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Generalized Durgapal–Fuloria relativistic stellar models

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Abstract. We present a new class of closed-form solutions to the Einstein–Maxwell system of equations for a static spherically symmetric anisotropic star in the presence of an electric field by generalizing earlier approaches. The field equations are integrated by specifying the form of the electric field, anisotropic factor, and one of the gravitational potentials which are physically reasonable. We regain some of the earlier solutions of Durgapal and Fuloria (1985), Gupta and Maurya (2011) and Maurya *et al.* (2015) as special cases. A detailed physical analysis of the solution indicates the physical viability for modelling anisotropic charged superdense stars.

Keywords. Einstein–Maxwell system—exact solution—relativistic stars.

1. Introduction

Theoretical modelling of compact stellar objects has been a field of active research for many decades. Current observations of mass-to-radius ratio of a wide range of pulsars have prompted many investigators to propose new theoretical models of compact stars by incorporating different types of matter composition. It is noteworthy that the choice of an equation of state (EOS) plays an important role in constraining the $M - R$ relationship of a star. In addition, factors such as electromagnetic field and local anisotropy also help in fine-tuning stellar observables like mass and radius. In this paper, we aim to develop new stellar models by incorporating these factors which would enable us to investigate their impact on the mass–radius relationship and other physical properties of a compact star.

The first factor that we would like to incorporate in our model is the presence of an electric field. The Einstein–Maxwell system has been taken up by many investigators in the past for the description of various astrophysical systems. Physical systems where such solutions might be relevant and a classification of the known solutions to the Einstein–Maxwell system describing spherically symmetric and static self-gravitating objects are available in the compilation by Ivanov (2002). The Einstein–Maxwell system has been extensively analysed to study the impact of the electromagnetic field on the gross physical behaviour of relativistic superdense stars [see, e.g., Sharma *et al.* (2001), Komathiraj and Maharaj (2007a, b, 2010), Maurya and Gupta (2012), Patel and Koppar (1987), Patel *et al.* (1997), Tikekar and Singh (1998), Gupta and Kumar (2005), Gupta and Maurya (2011), Kumar *et al.* (2018), Komathiraj and Sharma (2018), Thomas *et al.* (2005), Tikekar and Thomas (1998), Paul and Tikekar (2005) and references therein]. It is noteworthy that even though stellar objects are expected to be charge neutral, at certain evolutionary stages, a star may acquire a net charge. The electric field of a compact quark star composed of u, d and s quarks has been shown to be very high by Usov (2004). The structure and physical properties of quark stars in the presence of electromagnetic field have

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been studied by many investigators [see, e.g., Mak and Harko (2002), Bombaci (1984), Komathiraj and Maharaj (2007a, b), Thirukkanesh and Maharaj (2008) and references therein]. Such models, in the presence of an electromagnetic field, are shown to be consistent with the observational data available for different astrophysical sources like SAX J1808.4-3658, 4U 1538-52, PSR J1903+327, Vela X-1 and 4U1608-52.

Another important factor that seems to have a significant role in the modelling of a self-gravitating object is the presence of an anisotropic pressure. In an anisotropic system, the radial pressure differs from the tangential pressure. The origin of anisotropy and its role on stellar behaviour had been extensively analysed by Bowers and Liang (1974). The investigation of Ruderman (1972) has shown that nuclear matter might become anisotropic in the high-density regime where nuclear interactions should be treated relativistically. Kippenhahn and Weigert (1990) have showed that anisotropy might develop due to the existence of a solid core or presence of type $3A$ superfluid. Phase transition (Sokolov 1980) or pion condensation (Sawyer 1972) are some of the other driving factors for anisotropy. In fact, Ivanov (2010) has pointed out that in the case of a self-gravitating object, the role of shear or electromagnetic field can be interpreted by incorporating a gross anisotropic term in the energy momentum tensor. A large class of anisotropic stellar models have been developed till date. Dev and Gliser (2002, 2003) have studied the impact of anisotropy on the stability of a self-gravitating configuration. Making use of the Karmarkar's (1948) embedding condition, Maurya and Maharaj (2017) have developed an anisotropic stellar model to analyze the effect of anisotropy on its physical behaviour. Maurya *et al.* (2018) have studied anisotropic configurations by employing the Korkina and Orlyanskii (2017) ansatz. The geometric approach of Vaidya and Tikekar (1982) was used to analyze the impact of anisotropy on the maximum mass of a compact star by Karmakar *et al.* (2007). Komathiraj and Sharma (2018) have used the Durgapal and Bannerji (1983) transformations to generate new solutions for an anisotropic system. The stellar models developed by Maharaj and Mkhwanazi (1996) and John and Maharaj (2011) have been shown to be sub-classes of the solutions provided by Komathiraj and Sharma (2018). In the recent past, investigations on anisotropic stellar model have been carried out by Sharma *et al.* (2017), Deb *et al.* (2017), Thirukkanesh *et al.* (2018), Murad (2018), Ivanov (2017, 2018), Maurya *et al.* (2019a) and Maurya *et al.* (2019b).

A large class of analytical solutions corresponding to compact stellar configurations in the presence of both pressure anisotropy and electromagnetic are available in the literature [see, e.g., Thirukkanesh and Maharaj (2008), Varela *et al.* (2010), Rahaman *et al.* (2010), Esculpi and Aloma (2010), Feroze and Siddiqui (2011), Rahaman *et al.* (2012), Maharaj and Takisa (2012), Takisa and Maharaj (2013a, b), Maurya and Gupta (2013), Thirukkanesh and Ragel (2013), Maharaj *et al.* (2014), Takisa *et al.* (2014), Sunzu *et al.* (2014a, b), Takisa *et al.* (2014), Thirukkanesh and Ragel (2014), Matondo and Maharaj (2016), Takisa and Maharaj (2016), Bhar and Murad (2016), Maharaj *et al.* (2017), Sunzu and Danford (2017), Nasim and Azam (2018)]. In an earlier work, Gupta and Maurya (2011) have developed a charged analogue of the Durgapal and Fuloria (1985) solution describing an uncharged superdense star. Later, Maurya *et al.* (2015) proposed a new solution which has been shown to be well behaved and capable of describing realistic anisotropic stars. In a separate paper, Maurya *et al.* (2017a, b) have developed an algorithm for generating spherically symmetric charged anisotropic stellar solutions. Based on regularity and other physical requirements, they assumed a particular form of one of the metric potentials that enabled them to obtain a solution which has been found to be useful for the description of compact strange stars. In our investigation, we aim to adopt a technique which would allow us to analyse the effects of anisotropy and electromagnetic field on stellar properties distinctively. Accordingly, we intend to generalize the anisotropic stellar model developed by Maurya *et al.* (2015) by incorporating an electric field.

