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Accelerating universe with variable EoS parameter of dark energy in Brans–Dicke theory of gravitation

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Abstract. We have investigated the spatially homogeneous and isotropic Friedmann–Robertson–Walker (FRW) universe filled with barotropic fluid and dark energy in the framework of the Brans–Dicke theory of gravitation. Here we have discussed three models: (i) law of variation for Hubble's parameter, which leads to a constant value of deceleration parameter, (ii) hybrid expansion law model, and (iii) special form of deceleration parameter model. We have found that among all these derived models, the most suitable standard cosmological model according to the recent cosmological observations is the model with special form of deceleration parameter.

Keywords. FRW space-time—dark energy—statefinder parameters—Brans–Dicke theory.

1. Introduction

Many observational evidences such as data from type Ia supernovae (Riess *et al.* 1998; Perlmutter *et al.* 1999a; 2003; de Bernardis *et al.* 2000), Cosmic Microwave Background (CMB) (Spergel *et al.* 2003, 2007) and Sloan Digital Sky Survey (SDSS) (Tegmark *et al.* 2004a, b) have suggested to accept the fact that the universe is currently experiencing a phase of accelerated expansion. This expansion is due to unknown energy having negative pressure called dark energy (DE). The DE equation of state is $p = \omega_D \rho$, where $\omega_D(<0)$ is not necessarily constant. There are many candidates of DE such as cosmological constant Λ ($\omega_D = -1$) (Carroll 2001; Perlmutter *et al.* 2003; Astier *et al.* 2006), quintessence (Wetterich 1988; Ratra & Peebles 1988), K-essence (Chiba *et al.* 2000; Armendariz-Picon *et al.* 2001), phantom (Caldwell *et al.* 2003; Nojiri *et al.* 2006), quintom (Feng *et al.* 2005), tachyon (Padmanabhan 2002; Bagla *et al.* 2003; Guo & Zhang 2004; Copeland *et al.* 2005), holographic DE (Li 2004; Wang *et al.* 2005; Makarenko & Myagky 2018; Santhi *et al.* 2018), agegraphic DE (Cai 2007; Wei & Cai 2008), two fluid DE (Chirde & Shekh 2016; Katore & Kapse 2018), anisotropic DE (Akarsu & Kilinc 2010; Katore & Sancheti 2011; Katore & Hatkar 2015; Pawar & Solanke 2014; Mahanta & Sharma 2017) and many others.

In recent years, attention has been paid to the so-called 'scalar-tensor gravity', as the Einstein theory of gravity is having some problems in finding gravity accurately on all scales. One of the problems concerning general relativity was that it could not describe the accelerated expansion of the universe accurately (Perlmutter *et al.* 1999a, b; Riess *et al.* 2004). Besides, it is inconsistent with Mach's principle. The scalar tensor theory by Brans and Dicke (1961) accommodates the Mach's principle and it can pass the experimental tests from the solar system (Bertotti *et al.* 2003) and provide an explanation for the accelerated expansion of the universe (Mathiazhagan & Johri 1984; La & Steinhardt 1989; Das & Banerjee 2008). Recently, Reddy and Lakshmi (2015), Rao and Prasanthi (2016), Singh and Dewri (2016) and Naidu *et al.* (2018) have studied the cosmological models in the Brans–Dicke (BD) theory of gravitation.

The Friedmann–Robertson–Walker (FRW) cosmological models play an important role in cosmology. These models are established on the basis of isotropy and homogeneity of the universe. The FRW universe with two fluid DE has been extensively studied by several authors. Zang (2005) investigated an interacting two-fluid scenario for quintom DE. The tachyon cosmology in interacting and non-interacting cases in non-flat FRW universe was studied by Setare *et al.* (2009). An interacting and non-interacting two-fluid scenario for DE with constant deceleration parameter has been studied by Pradhan *et al.* (2011). An interacting two-fluid scenario for DE has been investigated by Amirhashchi *et al.* (2011a). Amirhashchi *et al.* (2011b) proposed an interacting and non-interacting two-fluid scenario for DE models with time-dependent deceleration parameter. Harko and Lobo (2011) investigated the possibility that dark matter is a mixture of two non-interacting perfect fluid with different four velocities and thermodynamics parameter. The two-fluid scenarios for DE models in an FRW universe have been studied by Saha *et al.* (2012). Amirhashchi *et al.* (2013) studied interacting two-fluid viscous DE models in a non-flat universe. Reddy and Santhi (2013) studied the evolution of two-fluid scenario for DE model in scalar-tensor theory of gravitation formulated by Saez and Ballester. Motivated by the above discussions, in this paper, we have considered the spatially homogeneous and isotropic FRW universe filled with barotropic fluid and DE in BD theory of gravitation.

This paper is organized as follows. In Section 2, we have proposed the metric and field equations for FRW space-time in the BD theory. The solutions of the field equations are obtained in Section 3. In Section 4, we have discussed the physical and geometrical properties of the derived models. Finally, conclusions are made in Section 5.

2. The metric and Brans–Dicke field equations

The line element for the homogeneous and isotropic FRW space-time is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2}+r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\Phi^2)\right], \tag{1}$$

where $a\,(t)$ is the scale factor and k is the curvature constant, $k=-1$, $k=0$, $k=+1$ indicate open, flat and closed universe respectively.

