entropy increases through the fast shock.

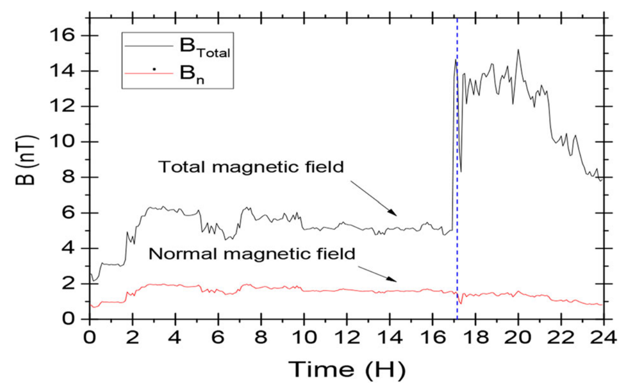

Figure 6. Plot of the total magnetic field and the normal component of the magnetic field.

For an abrupt adiabatic compression from the initial pressure and density on the upstream side to final pressure and density on the downstream side, the specific entropy is given by

$$\Delta S = c_{\rm V} Ln\left(\frac{P_{\rm d}}{P_{\rm u}}\left(\frac{\rho_{\rm mu}}{\rho_{\rm md}}\right)^{\gamma}\right), \quad (14)$$

where $c_{\rm V}$ is the specific heat capacity per unit volume. To evaluate the entropy change using the above equation, the first step is to compute the pressure ratio given by

$$\frac{P_{\rm d}}{P_{\rm u}} = 1 + \gamma \frac{U_{\rm nu}^2}{V_{\rm su}^2}\frac{(r-1)}{r} \left[1 - \frac{r V_{\rm Au}^2\left[(r+1)\,U_{\rm nu}^2 - 2r V_{\rm Au}^2 \cos^2\theta_{\rm u}\right]}{2\left(U_{\rm nu}^2 - r V_{\rm Au}^2\cos^2\theta_{\rm u}\right)^2}\sin^2\theta_{\rm u}\right]. \quad (15)$$

To compute the pressure ratio, we must solve the shock adiabatic for the shock speed $U_{\rm nu}$ and then substitute the shock speed into the above equation that this computation was done numerically. With this assumption, the computational results show that the value of specific entropy change is about 3.24 (normalized by $c_{\rm V}$) and this indicates that the entropy per particle increases across the shock for our case. Figure 3(c) shows the value of specific entropy change according to shock strength and parameters presented in Table 1. Finally, the angle between the magnetic field and the shock normal in the downstream region can be estimated by using the jump condition for the normal magnetic field. The estimated value is about 82 degrees ($\theta_{\rm d} \approx 82°$) which indicates an angle larger than $\theta_{\rm u}$ according to fast shock.

5. Summary

In this work, we have studied interplanetary shock parameters during high solar activity conditions on 7 June 2014. We used the solar wind parameters (density, speed, temperature and magnetic field) to estimate other parameters such as Alfven velocity and sound speed, thermal pressure (electron pressure and proton pressure), shock strength, entropy change and the angle (θ) between the upstream magnetic field direction and the shock normal.

The quantities for shock–shock speed were derived using the Rankine–Hugoniot equations and the estimation of $\theta_{\rm u}$ was done numerically. The computational results showed that (1) shock strength is about 3.88 which indicates a fast shock wave with $U_{\rm n} = 678$ km/s, (2) $\theta_{\rm u} \approx 71.8°$ indicates a quasi-perpendicular case, (3) the angle between the magnetic field and the shock normal in the downstream region was estimated as $\theta_{\rm d} \approx 82°$ which indicates an angle larger than $\theta_{\rm u}$ according to fast shock and (4) the change specific entropy was evaluated.

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