The paper is organized as follows: In Section 2, we lay down the Einstein–Maxwell equations as a set of differential equations where we utilize a specific transformation due to Durgapal and Bannerji (1983). To integrate the system, in Section 3, we assume a particular fall-off behaviour of the electric field, a generalized form of one of the metric potentials considered earlier by Durgapal and Fuloria (1985) and a more generalized form of the anisotropic parameter utilized by Maurya *et al.* (2015). These assumptions enable us to integrate the field equations which are given in Section 4. We express our solutions in terms of elementary functions and show that the solutions contain some of the earlier results. In particular, we demonstrate how our solutions can be utilized to regain the uncharged anisotropic solution of Maurya *et al.* (2015) and the uncharged isotropic solution of Durgapal and Fuloria (1985). In Section 5, based on physical grounds, we put bounds on our model parameters and show the physical viability of the model.

Finally, in Section 6, we summarize by discussing our main results and pointing out some interesting aspects of our investigation.

2. Interior spacetime

We assume a static spherically symmetric spacetime describing the interior of a compact relativistic star in the form

$$ds^2 = -e^{2\mu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where $\mu(r)$ and $\lambda(r)$ are the gravitational potentials. The energy momentum tensor for the interior matter distribution is assumed to be anisotropic and accordingly the Einstein–Maxwell field equations for the line element (1) are obtained as

$$\frac{1}{r^2}(1 - e^{-2\lambda}) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho + \frac{1}{2}E^2, \tag{2}$$

$$\frac{-1}{r^2}(1 - e^{-2\lambda}) + \frac{2\mu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2, \tag{3}$$

$$e^{-2\lambda}\left(\mu'' + \mu'^2 + \frac{\mu'}{r} - \mu'\lambda' - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2, \tag{4}$$

where a prime ($'$) denotes differentiation with respect to r. In Equations (2)–(4) ρ is the energy density, p_r is the radial pressure, p_t is the tangential pressure. These quantities are measured relative to the co-moving fluid velocity $u^i = e^{-\mu}\delta^i_0$. Note that we have used system of units where $8\pi G = 1 = c$. Due to spherical symmetry, the electric field E must be radial and consequently, the Maxwell stress-tensor has only two non-zero component F_{tr} and F_{rt}. The intensity of the electric field is given by $E^2 = -F_{tr}F^{tr}$. F_{ij} satisfies the Maxwell equations

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0. \tag{5}$$

We obtain the proper charge density

$$\sigma^2 = \frac{1}{r^4}e^{-2\lambda}[(r^2E)']^2, \tag{6}$$

from Equation (5).

A different but equivalent form of the field equations can be generated if we introduce a new independent variable x and introduce new functions y and Z proposed by Durgapal and Bannerji (1983):

$$x = Cr^2, \quad A^2y^2(x) = e^{2\mu(r)}, \quad Z(x) = e^{-2\lambda(r)}, \tag{7}$$

where A and C are arbitrary constants. Under the transformation (7), the system (2)–(6) becomes

$$\frac{\rho}{C} + \frac{E^2}{2C} = \frac{1-Z}{x} - 2\dot{Z}, \tag{8}$$

$$\frac{p_r}{C} - \frac{E^2}{2C} = 4Z\frac{\dot{y}}{y} + \frac{Z-1}{x}, \tag{9}$$

$$0 = 4Zx^2\ddot{y} + 2\dot{Z}x^2\dot{y} + \left(\dot{Z}x - Z + 1 - \frac{E^2x}{C} - \frac{\Delta x}{C}\right)y, \tag{10}$$

$$\sigma^2 = \frac{4CZ}{x}(x\dot{E} + E)^2, \tag{11}$$

where a dot (.) denotes differentiation with respect to the variable x. The quantity $\Delta = p_t - p_r$ is defined as the measure of anisotropy which is required to vanish at the centre. The mass of a gravitating object within a stellar radius r is given by

$$m(r) = \frac{1}{2}\int_0^r r^2\rho(r)dr. \tag{12}$$

We thus have a nonlinear system of four equations in seven unknowns ρ, p_r, E^2, σ, Δ, y and Z. Hence, to solve the system, we need to specify three of the quantities involved in the integration process.

3. Choice of potentials

In this work, we plan to solve the Einstein–Maxwell system by specifying physically reasonable forms of the gravitational potential Z, electric field E^2 and the measure of anisotropy Δ so that the system becomes integrable and provides a viable model of a superdense star.

For the metric function Z we make the following choice:

$$Z = 1 - \frac{kx(3+x)}{(1+x)^2}, \quad k \neq 0, \tag{13}$$

where k is a real constant. Note that Z is regular at the centre and is well behaved in the stellar interior for a wide range of values of k. We would like to point out here that the choice of Z is a generalization of earlier approaches which contains many previous studies as special cases. For example, for $k = 8/7$, we regain the potential form considered earlier by Maurya *et al.* (2015), Durgapal and Fuloria (1985) and Gupta and Maurya (2011).

Substitution of Equation (13) in (10) yields

$$4x(1+x)[1 + (2-3k)x + (1-k)x^2]\ddot{y} + 2kx(x-3)\dot{y}$$

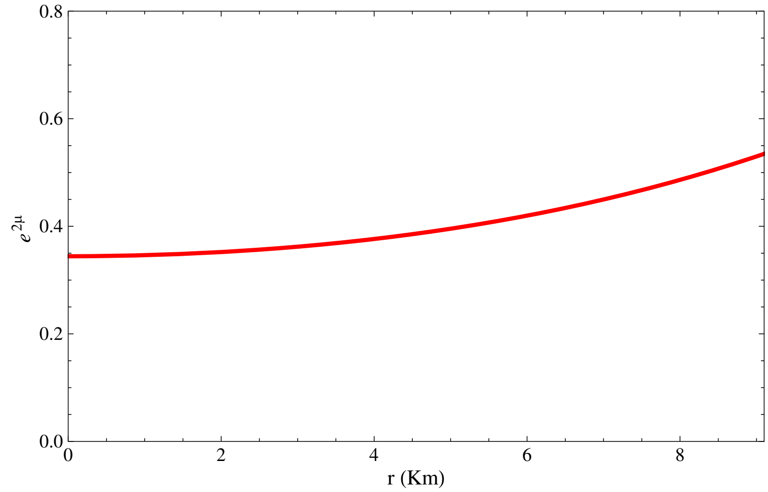

Figure 1. Radial variation of the metric potential $e^{2\mu}$.

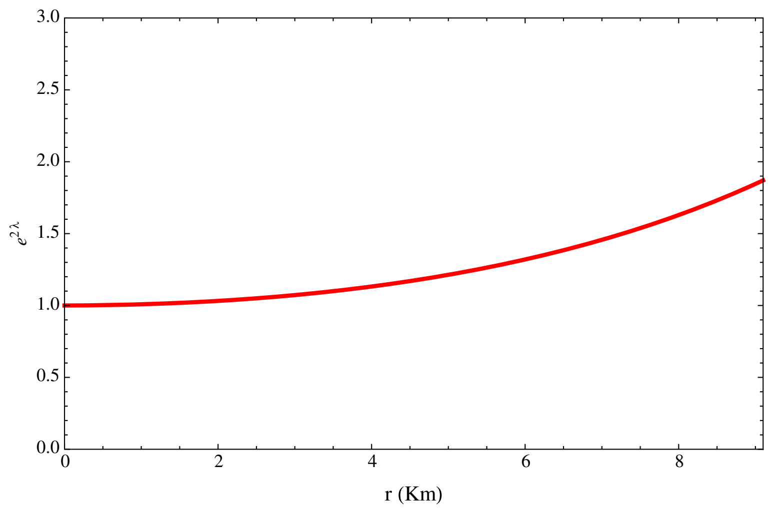

Figure 2. Radial variation of the metric potential $e^{2\lambda}$.

