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On the unified scheme for high-excitation galaxies and quasars in 3CRR sample

F. C. ODO*, P. E. OKOBI, J. A. ALHASSAN and A. A. UBACHUKWU

Department of Physics and Astronomy, University of Nigeria, Nsukka, Nigeria.
*Corresponding author. E-mail: finbarr.odo@unn.edu.ng

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Abstract. In this paper, we use the distributions of luminosity (P) and radio size (D) to re-examine the consistency of the unified scheme of high-excitation radio galaxies and quasars in the recently updated 3CRR sample. Based on a standard cosmology, we derive theoretically and show from observed data, the luminosity limit above which the 3CRR objects are well-sampled. We find, on average, a quasar fraction ~ 0.44 and galaxy-to-quasar size ratio ≈ 2. Assuming a relativistic outflow of jet materials, we find a mean angle to the line of sight in the range $35^\circ \leq \phi \leq 44^\circ$ for the quasars. On supposition of luminosity and orientation-dependent linear size evolution, expressed in a general functional form $D_{(P,z,\phi)} \approx P^{\pm q}(1+z)^{-w}\sin\phi$, we show that above the flux detection threshold of the 3CRR sample, high-excitation galaxies and quasars undergo similar evolution with $q = -0.5$; $w = -0.27$ and luminosity independent evolution parameter $x = 2.27$, when orientation effect is accounted for. The results are consistent with orientation-based unified scheme for radio galaxies and quasars.

Keywords. Galaxies: active—galaxies: quasars—general.

1. Introduction

Orientation-based unified scheme (OUS)—a theoretical framework that brings two apparently distinct subclasses of extragalactic radio sources (EGRSs), namely, radio galaxies and quasars, under one roof (Barthel 1989)—has been remarkably popular because of its simplicity. In this scheme, radio galaxies and quasars arise from the same parent population of EGRSs; only the orientation to observer's line of sight makes the difference. In particular, due to geometry of the universe and foreshortening arising from sharper inclination of the radio axes, both the numbers and radio sizes of quasars are expected to be systematically smaller compared to radio galaxies by factors dependent on the viewing angle (ϕ). Using the well-studied 3CRR sample of EGRSs, Barthel (1989) proposed that the observed number and size of quasars are typically half those of radio galaxies in the redshift (z) regime $0.5 \leq z \leq 1$, which led to the adoption of canonical $\phi \sim 45^\circ$ as the dividing line between the orientation of radio galaxies and quasars. Agreement with the OUS was argued by several authors in the past (Saikia and Kulkarni 1994; Gopal-Krishna and Mangalam 1994). However, outside the redshift regime ($0.5 \leq z \leq 1$), violations to the OUS scheme are well known (e.g. Kapahi 1989; Singal 1993; Singal and Singh 2013). In fact, Singal and Singh (2013) argued that the predicted foreshortening of linear sizes of quasars relative to radio galaxies is valid only for $z > 1$ in their sample. Indeed, for the lowest luminosity bin and $z < 0.4$ of the 3CRR sample, Singal (1993) suggested that the median linear size of quasars is larger than that of radio galaxies in the sense that is completely opposite to the predictions of the OUS. On the other hand, in a similar study, limited to sources which at the time have clear double morphology, with sharp outer boundaries, Nilsson *et al.* (1993) found results that were in contrast to Singal (1993): the median values of the linear size for quasars are smaller than for galaxies in all luminosity bins.

Furthermore, in addition to orientation effects, the observed linear sizes and number density of EGRSs are argued to be complex functions of the survey surface-brightness (SB) sensitivity, as well as the cosmological $(1+z)^{4+\alpha}$ dimming, where α is the spectral index ($S_\nu \sim \nu^{\pm\alpha}$) . Thus, if orientation to the line of sight alone makes the difference, remarkable similarities should exist in cosmological evolution as well as linear size–luminosity ($D - P$) correlation of radio galaxies and quasars in EGRS samples. The cosmological evolution of both the radio size and luminosity, as well as luminosity selection effect due to the SB sensitivity have often been used to interpret the observed angular size–redshift ($\theta - z$) data of extragalactic radio sources (Okoye and Onuora 1982; Oort *et al.* 1987; Ubachukwu 1995). In fact, Onuora (1991) pointed out in the study of $\theta - z$ relation of the original 3CRR sample that orientation and evolution are inherent in size distributions of the sample. Singal (1993) studied the linear sizes of a large, heterogeneous sample of radio galaxies and quasars and came out with a main result that both $D - P$ and $D - z$ correlations are significantly different for the two classes. The vast differences in cosmological evolution and opposite $D - P$ trends observed between radio galaxies and quasars in various samples (e.g. Oort *et al.* 1987; Singal 1993), apparently, do not fit in the OUS model. Recently, Onah *et al.* (2018) suggested that the disparity in $D - P$ relation of radio galaxies and quasars can be attributed to redshift dependence of both luminosity and linear size of EGRSs.

