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Transit cosmological models with perfect fluid and heat flow in Sáez-Ballester theory of gravitation

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Abstract. In this paper, the Bianchi-V universe has been applied to the transitional universe. Exact solutions of Einstein's modified field equations in the framework of Sáez-Ballester theory are obtained with heat conduction and perfect fluid. We have applied the hybrid expansion law for the average scale factor $a = kt^{\alpha}e^{\beta t}$, (where $\alpha \geq 0$, $k > 0$, and $\beta \geq 0$ are constants). This results into a new class of transit models from decelerating universe to the current accelerating universe. The present work also elucidates some of the physical, geometric and kinematic properties of the universe and found them in good agreement with recent observations.

Keywords. Sáez-Ballester theory—Bianchi type-V space-time—Exact solutions—Transit universe.

1. Introduction

Einstein's general relativity has been proved correct once again in the last few decades. As, its application, both theoretically and practically are proved. Researchers have been making efforts to bring in 'alternative theories' and theories like Brans and Dicke (1961) , Nordtvedt (1970), Wagoner (1970), Rose (1972), Dunn (1974), Sáez and Ballester (1986), Barber (1985), La and Steinhardt (1989) have cropped up to be the prominent ones. Basically, two categories of gravitational theories which involves a scalar field ϕ are important. The first category comprises of that in which the scalar field ϕ has the dimension as inverse of G, in which Brans-Dicke holds a place of prominence. The importance of Brans-Dicke is highlighted by the fact that it introduces an additional scalar field ϕ besides metric tensor g_{ij} and ω as a dimensionless coupling constant. The second category comprises of a dimensionless scalar field. The theory which has the coupling of dimensionless scalar field was proposed by Sáez and Ballester, which in addition describes the weak fields. This theory has an answer to the question of missing matter in FRW universe (non-flat). The importance of scalar tensor theories is highlighted by the fact that it solves the problem of the graceful exit in inflation era (Piemental 1997). In the framework of Sáez and Ballester theory many cosmological models have been studied (Reddy *et al.* 2006; Rao *et al.* 2017; Katore *et al.* 2010; Jamil *et al.* 2012; Pradhan *et al.* 2013).

Elizalde *et al.* (2004) have introduced a new dark energy model, with transient accelerated phase. Capozziello *et al.* (2006) have considered unified phantom cosmology from the perspective of dark matter. In their study they found that in the presence of scalar dependent function, the scalar field corresponds to the phantom, in early universe and DE phantom at the late universe. It describes the transition of universe from decelerating to accelerating phase. Nojiri and Odintsov (2004) have considered another method for early time and late time universe which is formulated on phantom cosmology by taking into consideration gravity-scalar system. These are the examples of standard scalar-tensor theory. Recently, Bamba *et al.* (2012) have reviewed different dark energy cosmologies. The dark fluid universe having equation of state (EoS) with inhomogeneous or imperfect fluid was investigated by them.

It is generally assumed that a perfect fluid describes the matter content of the universe, but at the early time of evolution of universe, the effect of heat conduction and magnetic fields may not be neglected. It is quite clear that thermal equilibrium was not there at the starting of the universe but that is attained at some stage later. Recent observations of Type Ia supernova (Perlmutter *et al.* 1999; Riess *et al.* 1998; Tonry *et al.* 2003;

Clocchiatti *et al.* 2006; Riess *et al.* 2004) and CMB anisotropies (Bennett *et al.* 2003; Bernardis *et al.* 2000; Hanany *et al.* 2000) shows that currently the universe is in accelerated expanding phase and had decelerated expansion in the past. So the deceleration parameter (DP) must show signature flipping (Padmanabhan *et al.* 2003; Amendola *et al.* 2003; Riess *et al.* 2001). This boasts the theory that DP is not constant but is time variable.

The anisotropic cosmological models (Bennett *et al.* 2003; Bernardis *et al.* 2000; Hinshaw *et al.* 2009; Dunkley *et al.* 2009) came into prominence due to CMBR and the formation of Helium at the early stages of the evolution of universe. Anisotropy of cosmic expansion is an important quantity. The critical arguments and experimental data gives the signs of existence of the anisotropic phase which approaches to isotropic one, in due course of time. So, it is of prime importance to consider the models with anisotropic background.