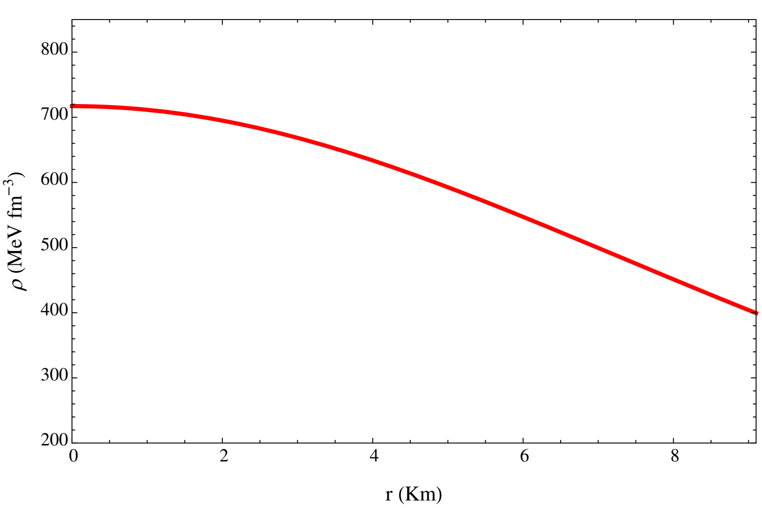

Figure 3. Radial variation of energy-density ρ.

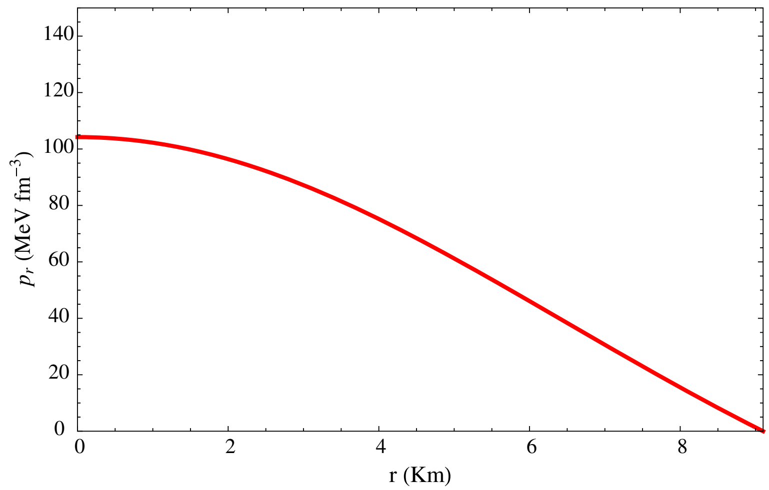

Figure 4. Radial variation of radial pressure p_r.

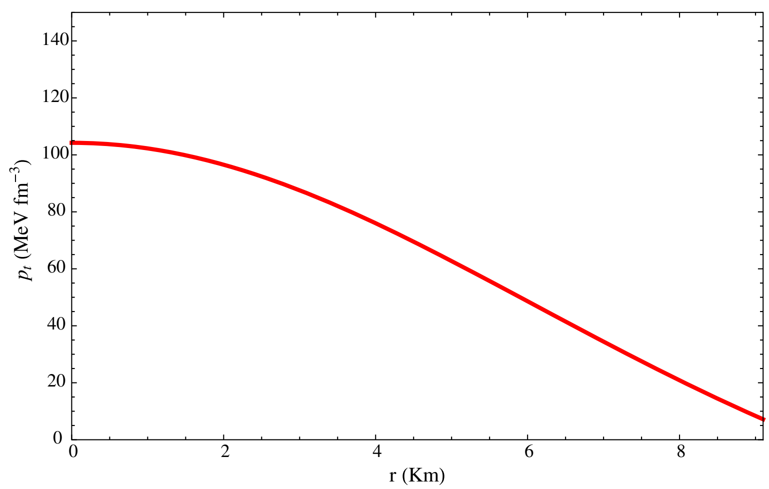

Figure 5. Radial variation of transverse pressure p_t.

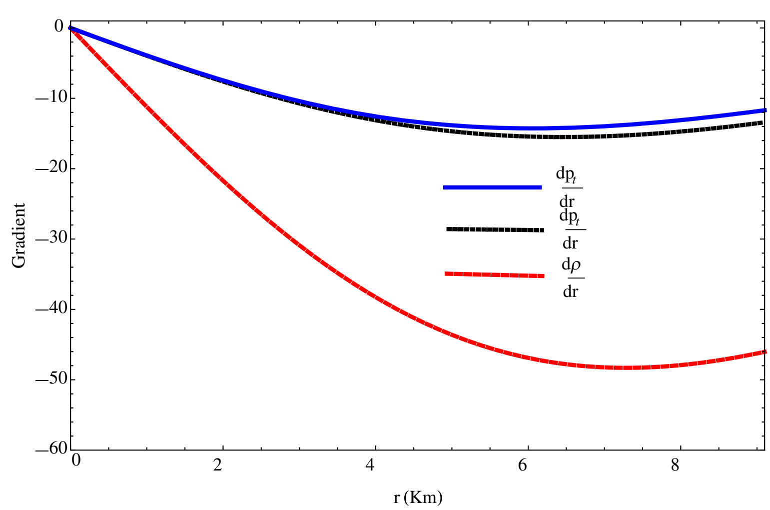

Figure 6. Radial variation of gradient of energy-density and two pressures.

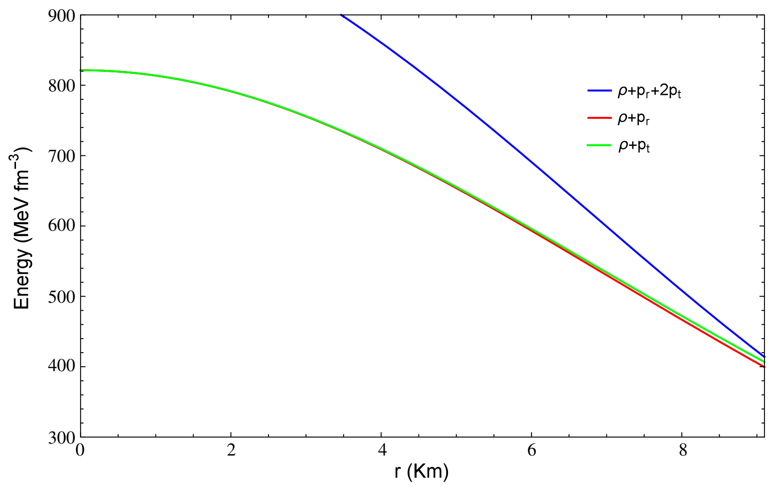

Figure 7. Fulfillment of weak ($\rho + p_r > 0$; $\rho + p_t > 0$) and strong ($\rho + p_r + 2p_t > 0$) energy conditions at the stellar interior.