However, Fanaroff and Riley (1974) showed that radio galaxies exhibit a dichotomy in radio morphology leading to a morphology-based classification into class I (FR I) and class II (FR II) radio galaxies, which was later found to be linked with their radio-to-optical luminosity (e.g. Ledlow and Owen 1996). The dividing line between FR I and FR II galaxies occurs around a monochromatic power at 1.4 GHz of $\sim 10^{25}$W Hz^{-1}, but occurs at higher radio luminosity for more optically-luminous sources. The division led to the hypothesis that FR I and FR II radio galaxies represent different populations of cosmologically evolving EGRSs (e.g. Jackson and Wall 1999). Similarly, there is a spectroscopic classification of the radio galaxies (Laing *et al.* 1994) into low-excitation radio galaxies (LERGs) and high-excitation radio galaxies (HERGs), based on optical emission line diagnostic, which does not correspond to the Fanaroff–Riley classification scheme. HERGs experience radiatively efficient accretion of gas via an accretion disk and show strong emission lines. They are thus believed to harbor a hidden-quasar in their nuclei (Laing *et al.* 1994). On the other hand, LERGs are radiatively inefficient (Hardcastle *et al.* 2009), showing weak or no emission lines, and thus, are not likely to appear like quasars (Ogle *et al.* 2006). The connection between Fanaroff–Riley and spectroscopic classification schemes has be a subject of several investigations and is yet to be fully understood (e.g. Buttiglione *et al.* 2010). While all FR I radio galaxies are known to be LERGs, not all FR II radio galaxies are HERGs (Laing *et al.* 1994; Buttiglione *et al.* 2010). Although a vast majority of FR II galaxies at high redshifts are HERGs, at low redshifts, FR II samples contain a significant number of LERGs which properties are very similar to those of FR I galaxies (e.g. Capetti *et al.* 2017). This implies that there is a fundamental difference between HERGs and LERGs (e.g. Williams et al. 2018) that is quite distinct from the radio morphological dichotomy. Hence, LERGs are not expected to be part of the OUS because they lack a hidden quasar in their nuclei (Hardcastle *et al.* 2009; Leipski *et al.* 2010).

Near infrared spectroscopy identifies a number of LERGs in the 3CRR radio galaxy sample (e.g. Grimes *et al.* 2004), which are not expected to be part of the OUS. These LERGs constitute a reasonable fraction ($\sim$ 20%) of the FR II radio galaxy population of the updated 3CRR sample (see Singal 2014). Exclusion of these LERGs from the 3CRR sample seems to pose a major challenge to the long-standing OUS (e.g. Singal 2014). In particular, Singal (2014) argues that at $z \leq 0.5$, there is apparent disappearance of the "must be" foreshortening of radio sizes of quasars relative to HERGs, suggestive that the size distributions of radio galaxies and quasars in the 3CRR sample is not consistent with the OUS when the LERGs are excluded from the sample.

A major issue with linear size determination is that, often, the morphology of the source is unclear and the source is poorly resolved, especially in complete flux-limited samples. In such samples, there is tight correlation between luminosity and redshift due to selection effect/evolution (Ubachukwu *et al.* 1993), which is expected to introduce some ambiguities in linear size data. Actually, results from new samples, such as that from Low Frequency Array (LOFAR), seem to provide substantial evidence that radio galaxies and quasars can be unified with an OUS after correcting for redshift dependence of the linear size (Morabito *et al.* 2017). Partly motivated by these findings from new samples, in this paper, we re-examine the 3CRR sample and show that luminosity dependence on redshift can explain why some previous results (e.g. Singal and Singh 2013; Singal 2014) found discordance with the OUS. In particular, we re-define the luminosity limit above which the 3CRR sample is complete and show that within this luminosity limit, the sample is consistent with OUS in number and size distributions, as well as cosmological evolution.

2. Redshift dependence of linear size

Orientation effects in EGRSs are often studied using the projected linear size (D) expressed in terms of the viewing angle (ϕ) and intrinsic linear size (D_o) in the form (e.g. Onuora 1991; Ubachukwu and Onuora 1993)

$$D = D_0 \sin\phi. \tag{1}$$

However, in the popular relativistic beaming and orientation model for EGRSs, at small angles to the line of sight, relativistic beaming is fundamentally characterized by a Doppler enhancement factor: $\delta = [\Gamma\,(1-\beta\cos\phi)]^{-1}$, where $\Gamma = \left(1-\beta^2\right)^{-1/2}$and β is the speed of the jet material in units of the speed of light. Hence, in a two component model, the observed radio flux from the core (S_{oc}) depends strongly on the viewing angle (ϕ) and can be expressed in terms of the intrinsic value (S_{ic})as $S_{oc} = S_{ic}\delta^{n+\alpha}$, where α is the spectral index ($S_\nu \sim \nu^{\pm\alpha}$) and n is a jet model-dependent parameter: n= 2 for continuous jet model, while n = 3 for a jet with distinct blobs (Lind and Blandford 1985). Radio emission from the core is characterized by flat spectra ($0 \leq \alpha < 0.5$), while radio lobe emission is characterized by steep spectra ($0.5 \leq \alpha \leq 1$). Orr and Browne (1982) introduced the ratio (R) of the beamed core to isotropic lobe emissions as orientation indicator, which, following from above, is given (e.g. Orr and Browne 1982; Ubachukwu and Chukwude 2002) as:

$$R = \frac{P_C}{P_E} = \frac{R_T}{2}\left[(1-\beta\cos\phi)^{-n+\alpha} + (1+\beta\cos\phi)^{-n+\alpha}\right], \tag{2}$$

where P_C and P_E are the core and lobe luminosities respectively, $R_T = R(\phi = 90°)$. The distributions of observed D and R for various samples have been shown by several authors in the past to be quite consistent with the OUS, via relativistic beaming model, for both high-luminosity (e.g. Ubachukwu and Chukwude 2002) and low luminosity sources (Odo *et al.* 2012).

