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Modified Chaplygin gas with bulk viscous cosmology in FRW (2+1)-dimensional spacetime

G. S. KHADEKAR[1], PRAVEEN KUMAR[1,*] and SAFIQUL ISLAM[2]

[1]Department of Mathematics, Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur, M.S. 440033, India.
[2]Harish-Chandra Research Institute, HBNI, Chhatnag Road, Jhunsi, Allahabad 211019, India.
*Corresponding author. E-mail: pkumar6743@gmail.com

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Abstract. In this paper we study the bulk viscous cosmology by considering modified Chaplygin gas in the framework of $(2 + 1)$-dimensional spacetime. For this we consider various form of bulk viscosity coefficient ζ and then obtain the physical parameters energy density ρ, Hubble and deceleration parameters, H and q, respectively. Finally we discuss the stability of the model by using the speed of sound.

Keywords. FRW cosmology—bulk viscosity—modified Chaplygin gas.

1. Introduction

General relativity in (2+1)-dimension spacetime is known to possess a number of special simplifying features: no gravitational waves, no black holes in the absence of negative cosmological constant, the Weyl curvature is identically zero and weak field limit of the theory does not correspond to Newtonian gravity in two space dimensions (Giddings *et al*. 1984; Barrow *et al*. 1986; Staruszkiewicz 1963; Gott & Alpert 1984; Deser *et al*. 1984; Deser & Mazur 1985; Deser 1985; Bañados *et al*. 1992). There simplifying features mean that considerable progress can be made in the search of the general cosmological solution of the 3- dimensional Einstein equations. The cosmological solution are rather cumbersome and dominated by non-integrability; in contrast, the theory in (2+1) dimension offers the possibility of finding the general solutions (Staruszkiewicz 1963). The unique status of Einstein's field equations in two space and one time dimensions provides the principal reason to focus on (2+1)-dimensions. Because the Einstein and Riemann tensors are equivalent in (2+1)-dimension, spacetime is flat outside sources; there is no gravitational field and no Newtonian limit.

The Riemann tensor is uniquely determined by the Ricci tensor in 3-dimension spacetime, so that any solution to the vacuum Einstein equations is flat, apart from possible singularities. The need to explore the stationary solutions in 3-dimension GR is emphasized in (Clément 1985).

All hydrostatic structures in (2+1) Einstein gravity contain matter-filled spaces with no matching to the external vacuum solution and thus represent static cosmologies (Cornish & Frankel 1991). Cosmological observations specify that there must be some kind of dark energy with a negative pressure in the universe. It has led to the study of cosmological scaling solutions of minimally coupled scalar fields in 3-dimensions (Martínez & Cruz 2000). The nucleation of a universe in a $(2 + 1)$-dimension gravity model with a negative cosmological constant has been explained by (Fujiwara 1991).

An interesting model to describe the dark energy is Chaplygin gas (CG) (Kamenshchik *et al*. 2001; Betnto *et al*. 2002) ,which emerged initially in cosmology from string theory (Barrow 1986, 1988), which are based on CG equation of state (EoS) and developed to the generalized Chaplyin gas (GCG) (Bilic *et al*. 2002). GCG was extended to modified Chaplygin gas (MCG)(Debnath *et al*. 2004).

In this paper we have studied the intrinsic details of the FRW bulk viscous cosmology in the presence of modified Chaplygin gas in (2+1)-dimensional spacetime and obtained the time-dependent energy density ρ for various form of bulk viscosity coefficient ζ.

The organization of this paper is as follows: In Section 2 we have obtained Einstein filed equations for flat

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FRW cosmological models and MCG in the framework of (2+1)-dimension spacetime. By considering various form of viscosity coefficient ζ we obtained the energy density ρ by using the method given earlier by Saadat & Pourhassan (2013) (Saadat & Pourhassan 2013). The stability of the system is discussed in Section 3. Concluding remarks are given in Section 4.

2. FRW Model and Friedmann equations

We consider the Friedmann–Robertson–Walker (FRW) universe in (2+1)-dimensional spacetime as (Saadat & Pourhassan 2013)

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2\right], \tag{1}$$

Here, $a(t)$ represents the scale factor. The coordinates $(t,\ r,\ \theta)$ represent the co-moving coordinates and the constant k denotes the curvature of the space $k = 0, 1, -1$ for flat, closed and open universes, respectively.