The BD field equations are

$$R_{ij} - \frac{1}{2}g_{ij}R - \bar{\omega}\varphi^{-2}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi^{,k}\varphi_{,k}\right) - \varphi^{-1}(\varphi_{i;j} - g_{ij}\varphi^{;k}_{;k}) = \varphi^{-1}(T_{ij}) \tag{2}$$

and

$$\phi^{k}_{;k} = \frac{T}{3+2\bar{\omega}}, \tag{3}$$

where R is the Ricci scalar, R_{ij} is the Ricci tensor, φ is the BD scalar field, $\bar{\omega}$ is the BD parameter and T is the trace of energy momentum tensor T_{ij}. For the line element (1), the field equations (2) and (3) lead to the following set of equations:

$$\frac{p_{\text{tot}}}{\varphi} = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} + \frac{\bar{\omega}}{2}\frac{\dot{\varphi}^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + 2\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}\right), \tag{4}$$

$$\frac{\rho_{\text{tot}}}{\varphi} = 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} - \frac{\bar{\omega}}{2}\frac{\dot{\varphi}^2}{\varphi^2} + 3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} \tag{5}$$

and

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} = \frac{\rho_{\text{tot}} - 3p_{\text{tot}}}{3+2\bar{\omega}}, \tag{6}$$

where $p_{\text{tot}} = p_m + p_D$ and $\rho_{\text{tot}} = \rho_m + \rho_D$. Here p_m and ρ_m are the pressure and energy densities of the barotropic fluid respectively, whereas p_D and ρ_D are the pressure and energy densities of the dark fluid respectively.

The energy conservation equation $T^{ij}_{;j} = 0$ leads to

$$\dot{\rho}_{\text{tot}} + 3\frac{\dot{a}}{a}(\rho_{\text{tot}} + p_{\text{tot}}) = 0. \tag{7}$$

The equation of state (EoS) parameters of the barotropic fluid and dark energy are given by

$$\omega_m = \frac{p_m}{\rho_m} \quad \text{and} \quad \omega_D = \frac{p_D}{\rho_D}. \tag{8}$$

Here the EoS parameter is assumed to be a constant with values 0, $\frac{1}{3}$ and $+1$ for dust, radiation and stiff matter dominated universe respectively. In general, it is a function of time or redshift.

We have assumed that there is no interaction between DE and barotropic fluid. Therefore, the energy conservation equation (7) leads to

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(1+\omega_m)\rho_m = 0 \tag{9}$$

and

$$\dot{\rho}_D + 3\frac{\dot{a}}{a}(1+\omega_D)\rho_D = 0. \tag{10}$$

Using Equation (9), the energy density for barotropic fluid is given by

$$\rho_m = \rho_0 a^{-3(1+\omega_m)}, \tag{11}$$

where ρ_0 is an integrating constant.

3. Solutions of field equations

There are three linearly independent equations (4)–(6) having six variables. The system is thus initially undetermined and we need additional constraint to solve it. In order to solve these field equations, we first assume the power-law relation between scale factor a and scalar field φ (Pimentel 1985; Johri & Desikan 1994) as $\varphi \propto a^{\varepsilon}$, where ε is any integer, which implies that

$$\varphi = \varphi_0 a^{\varepsilon}, \tag{12}$$

where $\varphi_0 > 0$ is the constant of proportionality.

Here we have discussed three models.

3.1 *Model-I* (*Constant deceleration parameter*)

We apply the special law of variation for the generalized Hubble's parameter, which leads to constant value of deceleration parameter q. We consider that the mean Hubble parameter H is related to the scale factor a by the relation (Berman 1983; Agrawal & Pawar 2017; Chirde & Shekh 2018)

$$H = Da^{-m}, \tag{13}$$

where $D > 0$ and $m \geq 0$ are constants.

From Equation (13), we obtain

$$\dot{a} = Da^{-m+1} \tag{14}$$

and

$$\ddot{a} = -D^2(m-1)a^{-2m+1}. \tag{15}$$

Here we obtain two cases, Case-I for $m \neq 0$ and Case-II for $m = 0$.

3.1.1 *Case-I*: *For $m \neq 0$ (i.e. power-law volumetric expansion).*

Integrating Equation (14), we obtain

$$a = (mDt + c_1)^{\frac{1}{m}}, \tag{16}$$

where c_1 is the constant of integration.

Using Equations (13), (14) and (15), the deceleration parameter q is given by

$$q = -\frac{\ddot{a}}{aH^2} = m - 1. \tag{17}$$

The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to the decelerating universe whereas the negative sign indicates the accelerating universe. From Equation (17), it is observed that the deceleration parameter q is negative in power-law model for $0 < m < 1$. This indicates that the universe is accelerating throughout the evolution of the universe. For $m > 1$, the deceleration parameter q is positive, hence the universe is decelerating. For $m = 1$, the deceleration parameter $q = 0$ which corresponds to the expansion with constant speed, which is not consistent with the present observations. Therefore, we restrict the value of m to $0 < m < 1$ in order to get an accelerating universe.