In the Friedmann–Robertson–Walker universe, the apparent angular size (θ) of a radio source is found to vary with redshift (z) in a form that depends on the value of density parameter, Ω_0, which suggests some evolutionary effects on these orientation parameters. In general, the angular size–redshift ($\theta - z$) relation of radio sources is given (e.g. Miley 1968) as

$$\theta = \frac{D(1+z)^2}{d_L}, \tag{3}$$

where d_L is the luminosity distance which depends on H_0 and Ω_0 and, following Mattig (1959), based on assumption of the simplest ($\Lambda \approx 0$) cosmology, can be written (e.g. Okoye and Onuora 1982) as:

$$d_L = \frac{2c}{H_0\Omega_0^2}\left\{\Omega_0 z + (\Omega_0 - 2)\left[(\Omega_0 z + 1)^{\frac{1}{2}} - 1\right]\right\}, \tag{4}$$

On supposition of a general Lambda-CDM cosmology, with $\Omega_0 = \Omega_m + \Omega_\Lambda = 1$ ($\Omega_\Lambda \neq 0$) which appears to be widely favored by the community consensus, equation (3) can be written as (Ubachukwu and Onuora 1993)

$$\theta = \frac{DH_0\,(1+z)^2}{2c\left\{(1+z) - \sqrt{1+z}\right\}}. \tag{5}$$

However, for a random radio source orientation, then, the linear size of all radio sources lying at small angles to the line of sight would appear foreshortened due to projection effects. Thus, using Equation (1) in (5) yields (e.g. Ubachukwu and Onuora 1993):

$$\theta = \frac{D_0 H_0\,(1+z)^2\sin\phi}{2c\left\{(1+z) - \sqrt{1+z}\right\}}. \tag{6}$$

Equation (6) therefore gives the $\theta - z$ relation for a radio source for which the effect of orientation has been admitted. But if linear size evolution of the form $D \approx (1+z)^{-x}$ is present, Equation (6) can be modified as

$$\theta = \frac{D_0 H_0\,(1+z)^{2-x}\sin\phi}{2c\left\{(1+z) - \sqrt{1+z}\right\}} \tag{7}$$

Furthermore, Masson (1980) introduced the idea that the $\theta - z$ data of radio sources can also be interpreted in terms of linear size–luminosity ($D - P$) correlation. Thus, an alternative to linear size evolution for radio sources is that linear sizes may be inversely correlated with luminosity. In particular, Ubachukwu (1995) argues that the observed $\theta - z$ variation in quasars could be more generally explained in terms of this dependence on luminosity rather than linear size evolution. The opposite behavior of radio galaxies and quasars on the $D - P$ plane actually resulted in adoption by the community, of a general power law $D - P$ relation of the form $D \sim P^{\pm q}$ (e.g. Mason 1980; Aird *et al.* 2010). It is thus suggested that in addition to orientation, linear size of a radio source also depends on redshift and luminosity and can be expressed in a general form (e.g. Onah *et al.* 2018) as:

$$D_{(P,z,\phi)} \approx P^{\pm q}(1+z)^{-w}\sin\phi, \tag{8}$$

where $w = 2–x$.

Although the angular diameter test appears to have fallen out of favor in the past decades, it is invoked in this paper to argue for a region of parameter space, in redshift and luminosity, over which the 3CRR sample is complete for a reliable investigation of the OUS.

3. On the luminosity limit

The spectral luminosity (P) of a radio source at redshift (z) is related to its spectral flux density (S_ν) at observing frequency (ν) according to the relation:

$$P = S_\nu d_L^2 (1+z)^{\alpha-1}, \quad (9)$$

However, for a complete sample with a flux density cut-off at $S_\nu = S_c$, Equation (9) can be written in the form (e.g. Ubachukwu and Onuora 1993; Alhassan *et al.* 2013):

$$P = 4\pi d_L^2 S_\nu H(S_\nu - S_c)(1+z)^{\alpha-1}, \quad (10)$$

where $H(S_\nu - S_c)$ is the Heaviside step function defined by:

$$H(S_\nu - S_c) = \left\{ \begin{matrix} 0 & if\, S_\nu < S_c \\ 1 & if\, S_\nu > S_c \end{matrix} \right\}.$$

Equation (10) can be used to show a simple power law $P - z$ relation (e.g. Ubachukwu and Onuora 1993) as

$$\log P = \log P_o + \beta \log(1+z) \quad (11)$$

where $P_0 \approx P_0\ (\Omega_0,\ H_{o,}c)$ and β is the slope of the $P - z$ data.

In flux density limited samples, sources appear to bunch up close to the flux limit due to Malmquist bias, with a strong correlation between P and z. Common practice has been to obtain β by fitting a log linear function to the observed $P - z$. In fact, it has been demonstrated (Ubachukwu 1995; Ubachukwu and Ogwo 1998; Onah *et al.* 2018) that cosmological evolution parameter x, independent of luminosity selection effect, is a linear function of β.