The Einstein field equations in (2+1)-dimension spacetime is given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 2\pi G T_{\mu\nu}, \tag{2}$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, and R is the Ricci scalar. In future, we consider $2\pi G = C = 1$.

Remark In usual Einstein theory, Einstein field equations can be written as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

but in our case, we are using (2+1)-dimensional theory proposed in (Cornish & Frankel 1991). In this theory, the Einstein field equations is in the form of Equation (2) as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2\pi G T_{\mu\nu}$$

The energy momentum tensor corresponding to the bulk viscous fluid is given by (Saadat & Pourhassan 2013)

$$T_{\mu\nu} = (\rho + \bar{p})u_\mu u_\nu - \bar{p}g_{\mu\nu}, \tag{3}$$

where ρ is the energy density and u^μ is the velocity three vector with $u^\mu u_\nu = -1$. The total pressure and the proper pressure involve bulk viscosity coefficient ζ and Hubble expansion parameter $H = \dot{a}/a$ are given by the following equations:

$$\bar{p} = p - 2\zeta H, \tag{4}$$

The modified Chaplygin gas equation of state can be written as

$$p = \gamma\rho - \frac{B}{\rho^\alpha}, \tag{5}$$

with $B > 0$ and $0 < \alpha \leq 1$. Here γ and B describes the features of dark energy models and Chaplygin gas respectively.

The field equations (2) with the help of line element (1) for flat space universe in $(2+1)$ dimensions are given (Betnto *et al*. 2002) by

$$\frac{\dot{a}^2}{a^2} = \frac{\rho}{2}, \tag{6}$$

and

$$\frac{\ddot{a}}{a} = -\bar{p}, \tag{7}$$

where $\cdot$ denotes derivative with respect to cosmic time t.

The energy–momentum conservation equation in (2+1)-dimensional spacetime is given by

$$\dot{\rho} + 2\frac{\dot{a}}{a}(\rho + \bar{p}) = 0. \tag{8}$$

By using Equations (4), (5) and (6) in the conservation Equation (8), we get the reduced form as

$$\dot{\rho} + \sqrt{2}(\gamma+1)\rho^{3/2} - 2\zeta\rho - \sqrt{2}B = 0. \tag{9}$$

We solve this equation for three different cases: namely, $\zeta = 0$, $\zeta \neq 0$ (constant) and when ζ is a linear combination of two terms, one constant and other is a linear combination of square root of energy density ρ :

Case (i): $\zeta = 0$

In this case from Equaton (9), we get

$$\dot{\rho} + \sqrt{2}(\gamma+1)\rho^{3/2} = \sqrt{2}B, \tag{10}$$

After integrating this equation, we get

$$\rho(a) = \left[\frac{1}{(\gamma+1)}\left(B - \frac{c}{a^{3(\gamma+1)}}\right)\right]^{2/3}, \tag{11}$$

where c is a constant of integration.

By using the value of ρ from Equation (11) in Equation (6), we get the Hubble parameter H as

$$H = \frac{1}{\sqrt{2}}\left[\frac{1}{(\gamma+1)}\left(B - \frac{c}{a^{3(\gamma+1)}}\right)\right]^{1/3}, \tag{12}$$

From Equation (11), it is observed that the energy density ρ increases when the scale factor a increases, which is shown in Figure 1.

From Equation (12), we observe that the Hubble parameter H increases when the scale factor a increases

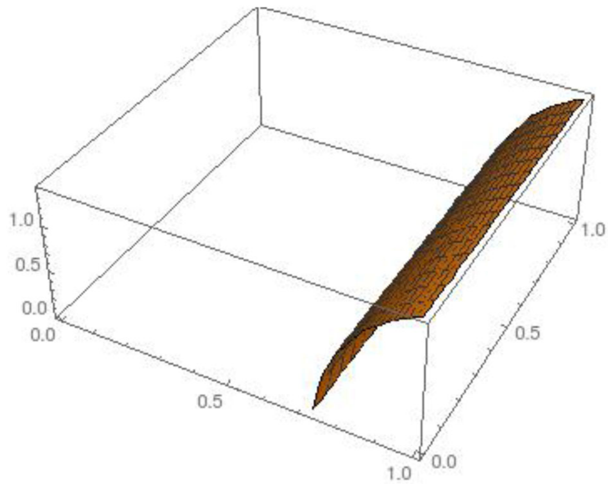

Figure 1. The energy density is shown against the scale factor a for values of the constants $\gamma = 0.3$, $B = 3.4$, $c = 1$.