Using Equations (12) and (16), the Brans–Dicke scalar field φ is given by

$$\varphi = \varphi_0(mDt + c_1)^{\frac{\varepsilon}{m}}. \tag{18}$$

From Equations (11) and (16), the energy density for barotropic fluid is obtained as

$$\rho_m = \rho_0(mDt + c_1)^{-\frac{3}{m}(1+\omega_m)}. \tag{19}$$

The energy density for DE can be obtained by using Equations (5) and (16) as

$$\rho_D = \varphi_0(mDt + c_1)^{\frac{\varepsilon}{m}} \left\{3\left(1 - \frac{\bar{\omega}}{6}\varepsilon^2 + \varepsilon\right)(mDt + c_1)^{-2} + 3k(mDt+c_1)^{\frac{-2}{m}}\right\} - \rho_0(mDt + c_1)^{-\frac{3}{m}(1+\omega_m)}. \tag{20}$$

The pressure p_D of DE using Equation (4) is given by

$$p_D = -\varphi_0(mDt + c_1)^{\frac{\varepsilon}{m}} \left\{D^2\left(3 - 2m + \left(1 + \frac{\bar{\omega}}{2}\right)\varepsilon^2 - (m-2)\varepsilon\right) \times (mDt + c_1)^{-2} + k(mDt + c_1)^{\frac{-2}{m}}\right\} - \omega_m \rho_0(mDt + c_1)^{-\frac{3}{m}(1+\omega_m)}. \tag{21}$$

Using Equations (20) and (21), we obtain the EoS parameter ω_D of DE as

$$\omega_D = -\frac{\left\{\begin{array}{l}\varphi_0(mDt + c_1)^{\frac{\varepsilon}{m}}\left\{D^2\left(3 - 2m + \left(1 + \frac{\bar{\omega}}{2}\right)\varepsilon^2 - (m-2)\varepsilon\right)(mDt + c_1)^{-2} + k(mDt + c_1)^{\frac{-2}{m}}\right\} \\ +\omega_m \rho_0(mDt + c_1)^{-\frac{3}{m}(1+\omega_m)}\end{array}\right\}}{\varphi_0(mDt+c_1)^{\frac{\varepsilon}{m}}\left\{3\left(1 - \frac{\bar{\omega}}{6}\varepsilon^2 + \varepsilon\right)(mDt + c_1)^{-2} + 3k(mDt + c_1)^{\frac{-2}{m}}\right\} - \rho_0(mDt + c_1)^{-\frac{3}{m}(1+\omega_m)}}. \tag{22}$$

The barotropic matter energy density parameter Ω_m and dark energy density parameter Ω_D are respectively given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{\rho_0 (mDt + c_1)^{-\frac{3}{m}(1+\omega_m)+2}}{3D^2} \tag{23}$$

and

$$\Omega_D = \frac{\rho_D}{3H^2} = \frac{\left\{ \begin{array}{l} \varphi_0 (mDt + c_1)^{\left(\frac{\varepsilon}{m}+2\right)} \left\{ 3\left(1 - \frac{\bar{\omega}}{6}\varepsilon^2 + \varepsilon\right)(mDt + c_1)^{-2} + 3k(mDt + c_1)^{\frac{-2}{m}} \right\} \\ -\rho_0 (mDt + c_1)^{-\frac{3}{m}(1+\omega_m)+2} \end{array} \right\}}{3D^2}. \tag{24}$$

Using Equations (23) and (24), the total energy density parameter is obtained as

$$\Omega = \Omega_m + \Omega_D = \frac{\varphi_0 (mDt + c_1)^{\left(\frac{\varepsilon}{m}+2\right)} \left\{ 3\left(1 - \frac{\bar{\omega}}{6}\varepsilon^2 + \varepsilon\right)(mDt + c_1)^{-2} + 3k(mDt + c_1)^{\frac{-2}{m}} \right\}}{3D^2}. \tag{25}$$

The distance modulus curve for power-law volumetric expansion model. The distance modulus curve is stated as

$$\mu = 5\log d_L + 25, \tag{26}$$

where d_L is the luminosity distance, which is calculated by

$$d_L = r_1(1+z)a_0. \tag{27}$$

Here z is the redshift parameter, r_1 is the radial co-ordinate and a_0 is the present scale factor.

We assume that $T = mDt + c_1$. Therefore, Equation (8) can be written as

$$a = T^{m_1}, \tag{28}$$

where $m_1 = \frac{1}{m}$.

For determination of the radial co-ordinate r_1, we assume that a phantom emitted by a source with co-ordinate (r, T_1) is received at a time T_0 by an observer located at $r = 0$. Then we determine r_1 from following relation:

$$r_1 = \int_{T_1}^{T_0} \frac{\mathrm{d}T}{a}. \tag{29}$$

Solving Equations (26) to (29), we obtain the expression for distance modulus μ in terms of the redshift parameter z as

$$\mu = 5\log\left(\frac{D}{(1-m_1)H_0}(1+z)^{m_1}((1+z)^{1-m_1} - 1)\right) +25, \tag{30}$$

where H_0 is in unit of km s^{-1} Mpc^{-1}.

3.1.2 *Case-II*: *For $m = 0$ (i.e. exponential volumetric expansion)*. Integrating Equation (14), we obtain

$$a = c_2 e^{Dt}, \tag{31}$$

where $c_2 > 0$ is the constant of integration.