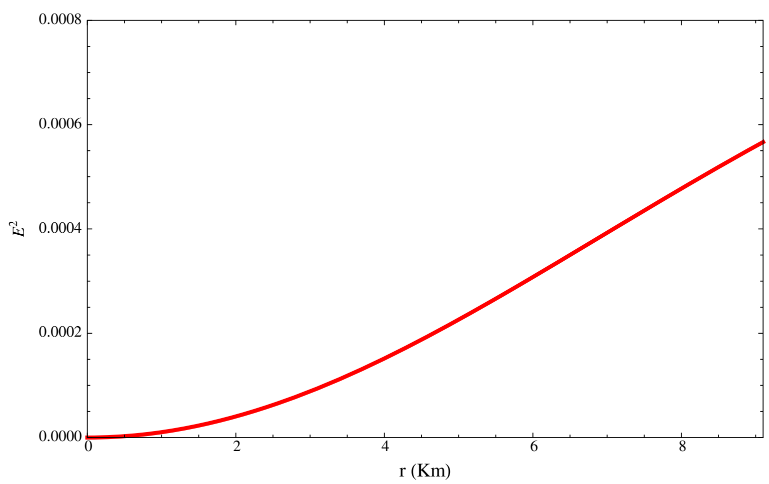

Figure 8. Radial variation of the electric field E.

$$+\left(kx(5+x) - \frac{E^2}{C}(1+x)^3 - \frac{\Delta}{C}(1+x)^3\right) y = 0. \tag{14}$$

To integrate Equation (14), we need to specify E^2 and Δ. Even though a variety of choices for E^2 and Δ can be made, it is important to note that such choices must be physically reasonable and should provide closed form solutions. Keeping these in mind, we first reduce Equation (14) to an integrable form by assuming

$$\frac{E^2}{C} = \frac{(8-7k)x}{(1+x)^2}, \tag{15}$$

$$\frac{\Delta}{C} = \frac{kx(1-a)[7-5a-(15+a)x-2x^2]}{(1+x)^3(a+x)^2}. \tag{16}$$

Note that E^2 vanishes at the centre, remains continuous and bounded in the interior of the star for a wide range of values of the parameter k and takes positive values in the interior of the star for $k < 8/7$. The choice of Δ is also reasonable in the sense that it vanishes at the centre and it ensures that we regain isotropic pressure for $a = 1$. It is noteworthy that when $k = 8/7$, the electric field vanishes and consequently we regain the anisotropic stellar model studied by Maurya *et al.* (2015) by setting $a = 1-\alpha$. Our choices of Equation (15) for the electric field and Equation (16) for the anisotropic factor provide a more general class of charged anisotropic relativistic spheres with desirable physical features.

Substitution of Equations (15) and (16) in (14) yields

$$4(1+x)[1+(2-3k)x+(1-k)x^2]\ddot{y} + 2k(x-3)\dot{y} + (8(k-1)x + 4(3k-2) + \frac{k(1-a)[2x^2+(15+a)x+5a-7]}{(a+x)^2}\Bigg) y = 0, \tag{17}$$

which is the master equation for the system.

4. Exact models

Closed form solutions of equation (17) is difficult to obtain. However, the equation permits a particular solution

$$y_1 = (a+x)^2.$$

Making use of this particular solution, the general solution of (14) can be written using the order reduction method of second order differential equations in the form

$$y_x = y_1\left[P + S\left(\frac{[c_1 + c_2(a+x) + c_3 y_1]f_x}{(a+x)^3} + w_x\right)\right], \tag{18}$$

where

$$f_x = \sqrt{1 + (2-3k)x + (1-k)x^2}, \tag{19}$$

$$w_x = \frac{c_4}{\sqrt{c_5}} \log\left[\frac{w(x)}{(a+x)}\right],$$
$$w(x) = (2(1-a) + 3ak + [2(1-a) - k(3-2a)]x) + 2\sqrt{c_5} f_x, \tag{20}$$

and P and S are arbitrary constants with

$$c_1 = -\frac{1-a}{3[1 - a(2-3k) + a^2(1-k)]},$$

$$c_2 = \frac{4 - 15k - a(8-7k) + 4a^2(1-k)}{12[1 - a(2-3k) + a^2(1-k)]^2},$$

$$c_3 = -\frac{1}{24[1 - a(2-3k) + a^2(1-k)]^3}\left[8 - 110k + 135k^2 - 3a(8 - 44k + 35k^2) + 2a^2(12 - 19k + 7k^2) - 8a^3(1-k)^2\right],$$

$$c_4 = -\frac{1}{16[1 - a(2-3k) + a^2(1-k)]^3}\left[k[56 - 180k + 135k^2 - a(104 - 240k + 135k^2) + 4a^2(10 - 19k + 9k^2) + 8a^3(1-k)^2]\right],$$

$$c_5 = 1 - a(2-3k) + a^2(1-k).$$

Consequently, we write the exact solution to the Einstein–Maxwell system in the form

$$e^{2\lambda} = \frac{(1+x)^2}{1 + (2-3k)x + (1-k)x^2}, \tag{21}$$

$$e^{2\mu} = A^2 y_1^2\left[P + S\left(\frac{[c_1 + c_2(a+x) + c_3 y_1]f_x}{(a+x)^3} + w(x)\right)\right]^2, \tag{22}$$

$$\frac{\rho}{C} = \frac{18k + (11k-8)x + (9k-8)x^2}{2(1+x)^3}, \tag{23}$$

$$\frac{p_r}{C} = \frac{4[1 + (2-3k)x + (1-k)x^2]}{(1+x)^2}\left(\frac{y_{px}(a+x)^3 + 2y_x}{(a+x)y_x}\right) - \frac{6k - (8-9k)x}{2(1+x)^2}, \tag{24}$$

$$\frac{p_t}{C} = \frac{4[1 + (2-3k)x + (1-k)x^2]}{(1+x)^2}\left(\frac{y_{px}(a+x)^3 + 2y_x}{(a+x)y_x}\right) - \frac{6k - (8-9k)x}{2(1+x)^2} + \frac{kx(1-a)[7 - 5a - (15+a)x - 2x^2]}{(1+x)^3(a+x)^2}, \tag{25}$$

$$\frac{E^2}{C} = \frac{(8-7k)x}{(1+x)^2}, \tag{26}$$

$$\frac{\Delta}{C} = \frac{kx(1-a)[7 - 5a - (15+a)x - 2x^2]}{(1+x)^3(a+x)^2}, \tag{27}$$

$$\sigma^2 = \frac{C^2(8-7k)(3+x)^2}{(1+x)^6}[1 + (2-3k)x + (1-k)x^2], \tag{28}$$

in terms of elementary functions, where

$$y_{px} = \frac{S(1+x)}{(a+x)^4\sqrt{1 + (2-3k)x + (1-k)x^2}}. \tag{29}$$

The mass function takes the following form

$$m(r) = \frac{C}{4}\left[\frac{(9k-8)r}{C} - \frac{4kr}{C(1+Cr^2)^2} + \frac{(11k-8)r}{2C(1+Cr^2)} - \frac{3(7k-8)\arctan\sqrt{C}r}{2C^{\frac{3}{2}}}\right], \tag{30}$$

with $C > 0$.