However, since the present analysis is concerned with the unification of radio galaxies and quasars via orientation at low redshifts below z = 0.5 (the region of parameter space over which discordance of the OUS has been widely reported in the 3CRR sample), we can consider a cosmology with $\Omega_0 = 0$, which offers the simplest form of d_L as a function of z in the form (e.g. Ubachukwu and Onuora 1993):

$$d_L = \frac{2c}{H_0}(1+z)^2 - 1, \quad (12)$$

so that, assuming $\alpha = 0$ for sources oriented at small angles to the line of sight, the spectral luminosity can be written as a function of z (Odo *et al.* 2014) as:

$$P = P_0\left(j^3 - 2j + \frac{1}{j}\right), \quad (13)$$

where $P_0 = 4\pi S_\nu H(S_\nu\_S_c)$ and $j = 1 + z$. The slope (β) of log P-log j data therefore yields (e.g. Odo *et al.* 2014):

$$\beta = \frac{d\log P}{d\log j} = \frac{3j^4 - 2j^2 - 1}{j\left(j^2 - 1\right)^2}. \quad (14)$$

Equation (14) shows that β is not a constant over all values of j. Actually, β is expected to decrease monotonically from ∞ at $j = 1$ down to some critical value, β_c at $j = j_c(z = z_c)$ and thereafter remains fairly constant. Thus, we suggest in this paper that P_0 represents the critical power (P_c) above which radio sources can be detected as a function of z (i.e. their total flux is above the detection threshold). This should correspond to a given radio power at z = z_c. Thus, z $\geq$ z_c; $P \geq P_c$ represents the parameter space over which the OUS trend is expected. Theoretically, Odo *et al.* (2014), using this method, demonstrated unambiguously that $z_c = 0.3$. Thus, in this paper, we adopt $z_c = 0.3$ and extend the investigation of OUS down to z = 0.3 for the 3CRR sample.

4. Analyses and results

The sample used in this study is the updated 3CRR sample of extragalactic radio sources for which a discordance of the OUS has been reported in a recent analysis by some authors (e.g. Singal 2014). The sample contains 130 EGRSs which include 85 radio galaxies and 45 quasars. Out of the 85 radio galaxies, 68 objects are HERGs and 17 are LERGs. With the current stipulation that LERGs do not contain a hidden quasar and ought not to partake in OUS, the 17 LERGs are excluded from this analysis leaving the current sample with 113 objects: 68 HERGs and 45 quasars. All the 113 objects in the sample are steep spectrum sources ($\alpha \geq 0.5$; $S_\nu \sim \nu^{-\alpha}$) with complete data on size, redshift and 408 MHz radio luminosity, assuming H_0 = 71 kms^{-1}Mpc^{-1} and Lambda-CDM cosmology with $\Omega_0 = \Omega_m + \Omega_\Lambda = 1$ ($\Omega_m = 0.27$; $\Omega_\Lambda = 0.73$).

All relevant information on this sample is available in Singal (2014).

We show for the current 3CRR objects, the scatter plot of the radio luminosity as a function of redshift (z) on log–log scales in Figure 1. The data is truncated with dotted (vertical) line at log $z = 0.11$ ($z_c = 0.3$), which corresponds to log $P_c = 27$ W Hz^{-1}, shown with the dashed (horizontal) line. Thus, log $P \approx 27$ W Hz^{-1} represents in this paper, the minimum luminosity for which the total radio flux is above the detection threshold of the sample. A close observation of the plot actually suggests a change in the $P - z$ slope near z = 0.3, quite

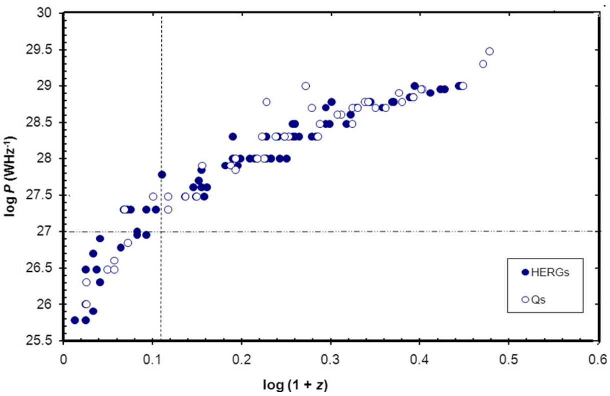

Figure 1. Plot of 408 MHz total luminosity P as a function of (1+z), on logarithmic scales, for the HERGs.

consistent with the theoretical prediction of $z_c = 0.3$. In fact, regression analysis of the P–z data yields $\beta \sim 8.6$, with a correlation coefficient $r \approx 0.8$ for $z < 0.3$ and $\beta \sim 1.1$, with $r \approx 0.6$ for $z \geq 0.3$. For all objects in the sample a quasar fraction ~ 0.40 (i.e. 45/68 + 45) is obtained. For $z \geq z_c = 0.3$, there are 49 HERGs and 37 quasars giving a quasar fraction ~ 0.44, while for $z < z_c$, the quasar fraction ~ 0.33. However, it is interesting to observe that the two truncation lines divide the $P - z$ data into four distinct panels, namely, the low z-low P; low z-high P, high z-low P and high z-high P panels. Similar analysis in the different panels yields quasar fraction ~ 0.42 for low z-low P; 0.40 for low z-high P and 0.33 for high z-high P. There is no tendency for HERGs and quasars to be located at low z-high P panel.

For the high z-high P panel, where the sources may be well sampled, we show the luminosity distributions of HERGs and quasars in Figure 2. The distribution gives a mean value of 28.30 ± 0.01 for HERGs and 28.46 ± 0.03 for quasars, on logarithmic scales. This does not apparently show that quasars are more luminous than radio galaxies in this redshift bin; rather, there is intrinsic spread in luminosity of the two populations of sources. Two sample Kolmogorov–Smirnoff (K–S) test on the luminosity data yields a chance probability $\rho \sim 0.08$ which actually shows that at 5% level, there is no statistical difference between the underlying distributions of HERGs and quasars in total luminosity: any observed difference in luminosity of the HERGs and quasars could have arisen by chance. Similarly, we show in Figure 3, the distributions of the objects in linear size. There is a wind range of linear size from less than 10 kpc to over 3000 kpc. Apparently, the distributions for individual sub-samples actually overlap, with quasars displaced to lower values, but do not seem to convincingly show that quasars are inclined at smaller viewing