The space-time is normally defined by a FLRW cosmology because the observed universe is almost isotropic and homogeneous. The problem is that, in the early universe the matter description was not correctly characterize by FLRW model. The existence of anisotropic phase, which turns into an isotropic one is supported by the modern experimental data and the theoretical arguments (Misner 1968). In creating arbitrary ellipsoidality of the universe and to fine tune the observed CMBR anisotropies of the DE, the framework of Bianchi type universe is found to be useful. The cosmological models with anisotropic EoS, utilizing the present SN Ia data, which allows large anisotropy was investigated by Koivisto and Mota (2008) .

In addition, as of late, Bianchi universes have been picking up increasing enthusiasm of observational cosmology, since the WMAP information (Hinshaw *et al.* 2007) appear to require an expansion to the standard cosmological model with positive cosmological constant that takes after the Bianchi morphology (Jaffe *et al.* 2006; Campanelli *et al.* 2007; Hoftuft *et al.* 2009). It is notable that the consideration of a positive cosmological constant, which is numerically comparable to the vacuum energy with $p = -\rho$ and which is the most straight forward contender for the DE, may prompt the clarification of this observed acceleration of the universe. In this way, the Bianchi models which stays anisotropic are of rather academical intrigue.

Motivated by the above discussions, in the present paper, we propose to investigate Bianchi type-V universe with perfect fluid and heat flow in the framework of Sáez-Ballester theory of gravitation. The paper follows the following structure: In Section 1 the introduction has been given. In Section 2, all the theoretical calculations, including the metric, the field equations of Sáez-Ballester, variation law of scale factor and other physical and kinematic parameters are given. Section 3 is devoted to the discussions of the results obtained in Section 2. Conclusions are summarized in Section 4.

2. Definitions and theoretical calculations

2.1 *The metric*

We consider the spatially homogeneous and anisotropic Bianchi type-V line element of the form :

$$ds^2 = -A^2(t)dx^2 - e^{2mx} \times \left[B^2(t)dy^2 + C^2(t)dz^2\right] + dt^2, \quad (1)$$

where the functions $A(t)$, $B(t)$ and $C(t)$ are the three anisotropic directions of expansion in normal three dimensional space. Those three functions are equal in FRW models due to the radial symmetry and so we have only one function $a(t)$ there. The constant m corresponds to type III Bianchi models for $m = 0$, $m = 1$ corresponds to type V, $m = -1$ corresponds to type VI_0, and all other m give VI_h, where $m = h - 1$. This form of the metric is taken into consideration because it is one of the most effective generalization of Bianchi-I and FRW flat universe.

2.2 *Sáez-Ballester field equations*

The Lagrangian for the Sáez and Ballester (1986) theory is given as :

$$L = R - \omega\phi^k \phi_{,i}\phi^{,i}, \quad (2)$$

where ω, k are arbitrary dimensionless constants, R is the scalar curvature, and ϕ is a dimensionless scalar field and $\phi^{,i}$ is the contraction $\phi_{,\alpha}g^{\alpha i}$. Right here a comma (,) and a semicolon (;) stand for partial and covariant differentiation in the regular manner. For a scalar field having the dimension of G^{-1}, the Lagrangian is not physically admissible because the two terms on the right hand of Equation (2) have different dimensions. However, it is far suitable Lagrangian within the case of dimensionless scalar field.

For the Lagrangian given by Equation (2), the action for Sáez-Ballester theory is defined as:

$$I = \int_{\Sigma} (L + L_m)\,(-g)^{\frac{1}{2}} dx^1 dx^2 dx^3 dx^4, \quad (3)$$

here L_m represents the matter Lagrangian, g represents the determinant of the matrix g_{ij}, x^i are the coordinates, $\sum$ is an arbitrary region of integration (we use geometrized units). Our model, when $k = 0$, look like the Einstein reasoning with minimally massless coupled scalar field coupled to gravity.

