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Theoretical Σ-D relations for shell-type galactic supernova remnants

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Abstract. Relations between radio surface brightness (Σ) and diameter (D) of supernova remnants (SNRs) are important in astronomy. In this paper, following the work Duric and Seaquist (ApJ 301:308, 1986) at adiabatic phase, we carefully investigate shell-type supernova remnants at radiative phase, and obtain theoretical Σ-D relation at radiative phase of shell-type supernova remnants at 1 GHz. By using these theoretical Σ-D relations at adiabatic phase and radiative phase, we also roughly determine phases of some supernova remnant from observation data.

Keywords. Shell-type supernova remnants—radiative phase—surface brightness—Σ-D relation.

1. Introduction

Relations between radio surface brightness (Σ) and diameter (D) of supernova remnants (SNRs) are important in astronomy, and are usually used to determine distance of a SNR (Poveda and Woltjer 1968; Clark and Caswell 1976; Lozinskaya 1981; Huang and Thaddeus 1985; Duric and Seaquist 1986; Guseinov *et al.* 2003). There have been many works via statistical or analytical approaches to investigate Σ-D relations (e.g., Poveda and Woltjer 1968; Clark and Caswell 1976; Mills *et al.* 1984; Huang and Thaddeus 1985; Arbutina *et al.* 2004; Xu *et al.* 2005; Pavlovic *et al.* 2014, etc.). Among statistical results of Σ-D relations, one straight line is often obtained by authors (e.g., Poveda and Woltjer 1968; Huang and Thaddeus 1985; Arbutina *et al.* 2004; Pavlovic *et al.* 2013, 2014), while a broken fit line or a transition point is also usually seen. For example, Clark and Caswell (1976) and Allakhverdiyev *et al.* (1983, 1985) obtained a broken fit line in their statistical works. At 408 MHz, Clark and Caswell (1976) had a broken line with slopes of $\beta = -2.7/-10$ ($\Sigma \propto D^{\beta}$) at $D \leq 32$ pc/$D \geq 32$ pc, while Allakhverdiyev *et al.* (1983) had 30 pc at 408 MHz and 32 pc at 1 GHz for 15 shell-type remnants. For analytical Σ-D relations, Duric and Seaquist (1986) derived

$$\Sigma(D) = 4 \times 10^{-14} D^{-5}, \quad D \ll 1\,pc \tag{1}$$

$$\Sigma(D) = 4 \times 10^{-15} D^{-3.5}, \quad D \gg 1\,pc \tag{2}$$

On the other hand, galactic supernova remnants are usually classified into three types: Shell-type, Plerion-type and Composite-type. In our paper, for simplicity, we just focus on investigating shell-type galactic supernova remnants. Usually, shell-type galactic supernova remnants have four evolution stages: the free expansion phase, adiabatic or Sedov phase, radiative or snow-plough phase and the dissipation phase. Nearly all of detected shell-type SNRs are at adiabatic phase or radiative phase, since almost none is observed in the 1st and 4th phases due to fact that shell-type SNRs at these two phases are usually practically undetectable. Therefore, Σ-D relations of shell-type supernova remnants at adiabatic phase and radiative phase are interesting issues to investigate. Indeed, Duric and Seaquist (1986) have analytical derived Σ-D relation of shell-type supernova remnants at adiabatic phase, i.e. the above equation (2). In our paper, we will mainly focus on analytical investigations on Σ-D relations of shell-type supernova remnants. We obtain theoretical Σ-D relation at radiative phase of shell-type supernova remnants at 1 GHz, which is simply followed the work Duric and Seaquist (1986). We also have collected 57 shell-type galactic supernova remnants data where some data have been updated according to Green (2004, 2009 and 2014) and other new references. By using these theoretical Σ-D relations at adiabatic phase and radiative phase, we have roughly determined the phases of some supernova remnant among these 57 shell-type galactic supernova remnants data.

This article is organized as follows. In Section 2, after a brief review of the work Duric and Seaquist (1986) at adiabatic phase, we simply follow their work and further analytically investigate Σ-D relation at radiative phase of shell-type galactic supernova remnants at 1 GHz. In Section 3, we have collected 57 shell-type galactic supernova remnants data. By using these two theoretical Σ-D relations at adiabatic phase and radiative phase, we roughly determine the phases of some supernova remnant among these data. Finally, a brief conclusion and discussion are given in Section 4.