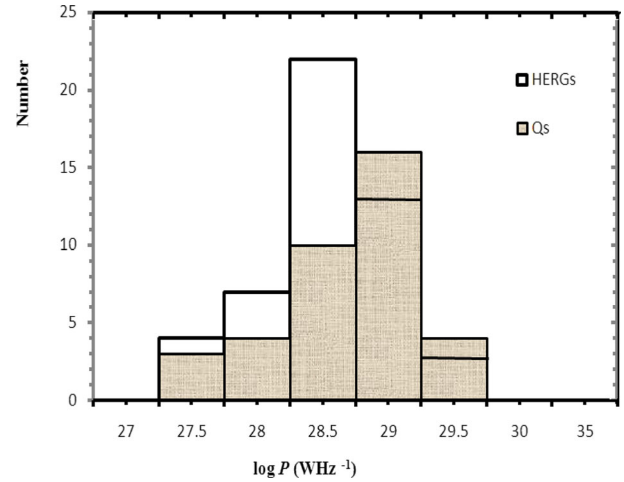

Figure 2. Distributions of the HERGs and quasars (Q) in 408 MHz total luminosity.

angles than radio galaxies, as expected in the OUS. However, Kapahi (1989) has shown that, in general, the mean/median values would be the best parameter for characterizing radio source properties since cosmological interpretation of the distributions of these properties assume that radio sources are randomly distributed in space. The mean value of the linear size data is 309 ± 5 kpc for the HERGs and 168 ± 4 kpc for quasars, which yields a size ratio of galaxies to quasars ($D_{\rm gal}/D_{\rm quas}$) of ~ 1.9. Two sample (K–S) test yields a chance probability $\rho \sim 0.038$, for the quasar/galaxy linear size distributions, which implies that there is less than 5% probability that the underlying distributions of the linear sizes of HERGs and quasars are statistically similar, with that of quasars, on average, being systematically smaller than HERGs.

In the current sample, 29 of the quasar sources with $z \geq z_c$ overlap with Fan and Zhang (2003) compilation of extragalactic radio sources. Thus, we obtain the core-to-lobe luminosity ratio R of these objects for $\alpha_E = 1$ (see Fan and Zhang 2003; Table 1), which distribution gives a mean value (R_m) of 0.16. A coarse treatment of Equation (2) suggests that once R_T is known, R_m can be used to estimate the mean viewing angle (ϕ_m) of a sample in the form (e.g. Odo *et al.* 2015):

$$\phi_m \approx \cos^{-1}\left[1 - \left(\frac{2R_m}{R_T}\right)^{-1/n+\alpha}\right]. \quad (15)$$

Obviously, Equation (15) shows that the viewing angle calculated from R is jet-model dependent (each of the jet models leads to a different value of ϕ_m). A more general situation is to adopt a range of values of ϕ_m bounded by the two models (e.g. Ubachukwu and Chukwude

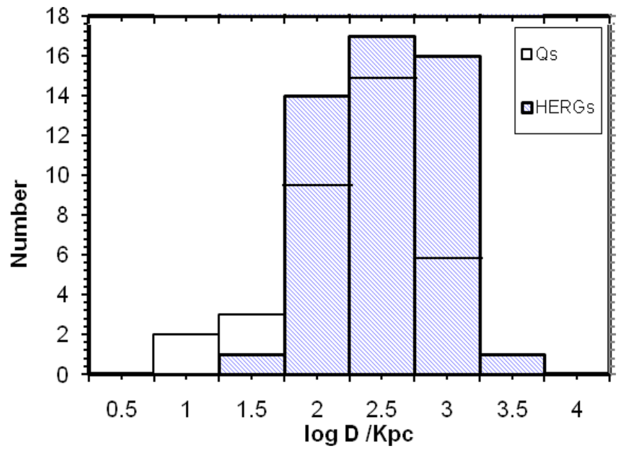

Figure 3. Distributions of the HERGs and quasars (Q) in linear size.

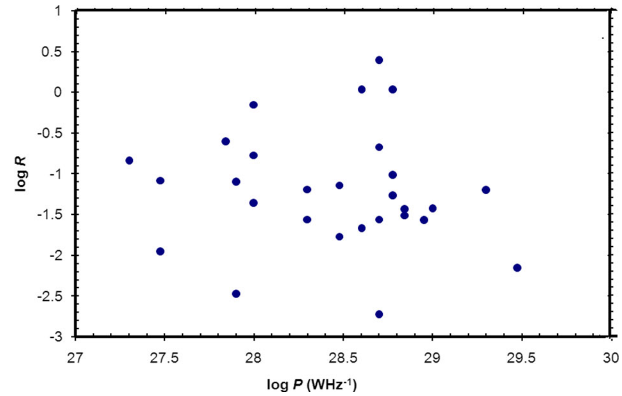

Figure 4. Scatter plot of core-to-lobe luminosity ratio (R) as a function of linear size (D) for the 29 quasars.

2002). Using the results of the R-distribution of the 28 quasars in our sample and $R_T = 0.002$, which appears to be consistent with quasar/ FR II galaxy unification (see Odo *et al.* (2015); Fan and Zhang (2003)), Equation (15) yields $\phi_m \sim 44°$ for $n = 3$ and $\approx 35°$ for $n = 2$. This implies that if radio galaxies are observed at the plane of the sky ($\phi = 90°$), corresponding to R_T, then, quasars are seen in projection at $35° \leq \phi \leq 44°$ to the line of sight, leading to foreshortening of their linear sizes. Thus, we argue that the distribution of linear sizes of quasars which is systematically smaller than radio galaxies, as observed in Figure 3, is attributable to average inclination of radio axes of quasars at $\leq 44°$.