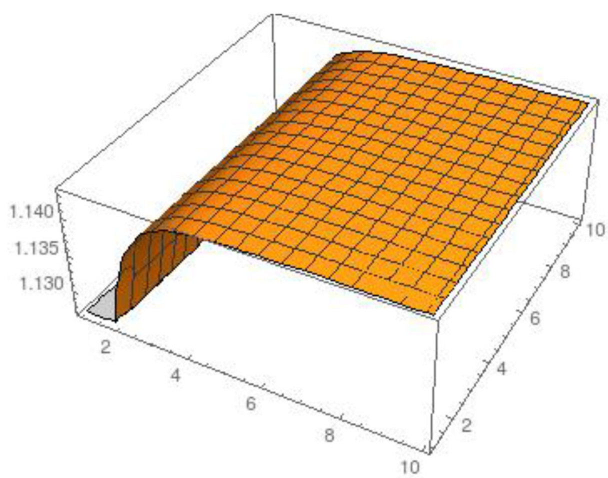

Figure 2. The Hubble parameter is shown against scale factor for the values of the constants $\gamma = 0.3$, $B = 3.4$, $c = 1$.

and approaches an infinitesimal constant at late time, which is shown in Figure 2.

Case (ii): $\zeta \neq 0$ (constant)
In this case we follow the particular form of ρ given earlier by Pourhassan and Saadat (Saadat & Pourhassan 2013) as

$$\rho = \frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}. \tag{13}$$

After using the value of ρ from Equation (13) in Equation (9), we get the differential equation of the form

$$\frac{d}{dt}\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right) + \sqrt{2}(\gamma+1)\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right)^{3/2} - 2\zeta\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right) - \sqrt{2}B = 0. \tag{14}$$

After a straightforward calculation and comparing likely coefficient on both sides, we get the unknown values of A, E, h, C and b as given in the following equations:

$$h = \sqrt{2}B, \tag{15}$$

$$A = \frac{2}{(\gamma+1)^2}, \tag{16}$$

$$E = \frac{2\zeta}{(\gamma+1)^2}, \tag{17}$$

$$C = \frac{(\gamma+1)^2}{6}\left[\frac{8\sqrt{2}\zeta^2}{(\gamma+1)^3} - \frac{3}{16}\frac{(\gamma+1)^4}{\zeta^2}\right], \tag{18}$$

$$b = \frac{2}{3}\zeta\left[\frac{2\sqrt{2}}{3}\zeta(\gamma+1)\left(\frac{2}{3}B(\gamma+1) - \frac{8}{3}\zeta^3\right) + \frac{9\sqrt{3}}{16\sqrt{2}}\left((\gamma+1) + \frac{7}{8}\right) + \frac{1}{9}\zeta^4 + o(\gamma^n)\right]\left[\frac{8\sqrt{2}}{\sqrt{3}}(\gamma+1)\left(\frac{16}{9\sqrt{3}}\zeta^4 - \frac{1}{72\sqrt{2}}(\gamma+1)^7\right)\right]^{-1}, \tag{19}$$

where

$$O(\gamma^n) = \frac{189}{64}\gamma^2 + \frac{189}{32}\gamma^3 + \frac{945}{128}\gamma^4 + \frac{189}{32}\gamma^5 + \frac{189}{64}\gamma^6 + \frac{27}{32}\gamma^7 + \frac{27}{256}\gamma^8,$$