By varying the action given by Equation (3) with respect to the metric and the scalar field by the variation principle $\delta I = 0$ gives the modified Einstein's field equations as :

$$G_{ij} - \omega\phi^k\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,l}\phi^{,l}\right) = T_{ij}, \tag{4}$$

and

$$2\phi^k\phi^{;i}_{;i} + k\phi^{k-1}\phi_{,l}\phi^{,l} = 0, \tag{5}$$

here T_{ij} is the stress-energy tensor of the matter Lagrangian L_m and $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor.

As we know the action I is a scalar, it is easy to show that the equations of motion

$$T^{ij}_{;i} = 0, \tag{6}$$

are consequences of the field equations.

For a perfect fluid with heat flow, the energy-momentum tensor has the form :

$$T_{ij} = (p+\rho)u_iu_j - pg_{ij} + h_ju_i + h_iu_j, \tag{7}$$

here ρ is the energy density, p the thermodynamic pressure, u_i the four-velocity of the fluid and h_i is the heat flow vector satisfying :

$$g_{ij}u^iu^j = 1, \tag{8}$$

and

$$h^iu_i = 0. \tag{9}$$

Assuming that the flow of heat is in x - direction only so that $h_i = (h_1, 0, 0, 0)$, h_1 is a function of time. For Equations (7) and (1), the modified Einstein's field Equations (4) and (5) yield the following six independent equations:

$$\frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{10}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{12}$$

$$-\frac{3m^2}{A^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = \rho - \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{13}$$

$$h_1 = m\left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \tag{14}$$

$$\frac{r}{2\phi}\dot{\phi}^2 + \ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \tag{15}$$

The energy-conservation law i.e., equation $T^{ij}_{;j} = 0$ gives:

$$\dot{\rho} + (p+\rho)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{2m}{A^2}h_1. \tag{16}$$

2.3 *Physical and kinematic parameters*

We now define the accompanying parameters to be utilized while solving Einstein's field equations for the metric (1).

The average scale factor a, volume scale factor V and the generalized mean Hubble parameter H for Bianchi type V space-time (1) are defined as :

$$a = (ABC)^{\frac{1}{3}}. \tag{17}$$

$$V = a^3 = ABC. \tag{18}$$

$$H = \frac{\dot{a}}{a} = \frac{1}{3ABC}\left(\dot{A}BC + \dot{B}AC + \dot{C}AB\right) = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{1}{3}(H_x + H_y + H_z), \tag{19}$$

where $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$ represents the directional Hubble parameters in x, y and z directions respectively. Here, and furthermore in what takes after, a dot shows ordinary differentiation with respect to t.

The deceleration parameter q is defined as:

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{20}$$

Next, we define the kinematic parameters, viz. expansion scalar (θ), shear scalar (σ^2) and anisotropy parameter (A_m) as follows :

$$\theta = u^i_{;i}, \tag{21}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{22}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{23}$$

Here $u^i = (0, 0, 0, 1)$ is 4-velocity vector for matter and

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}P^{\alpha}_j + u_{j;\alpha}P^{\alpha}_i\right) - \frac{1}{3}\theta P_{ij}. \tag{24}$$

Here the projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j. \tag{25}$$

In Bianchi type-V, these dynamical scalars θ and σ^2 have the forms :

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{26}$$

$$2\sigma^2 = \left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{3}. \tag{27}$$

Simplifying Equations (10)–(13) in terms of H, q, σ^2 and ϕ we found the following equations :

$$p = H^2(2q-1) - \sigma^2 + \frac{m^2}{A^2} + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{28}$$

$$\rho = 3H^2 - \sigma^2 - \frac{3m^2}{A^2} + \frac{1}{2}\omega\phi^r\dot{\phi}^2. \tag{29}$$

To get the exact solutions of the field equations (10)–(13) we use the well established method of quadrature. For this, subtracting Equation (10) from (11), (10) from (12) and (11) from (12), and also using the relation (17), we get :

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{dt}{a^3}\right), \tag{30}$$

$$\frac{A}{C} = d_2 \exp\left(k_2 \int \frac{dt}{a^3}\right), \tag{31}$$

$$\frac{B}{C} = d_3 \exp\left(k_3 \int \frac{dt}{a^3}\right), \tag{32}$$

here constants of integration are d_1, d_2, d_3 and k_1, k_2, k_3.