Note that the choice of the metric function (Equation 13) together with the electric field (Equation 15) and the anisotropic factor (Equation 16) have not been considered previously for the modelling of charged anisotropic stars. For $k \neq 8/7$, Equation (17) yields $E^2 \neq 0$, which suggests that the solution can be used to model an anisotropic sphere that is always charged. This feature is also shared by the Hansraj and Maharaj (2006) charged stellar model which is a generalization of the stellar model developed previously by Finch and Skea (1989). The Finch and Skea solution has been widely applied as a relativistic model for neutral stars with a superdense matter. The quark stellar models of Mak and Harko (2004), Komathiraj and Maharaj (2007a, b)

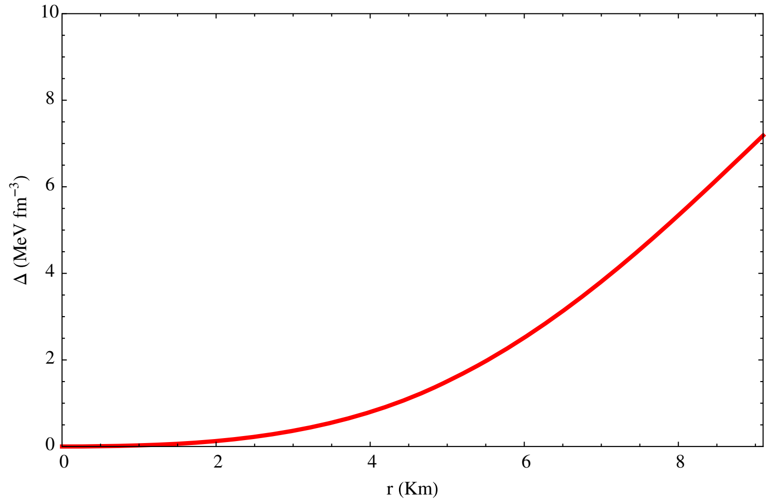

Figure 9. Radial variation of the anisotropy factor Δ.

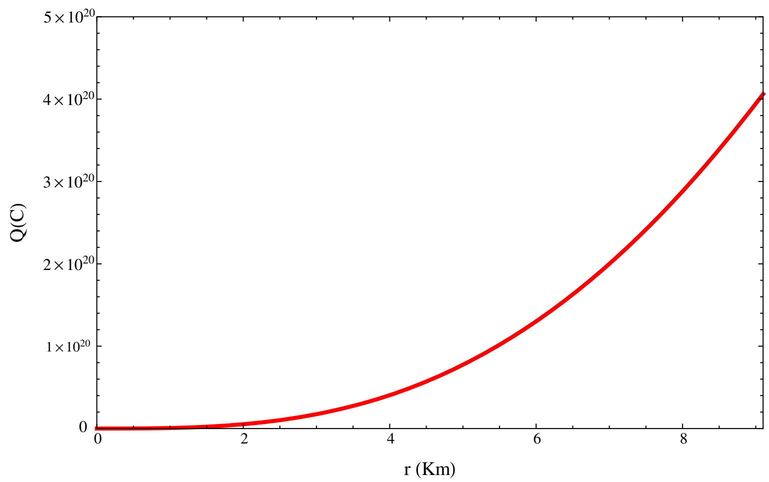

Figure 10. Distribution of the total charge Q.

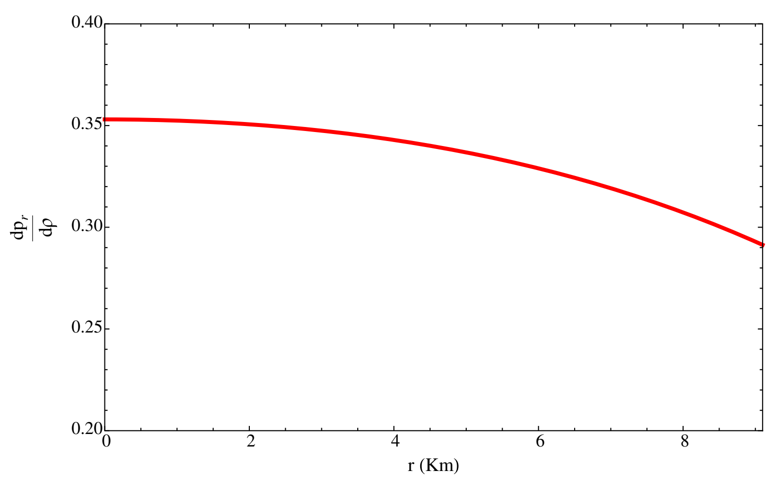

Figure 11. Variation of radial sound speed at the stellar interior.

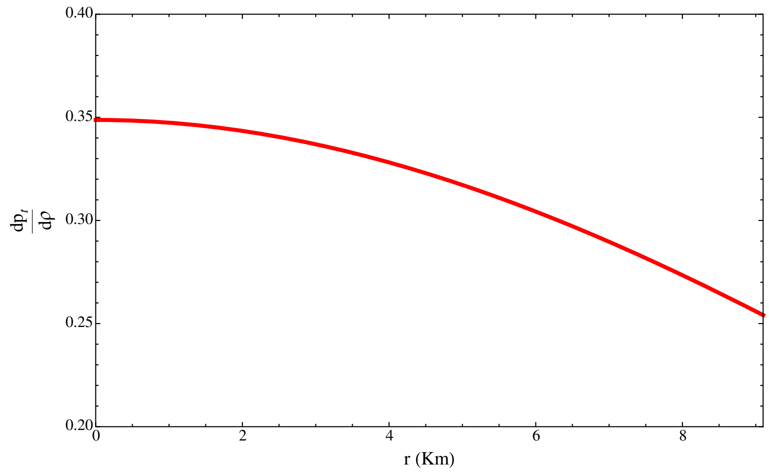

Figure 12. Variation of transverse sound speed at the stellar interior.

and Maharaj *et al.* (2014) also exhibit this property. For particular values of our model parameters, it is possible to regain the uncharged, anisotropic and isotropic models, as shown below.