2. Theoretical Σ-D relations at adiabatic phase and radiative phase

In this section, we mainly focus on theoretical Σ-D relations of shell-type supernova remnants. After making a brief review on the analytical work Duric and Seaquist (1986) at adiabatic phase, we theoretically derive Σ-D relation at radiative phase of shell-type supernova remnants.

2.1 *A brief review: work Duric and Seaquist (1986)*

Taking the linear diameter (D) of remnant in pc, time (t) in s, SNR initial explosion energy (E_0) in the unit of 10^{51} ergs with $E_0 = \varepsilon_{51} \times 10^{51}$, and ISM electron density (n_0) in cm^{-3}, from the standard Sedov solution, one has the following equation (Bignami *et al.* 1988; Zaninetti 2000; Völk *et al.* 2002; Ptuskin and Zirakashvili 2003)

$$D(t) = A_0 t^{2/5}, \tag{3}$$

where the coefficient is

$$A_0 = 5.4 \times 10^{-4} \left(\tfrac{\varepsilon_{51}}{n_0}\right)^{1/5}. \tag{4}$$

The shock wave velocity should be

$$\upsilon(t) = \frac{1}{2}\frac{d}{dt}D(t) = \frac{1}{5}A_0 t^{-3/5}. \tag{5}$$

At the adiabatic phase, the thickness of remnant is proportional to D, and the shell volume which contains all the radio-emitting particles is

$$V(D) = C_0 D^3, \tag{6}$$

here $C_0 = \frac{\pi}{6}(1 - (\frac{D_i}{D_0})^3) \simeq 0.37$ is the volume coefficient (Milne 1970). Notice that condition $D_i/D_o \sim 2/3$ has been assumed and D_i and D_o are the inner and outer diameter of the remnant shell respectively. Combining Equations (3) and (6), one obtains the volume of the shell with respect to t

$$V(t) = C_0 A_0^3 t^{6/5}. \tag{7}$$

As the shock waves of remnant travel, the ambient magnetic field B at the adiabatic phase will decrease with D according to (Duric and Seaquist 1986)

$$B(D) = B_0 \left(\frac{D_0}{D}\right)^2. \tag{8}$$

Substituting Equation (3) to it, we have

$$B(t) = B_0 D_0^2 A_0^{-2} t^{-4/5}. \tag{9}$$

Ginzburg and Syrovatskii (1965) and Bell (1978) show that the radio emissivity $\epsilon(B, \upsilon)$ of a shocked gas which are affected by a magnetic field to produce the synchrotron emission is expressed as (Arbutina *et al.* 2012)

$$\begin{aligned}\epsilon(\nu) = {} & 2.94 \times 10^{-34}(1.435 \times 10^5)^{0.75-\alpha}\xi(2\alpha+1) \\ & \times \left(\frac{n_0}{cm^{-3}}\right)\left(\frac{\alpha}{0.75}\right)\left(\frac{\upsilon}{10^4\ km\ s^{-1}}\right)^{4\alpha}\left(\frac{B}{10^{-4}G}\right)^{\alpha+1} \\ & \times \left(1 + \left(\frac{\upsilon}{7000\ Km\ s^{-1}}\right)^{-2}\right)^{\alpha}\left(\frac{\nu}{GHz}\right)^{-\alpha},\end{aligned} \tag{10}$$

where $\psi_e = 4$ and $\phi_e = 10^{-3}$ have been set as same as in Bell (1978), and unit of $\epsilon(\nu)$ is $W\,Hz^{-1}\,m^{-3}$, $\xi(\mu) = 11.7a(\mu)$, and $a(\mu)$ is the function tabulated by Ginzburg and Syrovatskii (1965). The velocities of shock waves in the second and third phase of SNRs are typically far less than 7000 $Km\ s^{-1}$. Thus, Equation (10) can be further simplified