After putting all the values of arbitrary constant in Equation (13), we get ρ as

$$\rho = \left(\frac{2}{(\gamma+1)^2}\right)\frac{1}{t^2} + \left(\frac{2\zeta}{(\gamma+1)^2}\right)\frac{1}{t} + \sqrt{2}Bt + \left(\frac{(\gamma+1)^2}{6}\left[\frac{8\sqrt{2}\zeta^2}{(\gamma+1)^3} - \frac{3}{16}\frac{(\gamma+1)^4}{\zeta^2}\right]\right) \times exp\left[\left(\frac{2}{3}\zeta\left(\frac{2\sqrt{2}}{3}\zeta(\gamma+1)\left(\frac{2}{3}B(\gamma+1) - \frac{8}{3}\zeta^3\right) + \frac{9\sqrt{3}}{16\sqrt{2}}\left((\gamma+1) + \frac{7}{8}\right) + \frac{1}{9}\zeta^4 + o(\gamma^n)\right) \times \left(\frac{8\sqrt{2}}{\sqrt{3}}(\gamma+1)\left(\frac{16}{9\sqrt{3}}\zeta^4 - \frac{1}{72\sqrt{2}}(\gamma+1)^7\right)\right)^{-1} t\right]. \tag{20}$$

By using this value in Equation (6), we get the Hubble parameter H as

$$H = \frac{1}{\sqrt{2}}\Bigg[\left(\frac{2}{(\gamma+1)^2}\right)\frac{1}{t^2} + \left(\frac{2\zeta}{(\gamma+1)^2}\right)\frac{1}{t} + \sqrt{2}Bt + \left(\frac{(\gamma+1)^2}{6}\left[\frac{8\sqrt{2}\zeta^2}{(\gamma+1)^3} - \frac{3}{16}\frac{(\gamma+1)^4}{\zeta^2}\right]\right) \times exp\left(\frac{2}{3}\zeta\left(\frac{2\sqrt{2}}{3}\zeta(\gamma+1)\left(\frac{2}{3}B(\gamma+1) - \frac{8}{3}\zeta^3\right) + \frac{9\sqrt{3}}{16\sqrt{2}}\left((\gamma+1)+\frac{7}{8}\right) + \frac{1}{9}\zeta^4 + o(\gamma^n)\right) \times \left(\frac{8\sqrt{2}}{\sqrt{3}}(\gamma+1)(\frac{16}{9\sqrt{3}}\zeta^4 - \frac{1}{72\sqrt{2}}(\gamma+1)^7)\right)^{-1} t\right)\Bigg]^{1/2}. \tag{21}$$

The deceleration parameter is given by

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right) = -\left[1 + \frac{1}{2}\left\{\left(\frac{-2A}{t^3} - \frac{E}{t^2} + h + Cbe^{bt}\right) \times \left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right)^{3/2}\right\}\right]. \tag{22}$$

In the absence of both bulk viscosity and Chaplygin gas, i.e. $B = 0$ and $\zeta = 0$, Equation (9) becomes

$$\dot{\rho} + \sqrt{2}(\gamma+1)\rho^{3/2} = 0, \tag{23}$$

After solving Equation (23), we get

$$\rho^{1/2} = \frac{\sqrt{2}}{(\gamma+1)t}, \tag{24}$$

Equation (24) becomes

$$\rho = \frac{2}{(\gamma+1)^2 t^2}, \tag{25}$$

From Equation (25), it is observed that energy density ρ decreases when t increases. It agrees with results (Saadat & Pourhassan 2013; Mazumder *et al*. 2012) where $\rho \propto t^{-2}$. However, for large bulk viscosity coefficient we get $b < 0$ and $\rho \propto \zeta/t$. For the late time behavior of the energy density 'ρ', the last term of Equation (13) is dominant, so we can say $\rho \sim Ce^{bt}$. It is indicative of the fact that the energy density is a decrementing function of time. This is dependent on the Hubble expansion parameter.

We plotted the Hubble expansion parameter in Figure 3 for $\gamma \simeq 1/3$. In this case, the modified Chaplygin gas model expresses the evolution of the macrocosm from the radiation regime to the Λ-cold dark matter scenario, where the fluid deports like a cosmological constant, and so there is an expedited expansion of the macrocosm. Numerically, we plotted the deceleration parameter in terms of time in Figure 4. It is shown that the deceleration parameter yields to -1 at the tardy time. In this case, in the values of $\zeta = 0.2, 0.4, 0.6, 1$ there is a minimum value of the deceleration parameter at $t \sim 1.15, 1.89, 1.34, 1.51$ respectively. After the minimum point, the deceleration parameter increments as the time increases.