4.1 $k = 8/7$, $a = 1 - \alpha$

By setting $k = 8/7$ and $a = 1 - \alpha$, the gravitational potentials are obtained as

$$e^{2\lambda} = \frac{7 + 14Cr^2 + 7C^2r^4}{7 - 10Cr^2 - C^2r^4}, \tag{31}$$

$$e^{2\mu} = c_{\alpha r}^2 \left[p - s\left(\frac{[c_{\alpha 1} + c_{\alpha 2}(1 - \alpha + Cr^2) + c_{\alpha 3}c_{\alpha r}]f_r}{(1 - \alpha + Cr^2)^3} + w_r\right)\right]^2, \tag{32}$$

where

$$f_r = \sqrt{c_{\alpha 5} - 2(4 + \alpha)(1 - \alpha + Cr^2) - c_{\alpha r}}, \tag{33}$$

$$w_r = \frac{c_{\alpha 4}}{\sqrt{c_{\alpha 5}}} \times \log\left[\frac{\{c_{\alpha 5} - (4 + \alpha)(1 - \alpha + Cr^2)\} + \sqrt{c_{\alpha 5}}f_r}{(1 - \alpha + Cr^2)c_{\alpha 5}}\right], \tag{34}$$

and $p = P - S\frac{c_{\alpha 4}\sqrt{7}}{\sqrt{c_{\alpha 5}}}\log\left(\frac{2c_{\alpha 5}}{7}\right)$, $s = S\sqrt{7}$ are new constants with

$$c_{\alpha r} = (1 - \alpha + Cr^2)^2,$$

$$c_{\alpha 1} = -\frac{c_1(a = 1 - \alpha,\ k = 8/7)}{7} = \frac{\alpha}{3(16 - 8\alpha - \alpha^2)},$$

$$c_{\alpha 2} = -\frac{c_2(a = 1 - \alpha,\ k = 8/7)}{7} = \frac{24 - 2\alpha + \alpha^2}{3(16 - 8\alpha - \alpha^2)^2},$$

$$c_{\alpha 3} = -\frac{c_3(a = 1 - \alpha,\ k = 8/7)}{7} = \frac{288 + 80\alpha - 10\alpha^2 + \alpha^3}{3(16 - 8\alpha - \alpha^2)^3},$$

$$c_{\alpha 4} = -c_4(a = 1 - \alpha,\ k = 8/7) = \frac{1536 + 384\alpha + 48\alpha^2 - 12\alpha^3}{3(16 - 8\alpha - \alpha^2)^3},$$

$$c_{\alpha 5} = 7c_5\,(a = 1 - \alpha,\ k = 8/7) = 16 - 8\alpha - \alpha^2.$$

This particular solution corresponds to the uncharged anisotropic model of Maurya *et al.* (2015). Note that in our solution we have corrected a misprint in the result of $c_{\alpha 4}$.

4.2 $k = 8/7$, $a = 1$

If we set $k = 8/7$ and $a = 1$ and follow the procedure outlined earlier, we obtain

$$e^{2\lambda} = \frac{7 + 14Cr^2 + 7C^2r^4}{7 - 10Cr^2 - C^2r^4}, \tag{35}$$

$$e^{2\mu} = (1 + Cr^2)^4 \times\left[l + m\left(\frac{(7 + 3Cr^2)\sqrt{7 - 10Cr^2 - C^2r^4}}{4(1 + Cr^2)^2} + \log\left[\frac{(3 - Cr^2) + \sqrt{7 - 10Cr^2 - C^2r^4}}{1 + Cr^2}\right]\right)\right]^2, \tag{36}$$

where we have set $l = P - \log(8/7)$ and $m = S$. Alternatively, we can obtain these results directly by setting $\alpha = 0$ in Case I. The exact solution thus obtained is the isotropic uncharged model of Durgapal and Fuloria (1985). Note that the Durgapal and Fuloria (1985) model satisfies all the physical requirements of an isolated spherically symmetric stellar body.

5. Physical viability

Let us now analyse the physical viability of the class of solutions obtained in this paper. We need to consider only those values of k for which the energy density ρ, the radial pressure p_r and the tangential pressure p_t are positive. The choice of k must ensure that the gravitational potential $e^{2\lambda}$ remains positive; the other potential $e^{2\mu}$ is necessarily positive.

We note that

$$e^{2\lambda}(r=0) = 1, \quad (e^{2\lambda})'(r=0) = 0,$$

$$e^{2\mu}(r=0) = A^2a^4\left[P + S\left(\frac{\{c_1 + c_2a + c_3a^2\}}{a^3} + \frac{c_4}{\sqrt{c_5}}\log\left[\frac{\{2(1-a)+3ak\} + 2\sqrt{c_5}}{a}\right]\right)\right]^2,$$

$$(e^{2\mu})'(r=0) = 0.$$

Obviously, the gravitational potentials are regular at the origin. Using Equation (24), we obtain the central density $\rho_0 = \rho(r=0) = 9kC$, which implies that we must have $k > 0$. Using Equation (25), we have

$$p_r(r=0) = p_t(r=0) = \frac{4SC}{a^2y(r=0)} + \frac{8C}{a} - 3kC > 0.$$

This immediately provides the bound

$$\frac{P}{S} < \frac{4}{a^3(3ak-8)} - \frac{\{c_1 + c_2a + c_3a^2\}}{a^3} - \frac{c_4}{\sqrt{c_5}}\log\left[\frac{\{2(1-a)+3ak\} + 2\sqrt{c_5}}{a}\right]. \tag{37}$$

The radial pressure and the tangential pressure will be positive if we choose our model parameters in such a manner that the condition (37) is satisfied.

For a charged and anisotropic system, the electric field E^2 should be finite and positive. Hence, using (26), we must have $0 < k < 8/7$.

For a realistic star of finite radius, the radial pressure should also vanish at some finite radial distance $r = R$ which yields

$$\frac{P}{S} = \frac{8(1+CR^2)\sqrt{1+(2-3k)CR^2+(1-k)C^2R^4}}{(a+CR^2)^3 l(CR^2)} - f(CR^2), \tag{38}$$

where

$$f(CR^2) = w(CR^2) + \frac{\{c_1 + c_2(a+CR^2) + c_3(a+CR^2)^2\}}{(a+CR^2)^3} \times \sqrt{1+(2-3k)CR^2+(1-k)C^2R^4},$$

$$l(CR^2) = (a+CR^2)[6k-(8-9k)CR^2] - 16[1+(2-3k)CR^2+(1-k)C^2R^4],$$

with $w(CR^2) = w_x(x = CR^2)$ given in (21).