However, since several observed properties of quasars are associated with relativistic beaming (e.g. Cohen *et al.* 1988), we tested these quasar sources for beaming effect by trying to find any correlation between R and D. The result did not yield any significant correlation, with correlation coefficient $r \sim 0.19$ at 95% confidence. The $R - D$ plot of the 29 quasars is shown in Figure 4.

5. *D–P/z* relation

In order to investigate the effects of cosmological linear size evolution and linear size–luminosity correlation in the linear size distributions of the HERGs and quasars in current sample, the observed sizes of quasars are de-projected using the upper limit $\phi = 44°$ as obtained in Section 4 above, to account for the orientation effect (c.f. Equation 10). The distributions of the linear size data after de-projection yields $D_m \sim 248$ Kpc for quasars, 302 Kpc for HERGs and 284 Kpc for the two subsamples taken together. Two sample K–S test yields $\rho = 0.556$ which suggests that when orientation effect is accounted for, there is over 55% probability that the underlying distributions of the linear sizes of HERGs and quasars are statistically similar.

To estimate the model parameters q and w, which characterize the $D-P/z$ relation of radio source samples, usual practice is to adjust w and q until a reasonable fit to the $\theta - z$ data is obtained for an assumed value of Ω (see Ubachukwu 1995). However, since P and $z$are not statistically independent in the redshift bin $z > z_c$ for current sample (P significantly correlates with z above z_c), we express their joint probability density function in the form (Le Cam 1990):

$$f(P, z) = \frac{1}{2\pi\sigma_P\sigma_z\sqrt{1-r^2}} \exp \left[-\frac{1}{2(1-r^2)} \left(\frac{(P-P_m)^2}{\sigma_P^2} - \frac{2r(P-P_m)(z-z_m)}{\sigma_P\sigma_z} + \frac{(z=z_m)^2}{\sigma_z^2} \right) \right], \tag{16}$$

where σ means the standard deviation of the parameter, P_m and z_m are the mean P and z data respectively, and r is the $P - z$ correlation coefficient. The joint probability allows the use of maximum likelihood estimation technique, which is a statistical technique that maximizes the probability density under the assumed relationship (c.f. Equation 8), with constantϕ. The maximum likelihood function can be expressed in general $x-$variable and μ- parameter (e.g. Le Cam 1990; Shannon and Cordes 2010) as:

$$L(\mu; x_1, \dots\dots\dots x_n) = f(x_1, \dots\dots x_n | \mu) = \prod_{i=1}^{n} f(x_i | \mu). \tag{17}$$

We use the maximum likelihood technique on current data to estimate the best-fit parameters of Equation (8)

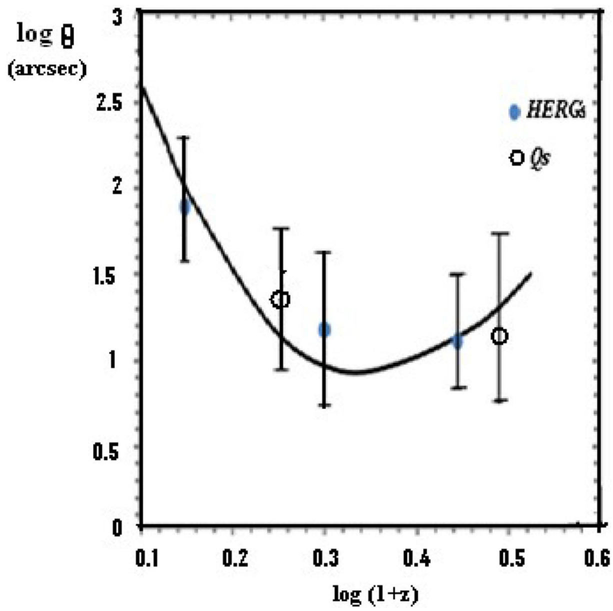

Figure 5. Plot of median angular size against redshift for HERGs and quasars (Q) above z_c.

for the de-projected data as $q = -0.5$ and $w = -0.27$ for the HERGs and quasars taken together above z_c. The result is thus consistent with a common evolution parameter $x = 2.27$ independent of luminosity effect, for both radio galaxies and quasars in the sample. This actually suggests that radio structures of both HERGs and quasars undergo similar evolution, which is in complete agreement with Barthel (1989) model of OUS. We show in Figure 5, the plot of median angular size as a function of redshift for the HERGs and quasars above z_c. In line with Equation (9) the median angular sizes of quasars have been de-projected using $\phi = 44°$. Superimposed on the plot is the theoretical curve (c.f. Equation 7) adjusted for $x = 2.27$. Obviously, this gives a reasonable fit to the observed θ- data of the sample and supports the opinion that HERGs and quasars undergo similar evolution in this redshift regime.