$$\begin{aligned}\epsilon(\nu) = {} & 2.94 \times 10^{-34} \times (1.435 \times 10^5)^{0.75-\alpha}\xi(2\alpha+1) \\ & \times(\frac{\alpha}{0.75})(0.7)^{4\alpha}(\frac{n_0}{cm^{-3}})(\frac{\upsilon}{7000\,km/s})^{2\alpha} \\ & \times(\frac{B}{10^{-4}G})^{\alpha+1}(\frac{\nu}{GHz})^{-\alpha} \\ = {} & 2.94 \times 10^{-34} \times (1.435 \times 10^5)^{0.75-\alpha}\xi(2\alpha+1) \\ & \times(\frac{\alpha}{0.75})(0.7)^{4\alpha}(\frac{n_0}{cm^{-3}})(\frac{\upsilon}{2.3 \times 10^{-10}\ pc/s})^{2\alpha} \\ & \times(\frac{B}{10^{-4}G})^{\alpha+1}(\frac{\nu}{GHz})^{-\alpha} \\ & (W\,Hz^{-1}\,m^{-3}).\end{aligned} \tag{11}$$

Note that, in the second line of this equation, shock velocity υ in unit of pc/s has been considered for the later convenience of discussion. Taking account of Equations (3) and (9) and the average value of the remnants spectral index $\alpha = 0.5$, we can get

$$\epsilon(D) = 2.25 \times 10^{-34} \left(\tfrac{D_0}{D}\right)^3 \left(\tfrac{B_0}{10^{-4}G}\right)^{3/2} \times \left(\tfrac{1}{5} A_0^{5/2} D^{-3/2} / (2.3 \times 10^{-10}\, pc/s)\right). \quad (12)$$

which is at 1 GHz and $a(2) = 0.103$ has been used in Ginzburg and Syrovatskii (1965). If the shell volume is considered to be encompassed by the radiating electrons, the surface brightness of remnant can be written as (Duric and Seaquist 1986)

$$\Sigma(t) = \frac{\epsilon(t) V(t)}{\pi^2 D^2(t)}. \quad (13)$$

Inserting Equations (3) (4) (7) and (12) into it, we obtain

$$\Sigma(D) = 2.25 \times 10^{-34} \frac{C_0 D_0^3}{\pi^2 D^2} \left(\tfrac{B_0}{10^{-4}G}\right)^{3/2} \times \left(\tfrac{1}{5} A_0^{5/2} D^{-3/2} / (2.3 \times 10^{-10}\, pc/s)\right). \quad (14)$$

Finally, one can get

$$\Sigma(D) = m_a D_{pc}^{-3.5}\ (Wm^{-2}\, Hz^{-1}\, sr^{-1}), \quad (15)$$

where

$$m_a = 2.25 \times 10^{-34} \frac{C_0 D_0^3}{\pi^2} \left(\frac{B_0}{10^{-4}G}\right)^{3/2} \times \left(\tfrac{1}{5} A_0^{5/2} / (2.3 \times 10^{-10}\, pc/s)\right) \times 3.08 \times 10^{16} = 3.88 \times 10^{-17}, \quad (16)$$

and $\Sigma(D)$ in Equation (14) in unit of ($Wm^{-3}\, Hz^{-1}\, pc\, sr^{-1}$) and $1 pc = 3.08 \times 10^{16}\, m$ have been considered, and some typical values of physical parameters of SNRs are taken: ISM density $n_0 = 0.1$ cm^{-3}, SNR initial explosion energy $E_0 = 10^{51}$ erg, the diameter and ISM magnetic field of remnant at the beginning of Sedov phase $D_0 = 2$ pc and $B_0 = 10^{-4}$ G, etc. Therefore, the analytically derived line of Σ-D relation at the second phase of shell-type SNR is

$$\Sigma(D) = 3.88 \times 10^{-17} D_{pc}^{-3.5}\ (Wm^{-2}\, Hz^{-1}\, sr^{-1}). \quad (17)$$

Note that, different typical values have been chosen, and hence coefficient in Equation (17) is a little different from that in work Duric and Seaquist (1986), but the power-law has the same exponent.