The solution of the Einstein–Maxwell system for $r > R$ is given by the Reissner–Nordström metric

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{39}$$

where $M = m(R)$ and $Q = E(R)R^2$ are the total mass and charge of the star. Matching the line element (Equation 1) with Equation (39) across the boundary R, we have

$$\left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right)^{-1} = \frac{(1+CR^2)^2}{1+(2-3k)CR^2+(1-k)C^2R^4},$$

$$1 - \frac{2M}{R} + \frac{Q^2}{R^2} = A^2(a+CR^2)^4\left[P + Sf(CR^2)\right]^2.$$

These matching conditions help us to determine the constant A as

$$A = \frac{\sqrt{1+(2-3k)CR^2+(1-k)C^2R^4}}{(1+CR^2)(a+CR^2)^2[P+Sf(CR^2)]}. \tag{40}$$

Using Equation (24), we obtain the gradient of the energy density

$$\frac{d\rho}{dr} = -\frac{C^2r[(8+43k) + 4kCr^2 + (9k-8)C^2r^4]}{(1+Cr^2)^4}.$$

Therefore, the energy density will decrease monotonically from the centre towards the boundary if the following condition is satisfied

$$8 + 43k + 4kCr^2 + (9k-8)C^2r^4 > 0. \tag{41}$$

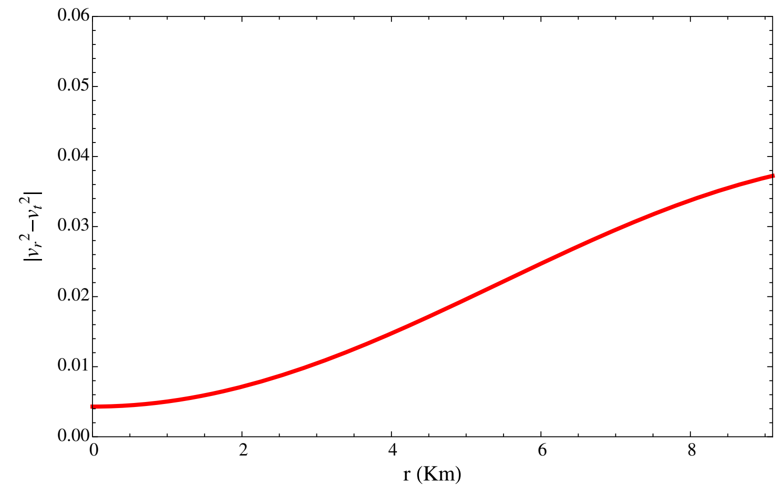

Figure 13. Variation of stability factor $v_r^2 - v_t^2$.

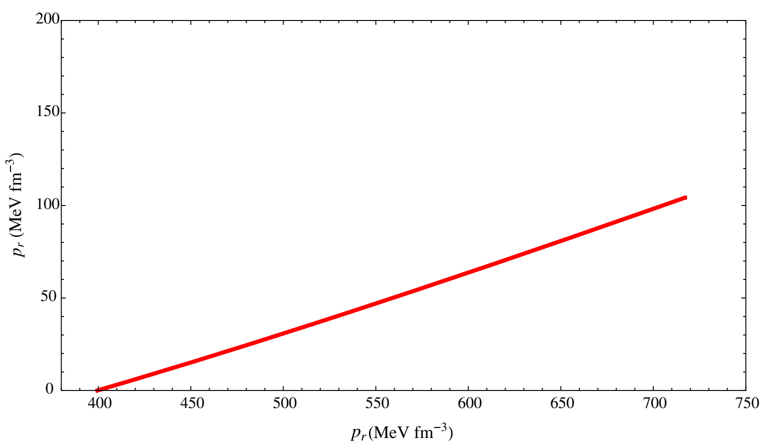

Figure 14. Equation of state $p_r = p_r(\rho)$.

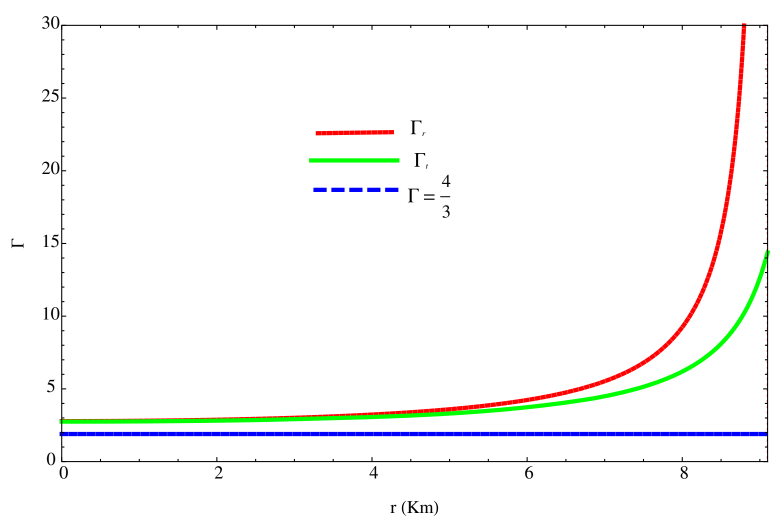

Figure 15. Radial variation of adiabatic index.

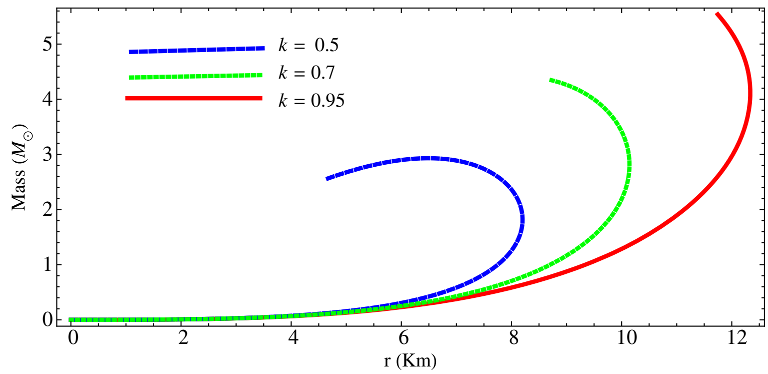

Figure 16. Mass-radius relationship for assumed surface density $\rho_R = 4.7 \times 10^{14}$ gm cm^{-3}.

Using Equations (25) and (26), we note that the gradient of radial pressure $\frac{dp_r}{dr}$ and the tangential pressure $\frac{dp_t}{dr}$ respectively will be negative if we have

$$\frac{dp_r}{dr} = \frac{C^2 r}{(1+Cr^2)^3}\big[8(1+Cr^2)[1+(2-3k)Cr^2 +(1-k)C^2r^4](y_{r1}-y_{r2}-y_{r3}) +[8+3k-(8-9k)Cr^2]\big] < 0, \quad (42)$$

$$\frac{dp_t}{dr} = \frac{C^2 r}{(1+Cr^2)^3}\big[8(1+Cr^2)[1+(2-3k)Cr^2 +(1-k)C^2r^4](y_{r1}-y_{r2}-y_{r3}) + [8+3k-(8-9k)Cr^2] - y_{r4}\big] < 0, \quad (43)$$

where

$$y_{r1} = \frac{2}{(a+Cr^2)^2},$$

$$y_{r2} = \left[\frac{2}{a+Cr^2} + \frac{(a+Cr^2)^2 y_{pr}}{y_r}\right]^2,$$

$$y_{r3} = \frac{k(3-Cr^2)}{2(1+Cr^2)[1+(2-3k)Cr^2+(1-k)C^2r^4]} \times \left[\frac{4}{a+Cr^2} + \frac{(a+Cr^2)^2 y_{pr}}{y_r}\right],$$

$$y_{r4} = \frac{k(1-a)[7-5a-2(a+15)Cr^2-6C^2r^4]}{(a+Cr^2)^2} - \frac{1}{(1+Cr^2)(a+Cr^2)^3} \big[kCr^2(1-a)(2+3a+5Cr^2) \times (7-5a-(15+a)Cr^2-2C^2r^4)\big],$$

with $y_r = y_x(x = Cr^2)$ given in Equation (19) and $y_{pr} = y_{px}(x = Cr^2)$ in Equation (30).