From above three equations (30–32), the metric functions A, B and C are obtained explicitly as:

$$A(t) = l_1 a \exp\left(\frac{X_1}{3} \int \frac{dt}{a^3}\right), \tag{33}$$

$$B(t) = l_2 a \exp\left(\frac{X_2}{3} \int \frac{dt}{a^3}\right), \tag{34}$$

$$C(t) = l_3 a \exp\left(\frac{X_3}{3} \int \frac{dt}{a^3}\right), \tag{35}$$

here

$$l_1 = \sqrt[3]{d_1 d_2}, \quad l_2 = \sqrt[3]{d_1^{-1} d_3}, \quad l_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$
$$X_1 = k_1 + k_2, \quad X_2 = k_3 - k_1, \quad X_3 = -(k_2 + k_3),$$

and the constants X_1, X_2, X_3 and l_1, l_2, l_3 satisfy the relations $X_1 + X_2 + X_3 = 0$ and $l_1 l_2 l_3 = 1$.

From Equation (15), the dimensionless scalar field function ϕ in terms of the quadrature expression is found as:

$$\phi = \left[\frac{\phi_0(r+2)}{2} \int \frac{dt}{a^3}\right]^{2/(r+2)}, \tag{36}$$

here ϕ_0 is a constant.

It is obvious from Equations (33)–(36) that once we get the estimation of the average scale factor a, we can without much of a stretch figure out the metric functions A, B, C and the scalar function ϕ explicitly.

2.4 *Law of variation of scale factor*

Since we are representing a transit cosmological model where the Universe has undergone a transition from early decelerating expansion phase to the current accelerating expansion phase, we consider Hybrid Expansion Law (HEL) (Akarsu 2014) for the average cosmological scale factor as a :

$$a(\tau) = a_0 \left(\frac{\tau}{\tau_0}\right)^\alpha e^{\beta\left(\frac{\tau}{\tau_0} - 1\right)}, \tag{37}$$

where $\alpha \geq 0$, $\beta \geq 0$ are constants, and a_0, τ_0 represent the present values of scale factor and the age of the universe respectively.

By using suitable transformations, Equation (37) can be reduced to the following form:

$$a(t) = (kt^\alpha e^{\beta t}), \tag{38}$$

where $k = \frac{a_0}{e^\beta} > 0$, $\frac{\tau}{\tau_0} \to t$, $\alpha \geq 0$ and $\beta \geq 0$ are constants.

The motivation to choose this form of average scale factor is that, it results into a time dependent deceleration parameter (DP), which for the suitable values of the constants leads to the observed and desired transit universe. Also, the HEL is a generalization of both the power law cosmology and the exponential law cosmology. For, if we take $\beta = 0$ HEL reduces to power law $a = k.t^\alpha$ and if take $\alpha = 0$, this reduces into exponential law $a = k.e^{\beta.t}$. The HEL, for $\alpha \neq 0, \beta \neq 0$ will have new directions to explore (Akarsu *et al.* 2014; Kumar *et al.* 2013; Yadav *et al.* 2015; Mahanta *et al.* 2014). The scale factor $a = (kt^\alpha e^{\beta t})$ is also a generalization of another scale factor $a = \alpha cosh(\beta t)$ recently used by Esmaeili *et al.* (2018).

The time dependent deceleration parameter for the scale factor (38), from (20) is obtained as :

$$q(t) = \frac{\alpha}{(\alpha + \beta t)^2} - 1. \tag{39}$$

For the scale factor (38), the metric functions (33)–(35) are calculated as:

$$A(t) = l_1(kt^{\alpha}e^{\beta t}) \exp\left(\frac{X_1}{3}\ F(t)\right), \tag{40}$$

$$B(t) = l_2(kt^{\alpha}e^{\beta t}) \exp\left(\frac{X_2}{3}\ F(t)\right), \tag{41}$$

$$C(t) = l_3(kt^{\alpha}e^{\beta t}) \exp\left(\frac{X_3}{3}\ F(t)\right). \tag{42}$$

where

$$F(t) = \int \frac{dt}{(kt^{\alpha}e^{\beta t})^3} = \frac{t^{3\alpha-1}e^{-3\beta t}}{k^3}\sum_{n=0}^{\infty}\frac{(3\alpha t)^n}{(1-3\alpha)(2-3\alpha)(n+1-3\alpha)}.$$

It should be mentioned here that the series in right hand side is convergent if $1-3\alpha > 0$. This implies $0 < \alpha < \frac{1}{3}$.