Using Equations (13), (14) and (15), the deceleration parameter q is given by

$$q = -1. \tag{32}$$

The Brans–Dicke scalar field φ is given by

$$\varphi = \varphi_0 (c_2 e^{Dt})^{\varepsilon}. \tag{33}$$

Using Equation (32), the energy density of the barotropic matter, energy density of DE and the pressure of DE are respectively given by

$$\rho_m = \rho_0 (c_2 e^{Dt})^{-3(1+\omega_m)}, \tag{34}$$

$$\rho_D = \varphi_0 (c_2 e^{Dt})^{\varepsilon} \left\{ 3D^2 \left(1 - \frac{\bar{\omega}}{6}\varepsilon^2 + \varepsilon\right) + 3k(c_2 e^{Dt})^{-2} \right\} - \rho_0 (c_2 e^{Dt})^{-3(1+\omega_m)}, \tag{35}$$

$$p_D = -\varphi_0 (c_2 e^{Dt})^{\varepsilon} \left\{ D^2 \left(3 + \frac{\bar{\omega}}{2}\varepsilon^2 + \varepsilon(\varepsilon + 2)\right) + k(c_2 e^{Dt})^{-2} \right\} - \omega_m \rho_0 (c_2 e^{Dt})^{-3(1+\omega_m)}. \tag{36}$$

Using Equations (35) and (36), we obtain the EoS parameter ω_D of DE as

$$\omega_D = -\frac{\varphi_0(c_2 e^{Dt})^{\varepsilon}\left\{D^2\left(3+\frac{\bar{\omega}}{2}\varepsilon^2+\varepsilon(\varepsilon+2)\right)+k(c_2 e^{Dt})^{-2}\right\}+\omega_m\rho_0(c_2 e^{Dt})^{-3(1+\omega_m)}}{\varphi_0(c_2 e^{Dt})^{\varepsilon}\left\{3D^2\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)+3k(c_2 e^{Dt})^{-2}\right\}-\rho_0(c_2 e^{Dt})^{-3(1+\omega_m)}}. \tag{37}$$

The barotropic matter energy density parameter and the dark energy density parameter are respectively given by

$$\Omega_m = \frac{\rho_0(c_2 e^{Dt})^{-3(1+\omega_m)}}{3D^2} \tag{38}$$

and

$$\Omega_D = \frac{\varphi_0(c_2 e^{Dt})^{\varepsilon}\left\{3D^2\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)+3k(c_2 e^{Dt})^{-2}\right\}-\rho_0(c_2 e^{Dt})^{-3(1+\omega_m)}}{3D^2}. \tag{39}$$

Using Equations (38) and (39), the total energy density parameter takes the form

$$\Omega = \frac{\varphi_0(c_2 e^{Dt})^{\varepsilon}\left\{3D^2\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)+3k(c_2 e^{Dt})^{-2}\right\}}{3D^2}. \tag{40}$$

The distance modulus curve for exponential expansion model. We have

$$a = c_2 e^{Dt}. \tag{41}$$

For determination of r_1, we assume that a phantom emitted by a source with co-ordinate (r, t_1) is received at a time t_0 by an observer located at $r = 0$. Then we determine r_1 from the following relation:

$$r_1 = \int_{t_1}^{t_0} \frac{dt}{a}. \tag{42}$$

Solving Equations (27), (28), (41) and (42), we obtain the expression for distance modulus μ in terms of the redshift parameter z as

$$\mu = 5\log\left(\frac{1}{H_0}(1+z)z\right)+25. \tag{43}$$

3.2 *Model-II* (*Hybrid law*)

We use the hybrid expansion law (Akarsu *et al.* 2014) for the scale factor a as

$$a(t) = nt^{\alpha}e^{\beta t}, \tag{44}$$

where n, α and β are positive constants. Equation (44) gives the exponential law when $\alpha = 0$ and the power-law when $\beta = 0$.

The deceleration parameter q is given by

$$q = -1+\frac{\alpha}{(\alpha+\beta t)^2}. \tag{45}$$

The Brans–Dicke scalar field φ is given by

$$\varphi = \varphi_0(nt^{\alpha}e^{\beta t})^{\varepsilon}. \tag{46}$$

Using Equation (44), the energy density of the barotropic matter, energy density of DE and pressure of DE are respectively given by

$$\rho_m = \rho_0(nt^{\alpha}e^{\beta t})^{-3(1+\omega_m)}, \tag{47}$$

$$\rho_D = \varphi_0(nt^{\alpha}e^{\beta t})^{\varepsilon}\left\{3\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)\left(\frac{\alpha+\beta t}{t}\right)^2 + \frac{3k}{(nt^{\alpha}e^{\beta t})^2}\right\} - \rho_0(nt^{\alpha}e^{\beta t})^{-3(1+\omega_m)}, \tag{48}$$

$$p_D = -\varphi_0(nt^{\alpha}e^{\beta t})^{\varepsilon}\left\{\left(3+\left(1+\frac{\bar{\omega}}{2}\right)\varepsilon^2+2\varepsilon\right)\times\left(\frac{\alpha+\beta t}{t}\right)^2-\frac{\alpha(\varepsilon+2)}{t^2}+\frac{k}{(nt^{\alpha}e^{\beta t})^2}\right\} - \omega_m\rho_0(nt^{\alpha}e^{\beta t})^{-3(1+\omega_m)}. \tag{49}$$