To fulfill the causality conditions $0 < \frac{dp_r}{d\rho} < 1$ and $0 < \frac{dp_t}{d\rho} < 1$, we must have

$$0 < \frac{(1+Cr^2)}{[(8+43k)+4kCr^2-(8-9k)C^2r^4]}\big[8(1 +Cr^2) \times [(k-1)C^2r^4+(3k-2)Cr^2-1](y_{r1} -y_{r2}-y_{r3}) - [8+3k-(8-9k)Cr^2]\big] < 1, \quad (44)$$

and

$$0 < \frac{(1+Cr^2)}{[(8+43k)+4kCr^2-(8-9k)C^2r^4]}\big[8(1 +Cr^2) \times [(k-1)C^2r^4+(3-2k)Cr^2-1](y_{r1} -y_{r2}-y_{r3}) - [8+3k-(8-9k)Cr^2] - y_{r4}\big] < 1,$$

throughout the interior of the star.

By providing the necessary bounds on the model parameters, we are now in a position to examine the physical viability of the solution. To illustrate that our model can describe realistic stars, we consider the pulsar $4U1820-30$ whose estimated mass and radius are $M = 1.58\ M_\odot$ and $b = 9.1$ km, respectively as cited by Gangopadhyay *et al.* (2013). Using these values as input parameters, the boundary conditions have been utilized to determine the constants. Note that the model has six parameters k, a, P, S, A and C with three boundary conditions. Accordingly, we choose three free parameters as $k = 0.95$, $a = 1.5$ and $P = 0.1$. The other parameters are then obtained as $C = 0.0027671$, $A = 1.89558$ and $S = -0.215221$. Note that the set of values thus obtained are consistent with the bounds discussed earlier. Values of the physical quantities, subsequently obtained by plugging in the values of G and c in appropriate places, are shown in Table 1. We illustrate graphically physical behaviour of the model in Figures 1–15. Figures 1 and 2 show that the gravitational potentials are continuous, regular and well-behaved in the interior of the star. Figure 3 shows that the energy density is positive, finite and decreases radially outward from its maximum value at the centre. Variation of radial and tangential pressures have been shown in Figures 4 and 5, respectively. In Figure 6, the gradients density and two pressures have been plotted which remain negative throughout the stellar configuration. Figure 7 shows that the weak energy conditions $\rho + p_r > 0$, $\rho + p_t > 0$ as well as the strong energy condition $\rho + p_r + 2p_t > 0$ are satisfied throughout the stellar configuration. In Figures 8 and 9, the respective behaviour of the electric field E and the anisotropic factor Δ have been shown. We note that E is positive and increases from the centre t o the boundary. The anisotropic factor Δ is positive and monotonically increases from the centre until it attains a maximum value at the boundary of the stellar object. This profile is similar to that obtained by Maurya and Maharaj (2017). Radial variation of total charge Q has been shown in Figure 10. In Figures 11 and 12, the sound speed in radial and transverse directions have been plotted which confirms that the causality condition is not violated in this model as pointed out by Delgaty and Lake (1998). For an anisotropic fluid sphere, a potentially stable configuration is ensured if we have $0 \le v_r^2 - v_t^2 \le 1$ (Abreu *et al.* 2007). Fulfillment of this condition is shown in Figure 13. Even though no barotropic EOS has been utilized in developing our model, in Figure 14, we have shown the $p_r = p_r(\rho)$ relationship which for our particular case turns out to be linear. It should be stressed here that stellar modelling is usually done by prescribing an EOS for the matter composition of the star. For example, Sharma and Maharaj (2007) have developed a stellar model where a linear EOS was assumed a priori. In the absence of adequate knowledge about the matter composition vis-a-vis EOS in the extremely high density regime, alternative approaches are often adopted to get a gross understanding about the physical behaviour of a compact star. Even though choice of an EOS essentially depends on the thermodynamic relationship between the energy-density ρ and the pressure p, in such a case, it is possible to generate a barotropic EOS from the knowledge of radial dependence of these quantities. By adopting this approach, Sharma and Ratanpal (2013) have demonstrated that the class of solution developed by them admits a quadratic EOS. In a series of papers, Maurya *et al.* (2017a, b) and Maurya *et al.* (2019a, b) have successfully generated barotropic EOS $p = p(\rho)$ making use of the solutions developed by them. However, due to involved nature of the mathetical expressions, it was not possible for us to generate an approximated EOS from the solution developed in this paper. Nevertheless, graphical representation shows that the EOS for the assumed configuration is almost linear.

Table 1. Values of central density $\rho(r = 0)$, surface density $\rho(r = R)$ and central pressure $p_r(r = 0)$ for a given stellar configuration of mass $M = 1.58\ M_\odot$ and radius $b = 9.1$ km.

M ($M_\odot$)	R (km)	$\rho(r=0)$ (MeV/fm^3)	$\rho(r=R)$ (MeV/fm^3)	$p_r(r=0)$ (MeV/fm^3)
1.58	9.1	717.13	104.23	399.51

The model parameters are $k = 0.95$, $a = 1.5$, $P = 0.1$, $A = 1.896$, $S = -0.215$ and $C = 0.0028$.

We have calculated the adiabatic index by using the relation

$$\Gamma = \frac{\rho + p_r}{p_r}\frac{dp_r}{d\rho},$$

and shown in Figure 15 that it remains greater than $4/3$ as expected. For an assumed value of the surface density ($\rho(r = R) = 4.7 \times 10^{14}$ gm cm^{-3}), we have also obtained the mass-radius ($M - R$) relationship for different values of model parameter k as shown in Figure 16. The figure clearly illustrates the dependence of the parameter k on the mass-radius relationship of a star. In other words, the value of k can be fixed by using the $M - R$ relationship.

6. Discussion

We have developed a static spherically symmetric stellar model in the presence of an electric field as well as anisotropy. In this model, the gravitational potential and matter variables are continuous and well behaved in the stellar interior. The solutions are expressed in terms of elementary functions which facilitate its physical study. An interesting feature of the model is that the choice of the parameter k can be utilized to fine-tune the mass-radius relation of a stellar body. It is observed that, for a given surface density, the higher value of k provides more mass within a given radius in this model. It is noteworthy that the parameter k is linked to the electromagnetic field and anisotropy of the composition. Moreover, it is to be stressed that by suitable adjustment of the model parameters in our model, it is possible to regain the uncharged stellar model of Maurya *et al.* (2015) and the anisotropic stellar model of Durgapal and Fuloria (1985). A different choice of the electric field, would perhaps enable us to regain other previously known charged stellar solutions in the presence or absence of anisotropic pressure. A paper in this direction is under preparation.

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