Figure 3 indicates that the Hubble parameter decreases with time and is almost constant at present day.

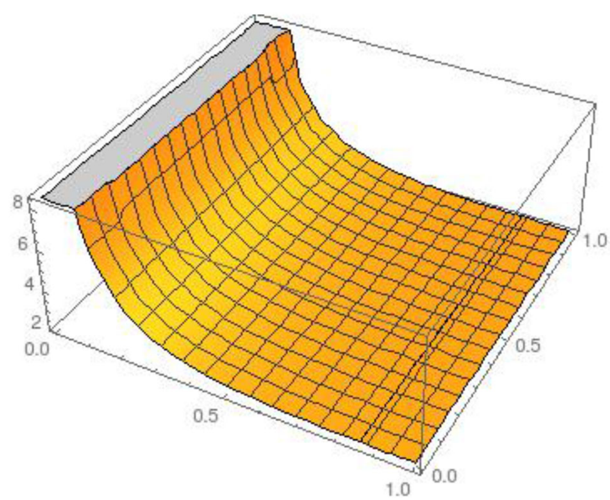

Figure 3. The Hubble parameter is shown against time for the values of the constants $A = 1.1834$, $E = 1.1834$, $h = 4.8083$, $C = 1.2996$, $b = 0.1997$.

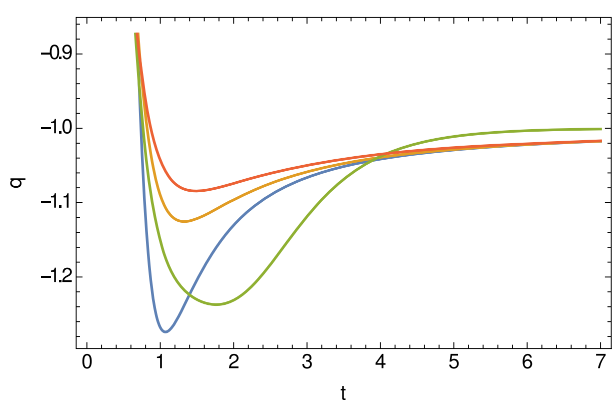

Figure 4. The deceleration parameter is plotted against time for various values of the constants [$blue(color)$: $E, C, b = 0.237, -3.713, -0.711$], [$yellow(color)$: $E, C, b = 0.473, -0.711, -2.989$], [$green(color)$: $E, C, b = 0.71, 0.103, 2.479$], [$orange(color)$: $E, C, b = 1.183, 1.299, 0.199$] alongwith $A = 1.183$, $h = 4.808$.

We observe from Figure 4 that the deceleration parameter first decreases and then increases till it attains a constant value –1 as $t \to \infty$.

Case (iii): $\zeta = \zeta_0 + \zeta_1 \rho^{1/2}$

In this case, ζ is a linear combination of two terms, one is constant and other is a linear combination of square root of energy density 'ρ'.
By putting these values in Equation (9) with the help of Equation (6), we get

$$\dot{\rho} + \rho_0 \rho^{3/2} - 2\zeta_0 \rho - \sqrt{2}B = 0. \tag{26}$$

where $\rho_0 = \sqrt{2}(\gamma+1) - 2\zeta_1$.

By the method given by Pourhassan and Saadat (Saadat & Pourhassan 2013) and using Equation (13), we get the differential Equation (26) of the form

$$\frac{d}{dt}\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right) + \rho_0\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right)^{3/2} - 2\zeta_0\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right) - \sqrt{2}B = 0. \tag{27}$$

where A, E, h, C and b are arbitrary constants whose values can be find by comparing the likely coefficients on both sides; we have

$$h = \sqrt{2}B, \tag{28}$$

$$A = \frac{4}{\rho_0^2} = \frac{4}{[\sqrt{2}(\gamma+1) - 2\zeta_1]^2} \tag{29}$$

$$E = \frac{4\zeta_0}{\rho_0^2} = \frac{4\zeta_0}{[\sqrt{2}(\gamma+1) - 2\zeta_1]^2} \tag{30}$$

$$C = \frac{\rho_0^2}{12}\left[\frac{32\zeta_0^2}{\rho_0^3} - \frac{3}{64}\frac{\rho_0^4}{\zeta_0^2}\right], \tag{31}$$