Using Equations (48) and (49), we obtain the EoS parameter ω_D of DE as

$$\omega_D = -\frac{\varphi_0(nt^{\alpha}e^{\beta t})^{\varepsilon}\left\{\left(3+\left(1+\frac{\bar{\omega}}{2}\right)\varepsilon^2+2\varepsilon\right)\left(\frac{\alpha+\beta t}{t}\right)^2-\frac{\alpha(\varepsilon+2)}{t^2}+\frac{k}{(nt^{\alpha}e^{\beta t})^2}\right\}+\omega_m\rho_0(nt^{\alpha}e^{\beta t})^{-3(1+\omega_m)}}{\varphi_0(nt^{\alpha}e^{\beta t})^{\varepsilon}\left\{3\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)\left(\frac{\alpha+\beta t}{t}\right)^2+\frac{3k}{(nt^{\alpha}e^{\beta t})^2}\right\}-\rho_0(nt^{\alpha}e^{\beta t})^{-3(1+\omega_m)}}. \quad (50)$$

The expression for barotropic matter energy density parameter and dark energy density parameter are respectively given by

$$\Omega_m = \frac{\rho_0(nt^{\alpha}e^{\beta t})^{-3(1+\omega_m)}}{3\left(\frac{\alpha+\beta t}{t}\right)^2}, \quad (51)$$

and

$$\Omega_D = \frac{\varphi_0(nt^{\alpha}e^{\beta t})^{\varepsilon}\left\{3\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)\left(\frac{\alpha+\beta t}{t}\right)^2+\frac{3k}{(nt^{\alpha}e^{\beta t})^2}\right\}-\rho_0(nt^{\alpha}e^{\beta t})^{-3(1+\omega_m)}}{3\left(\frac{\alpha+\beta t}{t}\right)^2}. \quad (52)$$

Using Equations (51) and (52), the total energy density parameter takes the form

$$\Omega = \frac{\varphi_0(nt^{\alpha}e^{\beta t})^{\varepsilon}\left\{3\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)\left(\frac{\alpha+\beta t}{t}\right)^2+\frac{3k}{(nt^{\alpha}e^{\beta t})^2}\right\}}{3\left(\frac{\alpha+\beta t}{t}\right)^2}. \quad (53)$$

3.3 *Model-III* (*special form of deceleration parameter*)

Here we consider the deceleration parameter form proposed by Singha and Debnath (2009) as

$$q = -\frac{\ddot{a}a}{\dot{a}} = -1 + \frac{\gamma}{1+a^{\gamma}}, \quad (54)$$

where $\gamma > 0$ is a constant.

Solving Equation (54), we obtain

$$a = (\exp(\gamma c_3 t) - 1)^{\frac{1}{\gamma}}, \quad (55)$$

where $c_3 > 0$ is the constant of integration.

The Brans–Dicke scalar field φ is given by

$$\varphi = \varphi_0 (\exp(\gamma c_3 t) - 1)^{\frac{\varepsilon}{\gamma}}. \quad (56)$$

Using Equation (55), the energy density of the barotropic matter, energy density of DE and pressure of DE are respectively given by

$$\rho_m = \rho_0(\exp(\gamma c_3 t) - 1)^{-\frac{3}{\gamma}(1+\omega_m)}, \quad (57)$$

$$\rho_D = \varphi_0(\exp(\gamma c_3 t) - 1)^{\frac{\varepsilon}{\gamma}}\left\{\frac{3c_3^2\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)\exp(2\gamma c_3 t)}{(\exp(\gamma c_3 t)-1)^2} + 3k(\exp(\gamma c_3 t)-1)^{\frac{-2}{\gamma}}\right\} - \rho_0(\exp(\gamma c_3 t) - 1)^{-\frac{3}{\gamma}(1+\omega_m)}, \quad (58)$$

$$p_D = -\varphi_0(\exp(\gamma c_3 t) - 1)^{\frac{\varepsilon}{\gamma}}\left\{\frac{c_3^2\left(3-2\gamma+\frac{\bar{\omega}}{2}\varepsilon^2+(2-\gamma)\varepsilon\right)\exp(2\gamma c_3 t)}{(\exp(\gamma c_3 t)-1)^2} + \frac{2c_3^2\gamma\exp(\gamma c_3 t)}{(\exp(\gamma c_3 t)-1)} + k(\exp(\gamma c_3 t)-1)^{\frac{-2}{\gamma}}\right\} - \omega_m\rho_0(\exp(\gamma c_3 t)-1)^{-\frac{3}{\gamma}(1+\omega_m)}. \quad (59)$$

Using Equations (58) and (59), we obtain the EoS parameter ω_D of DE as

$$\omega_D = -\frac{\left\{\begin{array}{l}\varphi_0(\exp(\gamma c_3 t)-1)^{\frac{\varepsilon}{\gamma}}\left\{\frac{c_3^2\left(3-2\gamma+\frac{\bar{\omega}}{2}\varepsilon^2+(2-\gamma)\varepsilon\right)\exp(2\gamma c_3 t)}{(\exp(\gamma c_3 t)-1)^2}+\frac{2c_3^2\gamma\exp(\gamma c_3 t)}{(\exp(\gamma c_3 t)-1)}+k(\exp(\gamma c_3 t)-1)^{\frac{-2}{\gamma}}\right\}\\ +\omega_m\rho_0(\exp(\gamma c_3 t)-1)^{-\frac{3}{\gamma}(1+\omega_m)}\end{array}\right\}}{\left\{\varphi_0(\exp(\gamma c_3 t)-1)^{\frac{\varepsilon}{\gamma}}\left\{\frac{3c_3^2\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)\exp(2\gamma c_3 t)}{(\exp(\gamma c_3 t)-1)^2}+3k(\exp(\gamma c_3 t)-1)^{\frac{-2}{\gamma}}\right\}-\rho_0(\exp(\gamma c_3 t)-1)^{-\frac{3}{\gamma}(1+\omega_m)}\right\}}. \tag{60}$$