2.2 *Analytical Σ-D relation at the radiative phase*

It should be pointed out that the above work Duric and Seaquist (1986) just analytically investigate the adiabatic phase of shell-type SNRs. In fact, we can also simply follow their work to analytically investigate Σ-D relation at radiative phase of shell-type SNRs. After setting the same choices of units as those in Section 2.1, the equation for shell-type SNRs at the radiative stage is (McKee and Ostriker 1977)

$$D(t) = A_1 t^{2/7}, \quad (18)$$

where A_1 is a constant

$$A_1 = 0.03 \left(\tfrac{\varepsilon_{51}}{n_0}\right)^{1/7}. \quad (19)$$

From which, we obtain the velocity of shock wave at the radiative phase

$$\upsilon(t) = \frac{1}{7} A_1 t^{-5/7}. \quad (20)$$

Same as the adiabatic phase, the volume of shell can be

$$V(D) = C_1 D^3. \quad (21)$$

If we roughly take $D_i/D_o \sim 3/4$, then the coefficient will be $C_1 = \frac{\pi}{6}(1 - (\frac{D_i}{D_0})^3) \simeq 0.3$. Changing the variant D to t, one can rewrite the volume of shell as

$$V(t) = C_1 A_1^3 t^{6/7}. \quad (22)$$

Note that the ambient magnetic field B of a remnant decreases with the diameter D at the adiabatic phase following Equation (8), while at the dissipation-phase it is $B(D) = B_1(D_1/D)^0$. Therefore, we can assume that the ambient magnetic field B at the radiative phase can be expressed as

$$B(D) = (\frac{D_1}{D})^\beta B_1, \quad (23)$$

where the parameter β ranges from 0 to 2. After substituting (18) to it, one gets

$$B(t) = \left(\frac{D_1}{A_1}\right)^\beta B_1 t^{-2\beta/7}. \quad (24)$$

Therefore, following the same steps as the above section and still taking $n_0 = 0.1$ cm^{-3}, we can obtain $\Sigma - D$ relation at radiative phase at 1 GHz

$$\Sigma(D) = 2.25 \times 10^{-34} \frac{C_1 D^3}{\pi^2 D^2} \left(\tfrac{B_1 D_1^\beta D^{-\beta}}{10^{-4}G}\right)^{3/2} \times \left(\tfrac{1}{7} A_1^{7/2} D^{-5/2} / (2.3 \times 10^{-10}\, pc/s)\right). \quad (25)$$

and this form is simply rewritten as

$$\Sigma(D) = m_r D^{-\frac{3}{2}(1+\beta)}\ (Wm^{-2}\, Hz^{-1}\, sr^{-1}), \quad (26)$$

where

$$m_r = 2.25 \times 10^{-34} \frac{C_1 D_1^{\frac{3}{2}\beta}}{\pi^2} \left(\tfrac{B_1}{10^{-4}G}\right)^{3/2} \times \left(\tfrac{1}{7} A_1^{7/2} / (2.3 \times 10^{-10}\, pc/s)\right) \times 3.08 \times 10^{16}. \quad (27)$$

Table 1. Σ is directly obtained from experimental data in Table 2. For each supernova remnant with observational diameter D, Σ_0 is obtained from theoretical Σ-*D* relation at adiabatic phase, while Σ_1 is from theoretical Σ-*D* relation at radiative phase. After making comparisons with Σ, and obtaining the deviations (i.e., deviation of Σ_0 is just simply calculated from $(\Sigma - \Sigma_0)/\Sigma$), we roughly determine supernova remnant at adiabatic phase or radiation phase.