$$b = \frac{2}{3}\zeta_0\left[\frac{2}{9}\zeta_0\rho_0(\sqrt{2}B\rho_0 - 4\zeta_0^3) + \frac{9\sqrt{3}}{256}(8\rho_0 - 7\sqrt{3}) + \frac{1}{9}\zeta_0^4 + o(\gamma^n)\right] \times (8\rho_0)\left(\frac{16}{81}\zeta_0^4 - \frac{1}{128}\frac{\rho_0^7}{27}\right)^{-1}, \tag{32}$$

where

$$O(\gamma^n) = \frac{189}{64}\gamma^2 + \frac{189}{32}\gamma^3 + \frac{945}{128}\gamma^4 + \frac{189}{32}\gamma^5 + \frac{189}{64}\gamma^6 + \frac{27}{32}\gamma^7 + \frac{27}{256}\gamma^8, \tag{33}$$

After putting all the values of arbitrary constant in Equation (13), we get ρ as

$$\rho = \left[\frac{1}{t^2}\left(\frac{4}{[\sqrt{2}(\gamma+1) - 2\zeta_1]^2}\right) + \frac{1}{t}\left(\frac{4\zeta_0}{[\sqrt{2}(\gamma+1) - 2\zeta_1]^2}\right) + (\sqrt{2}B)t + \frac{(\sqrt{2}(\gamma+1) - 2\zeta_1)^2}{12}\left[\frac{32\zeta_0^2}{(\sqrt{2}(\gamma+1) - 2\zeta_1)^3} - \frac{3}{64}\frac{(\sqrt{2}(\gamma+1) - 2\zeta_1)^4}{\zeta_0^2}\right] \times exp\left(\frac{2}{3}\zeta_0\left[\frac{2}{9}\zeta_0(\sqrt{2}(\gamma+1) - 2\zeta_1) \times \{\sqrt{2}B(\sqrt{2}(\gamma+1) - 2\zeta_1) - 4\zeta_0^3\}\right] + \frac{9\sqrt{3}}{256}\left[\{8(\sqrt{2}(\gamma+1) - 2\zeta_1) - 7\sqrt{3}\} + \frac{1}{9}\zeta_0^4 + o(\gamma^n)\right]\left(8(\sqrt{2}(\gamma+1) - 2\zeta_1)\right) \times \left(\frac{16}{81}\zeta_0^4 - \frac{1}{128}\frac{(\sqrt{2}(\gamma+1) - 2\zeta_1)^7}{27}\right)^{-1} t\right)\right]. \tag{34}$$

By substituting the value of ρ from Equation (33) in Equation (6), we get the Hubble parameter H as

$$H = \frac{1}{\sqrt{2}}\left[\frac{1}{t^2}\left(\frac{4}{(\sqrt{2}(\gamma+1) - 2\zeta_1)^2}\right) + \frac{1}{t}\left(\frac{4\zeta_0}{(\sqrt{2}(\gamma+1) - 2\zeta_1)^2}\right) + (\sqrt{2}B)t + \frac{(\sqrt{2}(\gamma+1) - 2\zeta_1)^2}{12}\left[\frac{32\zeta_0^2}{(\sqrt{2}(\gamma+1) - 2\zeta_1)^3} - \frac{3}{64}\frac{(\sqrt{2}(\gamma+1) - 2\zeta_1)^4}{\zeta_0^2}\right] \times exp\left(\frac{2}{3}\zeta_0\left[\frac{2}{9}\zeta_0(\sqrt{2}(\gamma+1) - 2\zeta_1) \times \{\sqrt{2}B(\sqrt{2}(\gamma+1) - 2\zeta_1) - 4\zeta_0^3\}\right] + \frac{9\sqrt{3}}{256}\left[\{8(\sqrt{2}(\gamma+1) - 2\zeta_1) - 7\sqrt{3}\} + \frac{1}{9}\zeta_0^4 + o(\gamma^n)\right]\left(8(\sqrt{2}(\gamma+1) - 2\zeta_1)\right) \times \left(\frac{16}{81}\zeta_0^4 - \frac{1}{128}\frac{(\sqrt{2}(\gamma+1) - 2\zeta_1)^7}{27}\right)^{-1} t\right)\right]^{1/2}. \tag{35}$$

and the corresponding deceleration parameter can be written as

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right)$$

$$= -\left[1 + \frac{1}{2}\left\{\left(\frac{-2A}{t^3} - \frac{E}{t^2} + h + Cbe^{bt}\right) \times \left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right)^{3/2}\right\}\right]. \quad (36)$$