The expression for matter energy density parameter and dark energy density parameter are given by

$$\Omega_m = \frac{\rho_0\,(\exp(\gamma c_3 t)-1)^{2-\frac{3}{\gamma}(1+\omega_m)}}{3c_3^2\exp(2\gamma c_3 t)} \tag{61}$$

and

$$\Omega_D = \frac{1}{3c_3^2\exp(2\gamma c_3 t)}\left\{\begin{array}{l}\varphi_0\,(\exp(\gamma c_3 t)-1)^{2+\frac{\varepsilon}{\gamma}}\left\{\frac{3c_3^2\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)\exp(2\gamma c_3 t)}{(\exp(\gamma c_3 t)-1)^2}+3k\,(\exp(\gamma c_3 t)-1)^{\frac{-2}{\gamma}}\right\}\\ -\rho_0\,(\exp(\gamma c_3 t)-1)^{2-\frac{3}{\gamma}(1+\omega_m)}\end{array}\right\}. \tag{62}$$

Using Equations (61) and (62), the total energy density parameter takes the form

$$\Omega = \frac{\varphi_0\,(\exp(\gamma c_3 t)-1)^{2+\frac{\varepsilon}{\gamma}}}{3c_3^2\exp(2\gamma c_3 t)}\left\{\frac{3c_3^2\left(1-\frac{\bar{\omega}}{6}\varepsilon^2+\varepsilon\right)\exp(2\gamma c_3 t)}{(\exp(\gamma c_3 t)-1)^2}+3k\,(\exp(\gamma c_3 t)-1)^{\frac{-2}{\gamma}}\right\}. \tag{63}$$

4. Results and discussion

The physical and geometrical behaviours of the above models are as follows:

(i) *The deceleration parameter* (q). The evolution of the deceleration parameter for power-law model ($m = 1/2$), exponential model ($m = 0$), hybrid law model ($\alpha = 1, \beta = 1$) and special form of deceleration parameter model ($\gamma = 3/2$) is shown in Figure 1. It is found that the universe has been accelerating throughout the evolution of the universe in the power-law model for $0 < m < 1$ and in the exponential model for $m = 0$, whereas the universe accelerates after an epoch of deceleration in the hybrid law model and special form of deceleration parameter model. For $\gamma = 3/2$, the deceleration parameter q is in the range $-1 \le q \le 0.5$ (shaded region in Figure 1) which matches with the observations (Perlmutter *et al.* 1998, 1999a, 2003; Riess *et al.* 1998, 2004; Tonry *et al.* 2003; Clocchiatti 2006).

(ii) *Energy density of the barotropic fluid* (ρ_m). The evolution of the energy density ρ_m of the barotropic fluid for all models is shown in Figure 2. It has been observed that when $t \to 0$, in the power-law model, $\rho_m \to \rho_0 c_1^{-3/m(1+\omega_m)}$, $a \to c_1^{1/m}$, $p_m \to \rho_0\omega_m c_1^{-3/m(1+\omega_m)}$ and in the exponential model, $\rho_m \to \rho_0 c_2^{-3/m(1+\omega_m)}$, $a \to c_2$, $p_m \to \rho_0\omega_m c_2^{-3/m(1+\omega_m)}$ which signify that there is no Big-Bang type of singularity. When $t \to 0$, in both hybrid law and special form of deceleration parameter models, $\rho_m \to \infty$, $a \to 0$, $p_m \to \infty$ which signify that there is a Big-Bang type of singularity. For all the models, $\rho_m \to 0$, $a \to \infty$, $p_m \to 0$ when $t \to \infty$ which implies that the models reduce to vacuum after very late time t.

(iii) *Equation of state parameter of DE* (ω_D). Figure 3 shows the evolution of EoS parameter of DE for power-law expansion, exponential expansion, hybrid law and special form of deceleration parameter. For all these models, at some finite time, the EoS parameter attains a constant value. In power-law expansion model (for $m = 0.5$), ω_D starts from the phantom region ($\omega_D < -1$) after some finite t attains the value $\omega_D = -1$ (cosmological constant) and then it enters in the quintessence region ($-1 < \omega_D < -1/3$). In case of exponential expansion model, hybrid expansion law model and special form of deceleration parameter model ω_D starts

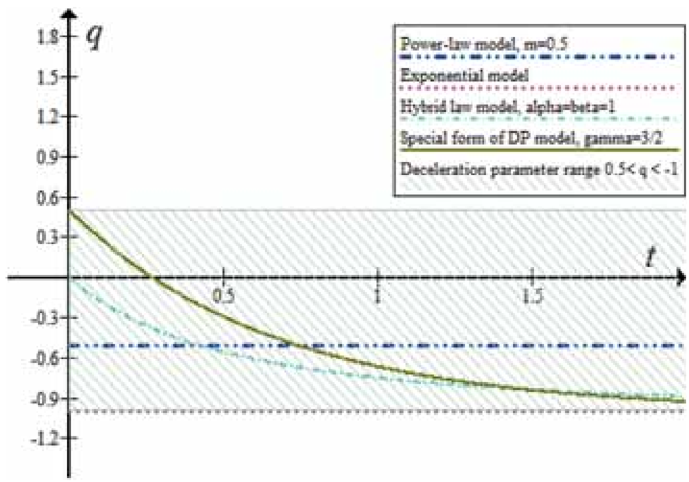

Figure 1. Evolution of the deceleration parameter q.