6. Discussion and conclusion

A great effort of research on EGRSs has been directed to the development of a unified scheme aimed at bringing the large number of classes and subclasses of EGRSs under one roof. In the popular orientation-based unified scheme (OUS), known for its simplicity, the observed properties of different classes of EGRSs could be explained as similar objects seen at different orientation angles to the line of sight (Barthel 1989). The OUS makes some simplistic predictions in terms of population statistics, luminosity and size distributions as well as cosmological evolution of the two broad categories of the EGRSs, namely, radio galaxies and quasars. Among the many samples of EGRSs that appear in literature, the 3CRR sample appears to be the most widely studied for the OUS and a wide range of conflicting results have been reported (Kapahi 1989; Singal 1993; Gopal-Krishna and Mangalam 1994; Ubachukwu and Ogwo 1998; Singal and Singh 2013), even with exclusion of LERGs from the scheme (Singal 2014). In particular, the disappearance of foreshortening of radio sizes of quasars compared to HERGs as recently argued by some authors (e.g. Singal 2014), which perhaps is a must in the OUS, seems to be a major challenge to the long-standing OUS and calls for a closer examination of various samples in this sense. Since a vast majority of objects in the 3CRR sample have been well studied and characterized, the sample continues to be a valuable resource for studies of radio sources for many cosmological applications.

We have shown in the results that $z \geq 0.3$; $\log P \geq 27$ W Hz^{-1} represents the parameter space over which the 3CRR sample is complete. Linear regression analysis of the $P - z$ data shows that the slope β changes from ~ 8.6, with a correlation coefficient $r \approx 0.8$ for $z < 0.3$ to ~ 1.1, with $r \approx 0.6$ for $z \geq 0.3$. Thus, in the 3CRR sample, below z = 0.3, the sources show steep dependence of radio luminosity on z, the effect of which could easily swamp those predicted based on source orientation. Padovani *et al.* (2015) argued that the number density of powerful radio loud AGNs evolve positively, peaking around $z = 0.5 \pm 0.1$ and thereafter evolves negatively (see also Rigby *et al.* 2015). This appears to be consistent with the steep increase in luminosity with redshift in this redshift regime. Although both radio galaxies and quasars undergo similar selection effect due to this bias, it is argued in this paper that the steep change in luminosity and number with redshift up to $z = 0.3$ may have affected the determination of the linear size, leading to the apparent disappearance of the expected foreshortening of the sizes of quasars relative to the HERGs in the sample. Alhassan *et al.* (2013) suggested that the steep $P - z$ dependence below $z = 0.3$ created an illusion that swamped relativistic beaming effects for a sample of radio galaxies and BL Lac objects. Perhaps, the linear size data of sources in this redshift range had contaminated the results of Singal (2014), leading to apparent disappearance of the foreshortening effect in quasars relative to HE galaxies, which is based on orientation. Actually, the results presented in this paper on the 3CRR sample above $z = 0.3$ show that on average, the linear sizes of quasars are smaller than radio galaxies by a factor of 1.9. Upon this, we argue that there is a foreshortening of the sizes of quasars compared to HERGs such that the sizes of quasars are smaller than radio galaxies by a factor of ~ 2. Morabito *et al.* (2017) also modeled the redshift dependence of linear sizes of new LOFAR sample and obtained a galaxy-to-quasar size ratio of $\sim$ 2, which is quite consistent with our result.

Another important result of our current analysis is that the average viewing angle of quasars is in the range of 35°–44°. The upper limit inclination angle $\phi \sim 44°$ obtained for quasars is quite consistent with $\phi \approx 45°$ adopted as the dividing line between radio galaxies and quasars (e.g. Barthel 1989; Saikia and Kulkarni 1994), and which is expected to yield a quasar fraction of half that of radio galaxies in both population statistics and size distributions. Actually, using the relative number densities and linear size distributions of radio galaxies and quasars, Barthel (1989) found an average inclination angle of $\sim$ 31° for the 3CRR quasars. A similar result $\phi \sim 25°$ was also obtained by Saikia and Kulkarni (1994). More recently, Morabito *et al.* (2017) found $\sim$ 27° for quasars in the new LOFAR sample. All these are in close agreement with our results.

We have also consistently shown in the results that in all redshift bins, the distributions of the updated 3CRR sample in number ratio of HERGs and quasars are consistent with OUS, with galaxy-to-quasar ratio of ~ 2. It is important to recall that Barthel (1989) proposed the OUS based on sources in the redshift range $0.5 \leq z \leq 1$. However, since radio sources are distributed over a wide redshift range, it is not likely that the OUS would be peculiar to the small redshift regime as used by Barthel (1989). Singal (2014) actually found agreement with the OUS for $z < 0.5$, which includes sources outside the range proposed by Barthel (1989), in both luminosities and relative numbers of HERGs and quasars, which are consistent with the results presented in this paper.

It is interesting to observe from the median values of angular size in Fig. 5 that both HERGs and quasars are fairly represented in the redshift range $z \geq 0.3$. We have demonstrated theoretically and also using observed data that $z = 0.3$ corresponds to the luminosity limit above which the 3CRR sample is complete. In this redshift range, the number of quasars, on average, is systematically smaller than HERGs. The distribution actually yields similar evolution $x = 2.27$ for both HERGs and quasars in the sample. The result appears not to be in agreement with Singal (1993) which argued that galaxies and quasars undergo different size evolution with $x \sim 3.0$ and 0.3 for radio galaxies and quasars respectively. However, Nilsson *et al.* (1993) had argued that there is no obvious difference in the $D - /z$ relations for both radio galaxies and quasars. Similarly, Okoye and Onuora (1982) suggested that depending on the value of cosmological density parameter (Ω_0), the linear size evolution parameter x is in the range $1 \leq x \leq 2$ for both radio galaxies and quasars. Oort *et al.* (1987) further obtained a steeper linear size evolution with $x = 3.0 \pm 0.5$ for radio galaxies. Ubachukwu and Onuora (1993) suggested a luminosity dependent size evolution $D \sim P^{\pm q}(1+z)^{-x}$ with $x = 2$ and 3 for radio galaxies and quasars respectively, which were disentangled by Ubachukwu and Ogwo (1998) with an introduction of density dependent size evolution. All these pictures are in agreement with our present results and are quite consistent with the Barthel (1989) model of OUS.