Determine adiabatic phase or radiation phase							
Source	Dia.	Σ	Σ_0	Deviation	Σ_1	Deviation	state
G8.7−0.1	51	5.95E−21	4.11E−23	99.31%	1.40E−21	76.42%	Radiative
G32.8−0.1	35	5.73E−21	1.53E−22	97.32%	4.34E−21	24.27%	Radiative
G33.6+0.1	23	3.01E−20	6.67E−22	97.79%	1.53E−20	49.21%	Radiative
G41.1−0.3	8	2.97E−20	2.69E−20	9.64%	3.63E−19	−1122.00%	Adiabatic
G43.3−0.2	10	3.97E−20	1.23E−20	69.03%	1.86E−19	−368.33%	Adiabatic
G49.2−0.7	52	2.68E−20	3.84E−23	99.86%	1.32E−21	95.06%	Radiative
G78.2+2.1	26	1.34E−20	4.34E−22	96.76%	1.06E−20	20.89%	Radiative
G109.1−1.0	24	4.22E−21	5.74E−22	86.40%	1.35E−20	−218.59%	Adiabatic
G111.7−2.1	5	1.32E−19	1.39E−19	−5.68%	1.49E−18	−1029.95%	Adiabatic
G127.1+0.5	69	8.91E−22	1.43E−23	98.40%	5.66E−22	36.51%	Radiative
G132.7+1.3	51	1.06E−21	4.11E−23	96.12%	1.40E−21	−32.51%	Radiative
G205.5+0.5	102	4.35E−22	3.63E−24	99.17%	1.75E−22	59.74%	Radiative
G284.3−1.8	20	2.87E−21	1.09E−21	62.17%	2.33E−20	−708.94%	Adiabatic
G327.4+0.4	29	1.02E−20	2.96E−22	97.11%	7.63E−21	25.51%	Radiative
G327.6+14.6	19	3.18E−21	1.30E−21	59.05%	2.71E−20	−753.50%	Adiabatic
G330.0+15.0	63	1.63E−21	1.96E−23	98.79%	7.44E−22	54.25%	Radiative
G332.4−0.4	9	4.21E−20	1.78E−20	57.79%	2.55E−19	−505.47%	Adiabatic
G337.8−0.1	27	9.29E−22	3.80E−22	59.06%	9.45E−21	−917.19%	Adiabatic
G346.6−0.2	19	1.88E−20	1.30E−21	93.08%	2.71E−20	−44.15%	Radiative
G349.7+0.2	9	1.20E−19	1.78E−20	85.23%	2.55E−19	−111.91%	Adiabatic

3. Roughly determine phases of some supernova remnants by using theoretical Σ-*D* relations

For these theoretical *Σ*-*D* relations at adiabatic and radiative phases, it will be highly interesting to identify some reasonable supernova remnants in adiabatic or radiative stages from observation data. Therefore, at first, some typical values of supernova remnants at radiative phase are also set as $B_1 = 10^{-6}$G, $D_1 = 20$ pc, $\beta = 1$. Then, theoretical Σ-D relation at radiative phase is

$$\Sigma(D) = 1.86 \times 10^{-16} D_{pc}^{-3} \; (Wm^{-2}\, Hz^{-1}\, sr^{-1}). \tag{28}$$

Second, 57 shell-type supernova remnants data in Galaxy at 1 GHz have been collected and listed in table 2. From these data, the 1 GHz surface brightness Σ_{1GHz} is obtained by (Clark and Caswell 1976)

$$\Sigma_{1GHz} = 1.505 \frac{S_{1GHz}}{\theta^2} \times 10^{-19} \; (Wm^{-2} Hz^{-1} sr^{-1}), \tag{29}$$

where S_{1GHz} is the 1 GHz flux density in jansky ($1\,Jy \equiv 10^{-26} Wm^{-2}\, Hz^{-1}$), and θ is the angular diameter in minutes of arc.

Finally, comparing Σ_{1GHz} with theoretical Σ-D relations both at adiabatic phase and radiative phase, we roughly determine phases of supernova remnants. From these comparisons, we find out that some of these supernova remnants indeed can be identified in adiabatic phase or radiative phase, which has been listed in Table 1.

4. Conclusion and discussion

In this paper, we have analytically investigated Σ-*D* relations of shell-type supernova remnants both at adiabatic phase and radiative phase. For convenience to compare with observation data, we have chosen some typical values of shell-type supernova remnants and also collected 57 shell-type supernova remnants data. By using these theoretical Σ-*D* relations and observation data, we have roughly identified some shell-type supernova remnants in adiabatic phase or radiative phase.

Table 2. Some physical parameters of 57 shell-type Galactic SNRs.