From Equation (33), it is noted that H is a decreasing function of t and is almost constant at present time, which is shown in Figure 5, and also we observe from Figure 6 that the deceleration parameter first decreases and then increases till it attains a constant value -1 as $t \to \infty$.

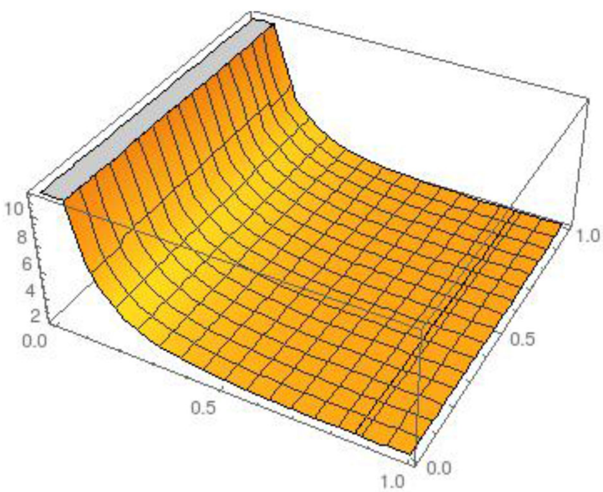

Figure 5. The Hubble parameter is shown against time for the values of the constants $A = 1.2096$, $E = 1.2096$, $h = 4.8083$, $C = 1.32517$, $b = 0.5625$.

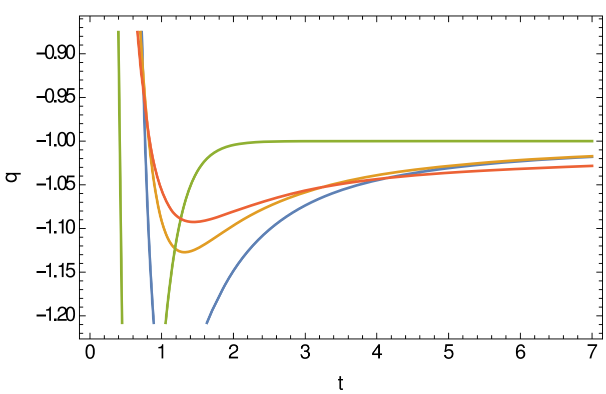

Figure 6. The deceleration parameter is plotted against time for various values of the constants [$blue(color)$: $E, C, b = 0.2419, -3.4728, -0.3513$], [$yellow(color)$: $E, C, b = 0.4838, -0.6482, -1.8353$], [$green(color)$: $E, C, b = 0.7258, 0.1355, 8.1727$], [$orange(color)$: $E, C, b = 1.2096, 1.3252, 0.5206$] along with $A = 1.2096$, $h = 4.8038$.

3. Stability

We investigate the stability of the system and find the sound speed C_s^2 in viscous medium. Using Equations (4) and (5), we get

$$C_s^2 = \frac{d\bar{p}}{d\rho}$$

$$C_s^2 = \left[\gamma + B\alpha\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right)^{-(\alpha+1)} - \frac{\zeta}{\sqrt{2}}\left(\frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}\right)^{-\alpha}\right]. \quad (37)$$

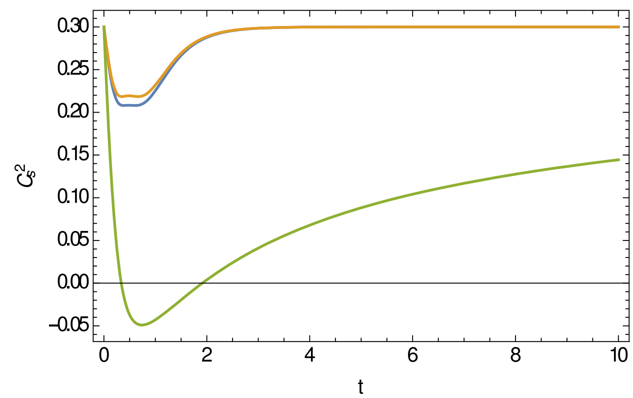

Figure 7. The stability of the system is shown against time for various values of the constants [$blue(color)$: $E, C, b, \zeta = 0.71, 0.331, 4.167, 0.65$], [$orange(color)$: $E, C, b, \zeta = 0.71, 0.331, 4.167, 0.6$], [$green(color)$: $E, C, b, \zeta = 1.183, 2.438, 0.075, 1.8$].