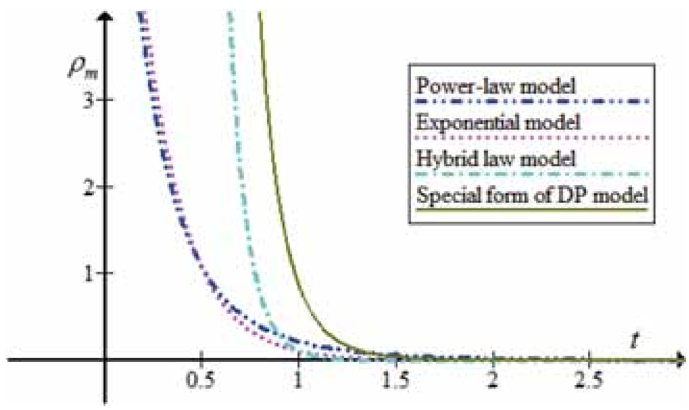

Figure 2. Evolution of energy density ρ_m for $m = 0.5$, $D = c_1 = c_2 = c_3 = n = \alpha = \beta = \gamma = 1$, $\rho_0 = 10$ and $\omega_m = 0$.

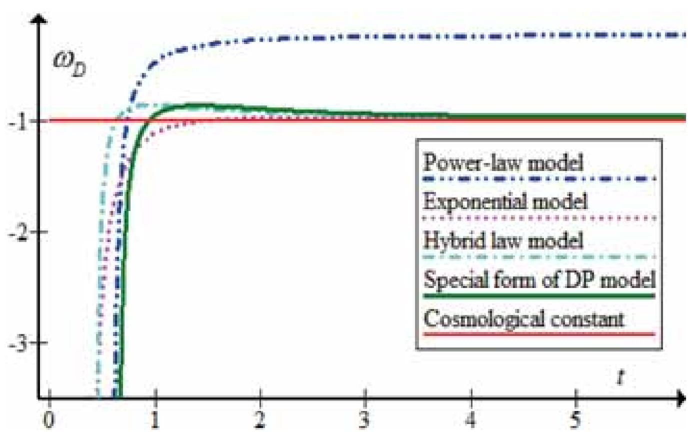

Figure 3. Evolution of DE EoS parameter ω_D for $m = 0.5$, $\varepsilon = 0.132124$, $\phi_0 = \bar{\omega} = D = c_1 = c_2 = c_3 = n = \alpha = \beta = \gamma = 1$, $\rho_0 = 10$, $k = \omega_m = 0$.

from the phantom region ($\omega_D < -1$) after some finite t attains the value $\omega_D \approx -1$ and remains the same for later time. Combining Planck data (Ade 2016) with other astrophysical data including Type Ia supernovae, the equation of state of dark energy is constrained to $\omega_D = -1.006 \pm 0.045$. Hence in our models, the EoS parameter is consistent with these observations.

(*iv*) *Statefinder parameters* (r, s) . A different DE model has emerged to explain the accelerating expansion

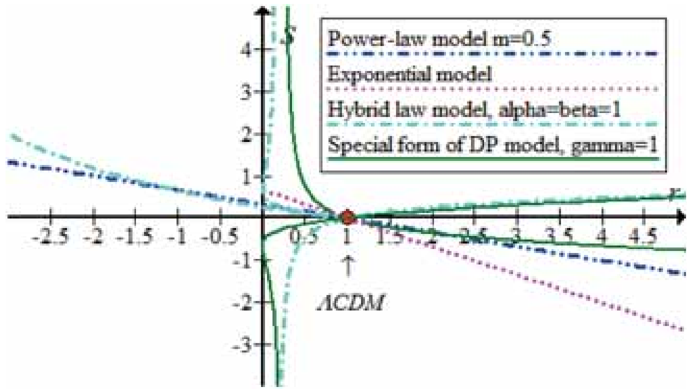

Figure 4. Evolution of statefinder parameters s vs. r.

Table 1. Comparison between the results of observed data and present models.