With these metric functions A, B and C, the line element (1) is given as:

$$ds^2 = dt^2 - l_1^2(kt^{\alpha}e^{\beta t})^2 \exp\left(\frac{2X_1}{3}\ F(t)\right)dx^2 - e^{2mx}\left[l_2^2(kt^{\alpha}e^{\beta t})^2 \exp\left(\frac{2X_2}{3}\ F(t)\right)dy^2 + l_3^2(kt^{\alpha}e^{\beta t})^2 \exp\left(\frac{2X_3}{3}\ F(t)\right)dz^2\right]. \tag{43}$$

3. Results and discussion

Equation (39) can be rewritten as :

$$q = -1 + \frac{\alpha}{H^2t^2} = -1 + \frac{\alpha}{\left(\beta+\frac{\alpha}{t}\right)^2 t^2}$$

Therefore, by the line element (43), the value of DP (present) is calculated as :

$$q_0 = -1 + \frac{\alpha}{H_0^2 t_0^2}, \tag{44}$$

where H_0 is the present value of Hubble's parameter and t_0 is the current age of the universe. From Equation (39), we observe that the transition point from decelerating to accelerating phase is at $t = \frac{\sqrt{\alpha}-\alpha}{\beta}$, which restricts the α in the range $0 \leq \alpha \leq 1$. We have also seen that $q \to -1$ as $t \to \infty$ which shows the inflationary behavior of the universe at late time. Our model is emerging from decelerating phase to accelerating phase, which now ia a well established fact. The recent observations of SNe Ia, uncover that the present universe is accelerating and the estimation of DP lies on some place in the range $-1 < q < 0$. It takes after that in our derived model, one can pick the estimation of DP steady with the observation. Figure 1 delineates the deceleration parameter (q) versus time which gives the behavior of q as transitional from decelerating to accelerating stage which is predictable with late observations of Type Ia supernovae (Perlmutter *et al.* 1999; Riess *et al.* 1998; Tonry *et al.* 2003; Clocchiatti *et al.* 2006; Riess *et al.* 2004).

When ($q > 0$) it shows that universe is decelerating and when ($q < 0$) then the universe is accelerating. Estimation of the deceleration parameter from the count magnitude relation for galaxies is a troublesome undertaking because of the developmental impacts. Recently, Santos *et al.* (2016) have obtained constraints on free parameters, based on data from Ia supernovae (SN Ia), (BAO), (CMB) and $H(z)$. The H(z) data shows affinity for H_0. Assuming a flat space geometry, $q_f = -1$, as in flat ΛCDM they obtained with 68% of confidence level, $q_0 = -0.54^{+0.05}_{-0.07}$, in excellent agreement with flat Λ CDM. Aubourg *et al.* (2015) and Riess *et al.* (2011) have also obtained $q_0 = -0.56^{+0.06}_{-0.05}$ and $q_0 = -0.59^{+0.06}_{-0.06}$ for the flat, with 95% and 99% of confidence level respectively.

First, we set $\alpha = 0.3$ and $\beta = 0.6$ in Equation (44), we obtain $q_0 = -0.63$. This value is in good agreement with recent observational values obtained in (Santos *et al.* 2016; Aubourg *et al.* 2015; Riess *et al.* 2011). Secondly, if we choose $\alpha = 0.3$ and $\beta = 0.6$, we see that every one of the estimations of physical and geometric parameters are effortlessly integrable. These are the two reasons for limiting to $\alpha = 0.3$ and $\beta = 0.6$ in the accompanying talks of the model and graphical show of physical parameters. It is also mentioned that the units of time in all figures have been taken in Billion Years (Gyr).

Figure 1 corresponding to Equation (39) depicts the plot of DP versus time t. From the figure also, we see that it justifies the above discussions.