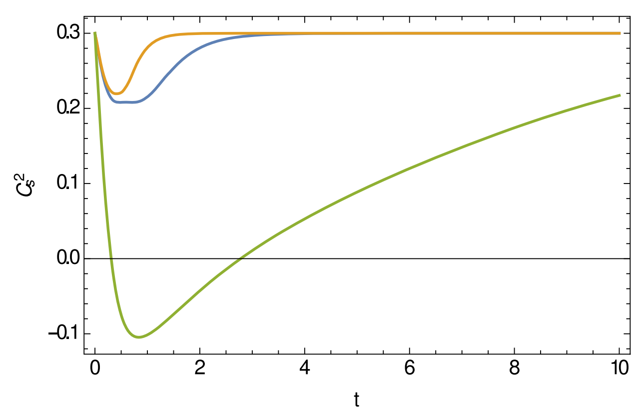

Figure 8. The stability of the system is shown against time for various values of the constants [$blue(color)$: $E, C, b, \zeta = 0.7862, 0.2852, 3.7731, 0.65$], [$orange(color)$: $E, C, b, \zeta = 0.7258, 0.1355, 8.173, 0.6$], [$green(color)$: $E, C, b, \zeta = 1.2096, 1.325, b = 0.5206, 2$].

For a particular choice of parameters $A = 0.788954$, $\gamma = 0.3, B = 3.4, h = 5.889$, the stability of the medium is graphically represented in Figure 7.

According to stability criteria (Sadeghi *et al.* 2010), $C_s^2 > 0$ indicates the stability region. We observe that the fluid is plenarily stable for $\zeta < 1.65$ and has an unstable region for $\zeta > 1.65$ as is evident from Figure 7.

For a particular choice of parameters $A = 1.2096$, $\gamma = 0.3, B = 3.4, h = 4.8083$, the stability of the medium is graphically represented in Figure 8. According to stability criteria (Mazumder *et al.* 2012), $C_s^2 > 0$ indicates the stability region. We observe that the fluid is completely stable for $\zeta < 1.62$ and has an unstable region for $\zeta > 1.62$ as is evident from the figure.

4. Conclusion

In this work, we studied the FRW bulk viscous cosmology with modified Chaplygin gas in (2+1)-dimensional spacetime. We obtained the modified Friedmann equations due to bulk viscous and Chaplygin gas coefficients. Then, we solved the field equations for various form of bulk viscous coefficient ζ and found the time-dependent energy density ρ.

The energy density ρ, for case (i) $\zeta = 0$, is dependent on the scale factor a, and the behavior of ρ for $\zeta = 0$ is shown in Figure 1, and the corresponding the Hubble parameter H shown in Figure 2, which increases as the scale factor a increases and approaches an infinitesimal constant at late time.

For case (ii) $\zeta \neq 0$, we again obtain the ρ and H as given in Equations (10) and (12). It is observed that the Hubble parameter decreases with cosmic time t and is almost constant at present day, as shown in Figure 3, and also we observed that the deceleration parameter first decreases and then increases till it attains a constant value –1 as $t \to \infty$, from Figure 4.

For case (iii) $\zeta = \zeta_0 + \zeta_1 \rho^{1/2}$, we again found density ρ and H, and it is noted that the Hubble parameter H decreases with cosmic time t and is almost constant at present day, as shown in Figure 5. We observed from Figure 6 that the deceleration parameter first decreases and then increases till it attains a constant value –1 as $t \to \infty$. We also studied stability of the model and observed that stability of the system strongly depends on viscosity coefficient. However, at late time, the model is stable and the speed of sound has constant real value.

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