Redshift (z)	Supernovae Ia (μ)	Power-law model (μ)	Exponential model (μ)
0.014	33.73	34.24	33.60
0.026	35.62	35.59	34.97
0.036	36.39	36.29	35.70
0.040	36.38	36.55	35.95
0.050	37.08	37.02	36.44
0.063	37.67	37.52	36.97
0.079	37.94	38.04	37.50
0.088	38.07	38.26	37.75
0.101	38.73	38.53	38.07
0.160	39.08	39.59	39.19
0.240	40.68	40.52	40.21
0.380	42.02	41.57	41.44
0.430	42.33	41.85	41.79
0.480	42.37	42.10	42.10
0.620	43.11	42.70	42.85
0.740	43.35	43.12	43.39
0.778	43.81	43.24	43.55
0.828	43.59	43.37	43.74
0.886	43.91	43.55	43.96
0.910	44.44	43.61	44.04
0.930	44.61	43.66	44.11
0.949	43.99	43.71	44.18
0.970	44.13	43.76	44.25
0.983	44.13	43.80	44.29
1.056	44.25	43.97	44.53
1.190	44.19	44.26	44.92
1.305	44.51	44.48	45.23
1.340	44.92	44.54	45.32
1.551	45.07	44.99	45.82

of the universe until now. We need a thorough investigation to differentiate these DE models. Sahni *et al.* (2003) introduced the parameter pair $\{r, s\}$, i.e., the so-called 'statefinder'. The statefinder pair $\{r, s\}$ is defined as follows:

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)}$$

The statefinder is a 'geometrical' diagnostic in the sense that it depends upon the expansion factor and hence upon the metric describing space-time. Trajectories in the $r-s$ plane corresponding to different cosmological models exhibit qualitatively different behaviours.

The statefinder parameters r and s for power-law model (i.e., model for $m \neq 0$) are given by

$$r = (m-1)(2m-1) \quad \text{and}$$

$$s = \frac{2\left((1-m)(1-2m)-1\right)}{3(2m-3)}.$$

The statefinder parameters r and s for exponential model (i.e., model for $m = 0$) are given by

$r = 1$ and $s = 0$.

The statefinder parameters r and s for hybrid law model are given by

$$r = 1 + \frac{2\alpha}{(\alpha+\beta t)^3} - \frac{3\alpha}{(\alpha+\beta t)^2} \text{ and}$$

$$s = \frac{2\left(2\alpha - 3\alpha\left(\alpha+\beta t\right)\right)}{\left(2\alpha - 3\left(\alpha+\beta t\right)^2\right)\left(\alpha+\beta t\right)}.$$

The statefinder parameters r and s for special form of deceleration parameter model are given by

$$r = 1 + \frac{\gamma^2 - 3\gamma}{1+a^\gamma} + \frac{\gamma^2}{(1+a^\gamma)^2} \text{ and}$$

$$s = \frac{2\left(\gamma^2 + \left(\gamma^2 - 3\gamma\right)\left(1+a^\gamma\right)\right)}{3\left(1+a^\gamma\right)\left(2\gamma - 3\left(1+a^\gamma\right)\right)}.$$

Figure 4 shows the evolving trajectory of this scenario in the $r-s$ plane which is quite different from those of

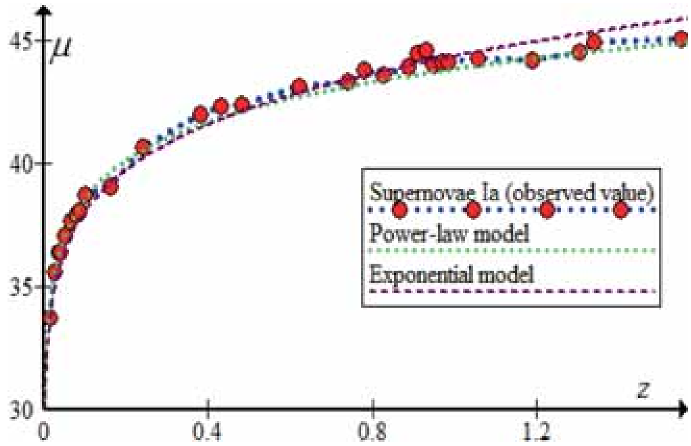

Figure 5. The plot of distance modulus μ vs. redshift z.

the other DE models. We hope that precise observations in future can determine these statefinder parameters and consequently explore the nature of DE.

(v) *The distance modulus curve for Model-I.* In the present analysis, we analysed 29 data sets out of the recently released 38 data sets of supernova Ia in the range $0.014 < z < 1.551$. The comparison between observed distance modulus μ and calculated distance modulus μ are shown in Table 1 and Figure 5. It is found that the distance modulus curve of the derived Model-I (power-law and exponential models) fit well with the observational data (see Table 1 and Figure 5) and which are physically realistic.

5. Conclusion

In this article, we have investigated the spatially homogeneous and isotropic Friedmann–Robertson–Walker (FRW) universe filled with barotropic fluid and dark energy in the framework of the Brans–Dicke theory of gravitation. The field equations have been solved by using the following assumptions: (i) law of variation for Hubble's parameter, (ii) hybrid expansion law, and (iii) special form of the deceleration parameter. Some physical and geometrical behaviour of the models are also discussed. It is found that there is no Big-Bang type of singularity for constant deceleration parameter models whereas there is a Big-Bang type of singularity in both hybrid law and special form of deceleration parameter models. It is observed that the EoS parameter ω_D is consistent with the observations made by the Planck data (Ade 2016) with Type Ia supernovae which restricts that it should be $\omega_D = -1.006 \pm 0.045$. If $\varphi \to 1$, the power-law model reduced to results of Pradhan (2011) and the hybrid expansion law model reduced to the results of Amirhashchi *et al.* (2011a) (for particular values of constants). Statefinder diagnostic is applied to each model in order to distinguish our DE model with other existing DE models. Among all the derived models, the best suitable standard cosmological model according to recent cosmological observations is the model with special form of deceleration parameter.

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