Source	Age/year	Dist./pc	Dia./pc	Size/arcmin	S_{1GHz}/Jy	References
G4.5+6.8	380	2900	3	3	19	Hatsukade *et al.* (1990) and Green (2004)
G7.7−3.7	–	4500	29	22	11	Milne *et al.* (1986)
G8.7−0.1	15800	3900	51	45	80	Gorham *et al.* (1996)
G18.8+0.3	16000	12000	57	17×11	33	Green (2004) and Tian *et al.* (2007)
G27.4+0.0	2700	6800	8	4	6	Green (2004) and Caswell *et al.* (1982)
G31.9+0.0	4500	7200	13	7×5	25	Chen and Slane (2001) and Green (2014)
G32.8−0.1	–	7100	35	17	11	Koralesky *et al.* (1998)
G33.6+0.1	9000	7800	23	10	20	Seward *et al.* (2003), Seward and Velusamy (1995) and Green (2004, 2014)
G39.2−0.3	1000	11000	22	8×6	18	Green (2014) and Caswell *et al.* (1982)
G41.1−0.3	1400	8000	8	4.5×2.5	25	Chen *et al.* (1999), Caswell *et al.* (1982), Binette *et al.* (1982) and Green (2014)
G43.3−0.2	3000	10000	10	4×3	38	Lacey *et al.* (2001) and Zhu *et al.* (2014)
G49.2−0.7	30000	6000	52	30	160	Koo *et al.* (1995) and Green (2004)
G53.6−2.2	15000	2800	24	33×28	8	Saken *et al.* (1995) and Green (2004)
G55.0+0.3	1100000	14000	71	20×15	0.5	Matthews *et al.* (1998)
G65.3+5.7	14000	1000	78	310×240	42	Green (2014) and Rosado (1981)
G73.9+0.9	10000	1300	8	27	9	Lorimer *et al.* (1998), Green (2014) and Leahy (1989)
G74.0−8.5	14000	400	23	230×160	210	Levenson *et al.* (1999), Stil and Irwin (2001) and Green (2004)
G78.2+2.1	50000	1500	26	60	320	Lorimer *et al.* (1998), Koo and Heiles (1991) and Green (2014)
G84.2−0.8	11000	4500	23	20×16	11	Matthews and Shaver (1980), Matthews *et al.* (1977) and Green (2004)
G89.0+4.7	19000	800	24	120×90	220	Leahy and Aschenbach (1996)
G93.3+6.9	5000	2200	15	27×20	9	Landecker *et al.* (1999) and Green (2004)
G93.7−0.2	–	1500	35	80	65	Uyaniker *et al.* (2002)
G109.1−1.0	17000	4000	24	28	22	Fesen and Horford (1995), Green (2004, 2014), Hughes *et al.* (1981) and Tian and Leahy (2012)
G111.7−2.1	320	3400	5	5	2720	Thorstensen *et al.* (2001)
G114.3+0.3	41000	700	15	90×55	5.5	Mavromatakis *et al.* (2002) and Green (2004, 2014)
G116.5+1.1	280000	1600	32	80×60	10	Green (2004, 2014) and Reich and Braunsfurth (1981)
G116.9+0.2	44000	1600	16	34	8	Koo and Heiles (1991) and Green (2004, 2014)
G119.5+10.2	24500	1400	37	90	36	Mavromatakis *et al.* (2000)
G120.1+1.4	410	2300	5	8	56	Hatsukade *et al.* (1990) and Green (2004)
G127.1+0.5	85000	5250	69	45	12	Green (2014) and Fürst *et al.* (1984)
G132.7+1.3	21000	2200	51	80	45	Green (2004) and Galas *et al.* (1980)