It is apparent from the distribution of R in Figure 4 that a vast majority of the quasars in the sample are lobe-dominated and thus are not selected on the basis of core radio emission, thus, clearing out any ambiguity that may have arisen from the illusion associated with relativistic beaming in the quasar sources. Furthermore, the lack of any significant correlation between R and D actually shows that relativistic beaming is less important in these sources.

In conclusion, we have re-examined the updated 3CRR sample for orientation-based unified scheme. We demonstrated theoretically and also using observational data that $z = 0.3$ corresponds to the flux limit above which the sample is complete and showed that the disappearance of the “must be” foreshortening of radio sizes of quasars relative to radio galaxies, reported earlier by some authors, could be attributed to an illusion due to the strong luminosity-redshift dependence below $z = 0.3$. The galaxy-to-quasar size ratio of $\sim$ 2 and similar evolution of HERGs and quasars obtained in this paper after correcting for the luminosity–redshift dependence suggest that the 3CRR sample is consistent with orientation-based unified scheme when the LERGs are excluded.

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References

Aird J., Wandra K., Laird E. S., Georgakakis A., Ashby M. 2010, MNRAS, 401, 2531

Alhassan J. A., Ubachukwu A. A., Odo F. C. 2013, JOAA, 34, 61
Barthel, P. D. 1989, ApJ, 33, 606
Buttiglione S., Capetti A., Celotti A., *et al.* 2010, A & A, 509, A6
Capetti A., Massaro F., Baldi R. D. 2017, A & A, 601, A81
Cohen M. H., Barthel P. D., Pearson T. J., Zensus J. A. 1988, ApJ, 329, 1
Fan J. H., Zhang J. S. 2003, A & A, 407, 899
Fanaroff B. L., Riley J. M. 1974, MNRAS, 167, 31P
Grimes J. A., Rawlings S., Willott C. J. 2004, MNRAS, 349, 503
Gopal-Krishna Kulkarni V. K., Mangalam A. V. 1994, MNRAS, 268, 459
Hardcastle M. J., Evans D. A., Croston J. H. 2009, MNRAS, 396, 1929
Jackson C. A., Wall J. V. 1999, MNRAS, 304, 160
Kapahi V. K. 1989, ApJ, 97, 1
Laing R. A., Jenkins C. R., Wall J. V., Unger S. W. 1994, in Bicknell, G. V., Dopita, M. A Quinn, P. J. ASP conference series, 54, 201
Le Cam L. 1990, ISI Review, 58, 153
Ledlow M. J., Owen F. N. 1996, AJ, 112, 9
Leipski C., Haas M., Willner S., *et al.* 2010, ApJ, 717, 766
Lind K. R., Blandford R. D. 1985, ApJ, 295, 358
Masson C. R. 1980, MNRAS, 187, 253
Mattig W. 1959, Astron. Nach., 285, 1
Miley G. K. 1968, Nature, 218, 93
Morabito L. K., Williams W. I., Duncan K. J., *et al.* 2017, MNRAS, 469, 1883
Nilsson K., Valtonen M. J. Kotilainen J., Jaakkola T. 1993, ApJ, 413, 453
Odo F. C., Ubachukwu A. A., Chukwude A. E. 2012, JAA, 33, 279
Odo F. C., Ubachukwu A. A., Chukwude A. E. 2014, Ap&SS, 349, 939
Odo F. C., Ubachukwu A. A., Chukwude A. E. 2015, Ap&SS, 357, 147
Ogle P., Whysong D., Antonucci R. 2006, ApJ, 647, 161
Okoye S. E., Onuora L. I. 1982, ApJ, 260, 37
Onah C. I., Ubachukwu A. A., Odo F. C., Onuchukwu C. C. 2018, RevMexAA, 54, 1
Onuora L. I. 1991, Ap. J., 377, 36
Onuora L. I., Okoye S. E. 1983, ApJ, 270, 360
Oort M. J. A., Kartgert P., Windhorst R. A. 1987, Nature, 328, 500
Orr M. J., Browne I. W. A. 1982, MNRAS, 200, 1067
Padovani P., Bonzini M., Kellermann K. I., Miller N., Mainieri V., Tozzi P. 2015, MNRAS, 452, 1263
Rigby E. E., Argyle J., Best P. N., Rosario D., Röttgering H. J. A. 2015, A & A, 581, A96
Saikia D. J., Kulkarni V. K. 1994, MNRAS, 270, 897
Shannon R. M., Cordes J. M. 2010, ApJ, 725, 1607
Singal A. K. 1993, MNRAS, 263, 139
Singal A. K. 2014, AJ., 148, 16
Singal A. K., Singh 2013, ApJ., 766, 37
Ubachukwu A. A. 1995, Ap & SS, 228, 195
Ubachukwu A. A., Ogwo J. N. 1998, AJP, 51, 143
Ubachukwu A. A., Onuora L. I. 1993, Ap & SS, 209, 169
Ubachukwu A. A., Okoye S. E., Onuora L. I. 1993, Proc. Of Nig. Acad. Of Sc., 5
Ubachukwu A. A., Chukwude A. E. 2002, JAA, 23, 253
Williams W. L., Rivera G. C. Best P. N., *et al.* 2018, MNRAS, 475, 3429