Table 2. continued

Source	Age/year	Dist./pc	Dia./pc	size/arcmin	S_{1GHz}/Jy	References
G156.2+5.7	26000	2000	64	110	5	Reich *et al.* (1992)
G160.9+2.6	7700	1000	38	140×120	110	Leahy and Aschenbach (1995)
G166.0+4.3	81000	4500	57	55×35	7	Koo and Heiles (1991), Green (2004) and Leahy (1989)
G166.2+2.5	150000	8000	186	90×70	11	Green (2014) and Routledge *et al.* (1986)
G182.4+4.3	3800	3000	44	50	0.4	Kothes *et al.* (1998) and Green (2014)
G205.5+0.5	50000	1600	102	220	140	Case and Bhattacharya (1999) and Green (2014)
G206.9+2.3	60000	7000	102	60×40	6	Green (2014) and Leahy (1986)
G260.4−3.4	3400	2200	35	60×50	130	Berthiaume *et al.* (1994) and Rosado and González (1981)
G266.2−1.2	680	1500	52	120	50	Kargaltsev *et al.* (2002) and Aschenbach *et al.* (1999)
G272.2−3.2	6000	1800	8	15	0.4	Duncan *et al.* (1997)
G284.3−1.8	10000	2900	20	24	11	Green (2014) and Ruiz and May (1986)
G296.5+10.0	20000	2000	44	90×65	48	Green (2014) and Matsui *et al.* (1988)
G296.8−0.3	1600000	9600	47	20×14	9	Gaensler and Johnston (1995) and Green (2004)
G299.2−2.9	5000	500	2	18×11	0.5	Slane *et al.* (1996)
G309.2−0.6	2500	4000	16	15×12	7	Rakowski *et al.* (2001)
G315.4−2.3	2000	2300	28	42	49	Dickel *et al.* (2001) and Green (2004)
G321.9−0.3	200000	9000	70	31×23	13	Green (2014), Shull *et al.* (1989) and Salter *et al.* (1989)
G327.4+0.4	–	4800	29	21	30	Seward *et al.* (1996), Green (2004, 2014) and Weiler and Sramek (1988)
G327.6+14.6	980	2200	19	30	19	Green (2004) and Srinivasan *et al.* (1984)
G330.0+15.0	–	1200	63	180	350	Knöedlseder *et al.* (1996)
G332.4−0.4	2000	3100	9	10	28	Carter *et al.* (1997), Green (2004) and Meaburn and Allan (1986)
G337.2−0.7	3250	15000	26	6	1.5	Rakowski *et al.* (2001) and Green (2014)
G337.8−0.1	–	12300	27	9×6	18	Koralesky *et al.* (1998)
G346.6−0.2	–	8200	19	8	8	Koralesky *et al.* (1998) and Dubner *et al.* (1993)
G349.7+0.2	14000	11500	9	2.5×2	20	Reynoso and Mangum (2001), Green (2004) and Tian and Leahy (2014)
G352.7−0.1	2200	8500	17	8×6	4	Kinugasa *et al.* (1998)

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[a] Many of the radio SNRs have more than one published value for distance and age. For these, we either chose the most recent estimates or used an average of the available estimates, or the most commonly adopted value.

[b] Diameters were calculated using from distances together with the angular sizes in Green (2004, 2009 and 2014) catalogue. In addition, some data have been updated according to the new results in Green (2014) and other new references.

[c] Some data regarding G349.7+0.2 (Tian and Leahy 2014), G43.3−0.2 (Zhu *et al.* 2014), G18.8+0.3 (Tian *et al.* 2007) and G109.1−1.0 (Tian and Leahy 2012) have been updated.

Some discussions related to our results are in the following. First, shock compression ratio may differ among supernova remnants, and hence spectral index α will be different from $\alpha = 0.5$. For simplicity and convenience, we also investigate the case with $\alpha = 0.75$, and theoretical Σ-D relation at adiabatic phase

$$\Sigma(D) = 1.06 \times 10^{-17} D_{pc}^{-19/4}\ (Wm^{-2}\ Hz^{-1}\ sr^{-1}), \tag{30}$$

while at radiative phase

$$\Sigma(D) = 5.52 \times 10^{-16} D_{pc}^{-9/2}\ (Wm^{-2}\ Hz^{-1}\ sr^{-1}). \tag{31}$$

From these equations, it will be easily found that effects from spectral index α on Σ-D relations are in fact huge, i.e. different power exponents. Second, not only spectral index α, our results are also dependent on other parameters such as volume coefficient C_0, mean electron density n_0, SNR initial explosion energy E_0, magnetic field at the beginning of the evolving second-stage and third-stage B_0 and B_1, and parameters D_0 and D_1. If those parameters are changed, our results may be also different. Therefore, further effects of parameters on theoretical Σ-D relations will be an interesting open issue, while maybe they are also the main reason that why just some of supernova remnants are roughly identified at adiabatic phase or radiative phase among 57 supernova remnants, and hence our results just shed some insights onto the possibility to identify the phase of supernova remnant through comparisons theoretical Σ-D relations with observation data. Third, the true physical process of supernova remnant at radiative phase is complicated, and we have assumed that the synchrotron radiation equation (11) is still valid in radiative stage. But yet, whether this assumption is correct or not is still an issue (e.g., discussions in Asvarov 2006). Comparison with observational data seem to support this assumption. Fourth, in principle, there is a simple and direct method to identify the phase of supernova remnant, i.e. comparisons $D(t)$ relations with observation data. However, this method may be not better than our method, and the reason may be that there is a larger uncertainty to decide the age of a supernova remnant than diameter. Finally, our results also predict that there will be a transition point between these two theoretical Σ-D relations. However, since $D(t)$ relations used in our paper are just statistical and not precise, the details of this transition point are still lacking.

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