From Equation (37), the scalar function ϕ is obtained as :

$$\phi = \left[\frac{\phi_0(r+2)}{2}\int\frac{dt}{(t^k e^t)^{3/n}}\right]^{2/(r+2)}. \tag{45}$$

Using Equations (40)–(42) into Equation (25), the heat flow function h_1 comes out to be :

$$h_1 = \frac{mX_1}{(kt^{\alpha}e^{\beta t})^3}. \tag{46}$$

The heat flow along the x-axes was most extreme in early universe, and it decreases as t increases. On large scale the space is assumed to be homogeneous and

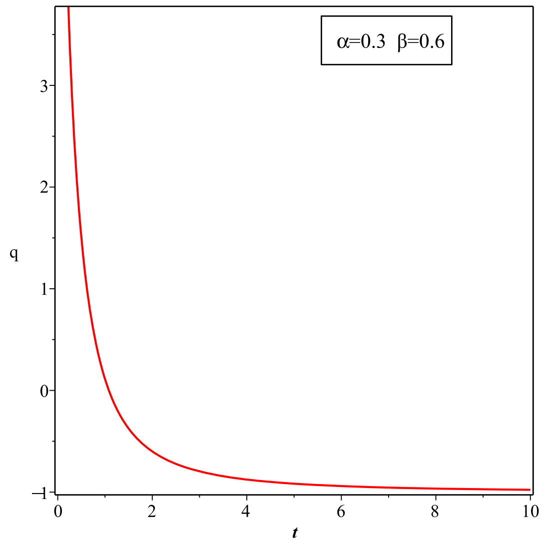

Figure 1. The graph of deceleration parameter q versus t.

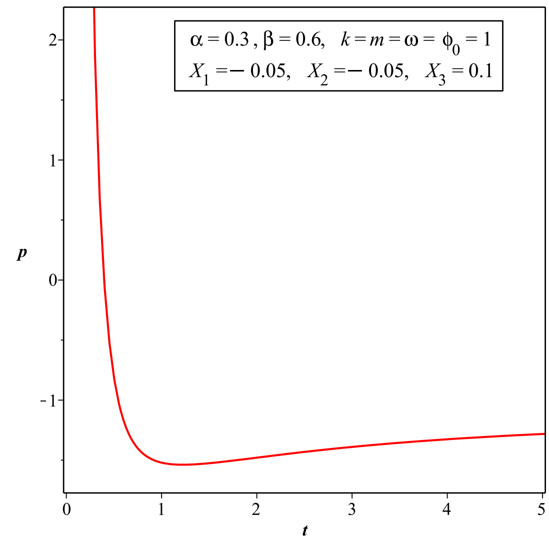

Figure 2. The plot of isotropic pressure p versus t.

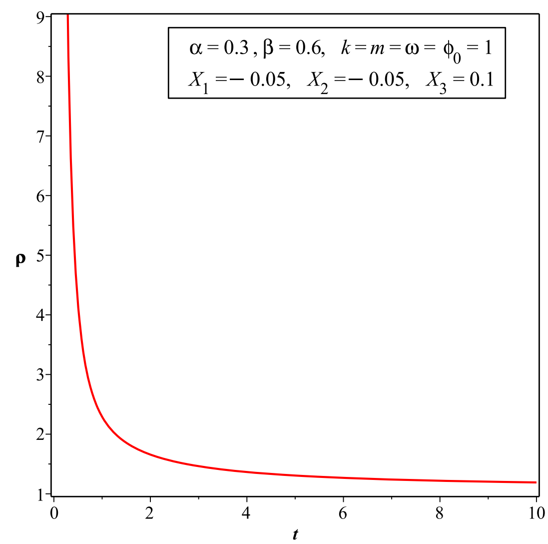

Figure 3. The plot of energy density ρ versus t.

isotropic which corresponds to $m = 0$. In this case $h_1 = 0$ for all t. However, if $m \neq 0$ we have anisotropic space-time and in this case we find $h_1 \neq 0$ for present epoch. As $t \to \infty$ the space-time becomes homogeneous and isotropic and in this case $h_1 \to 0$ for all values of m.

From Equations (46) and (52), it is also observed that $\frac{\sigma^2}{h_1^2}$ is constant, demonstrating that shear scalar is directly proportional to the heat conduction, i.e. greater the heat flow greater the shear.

The isotropic pressure (p) and the energy density (ρ) from Equations (28) and (29), for model (43), are obtained as :

$$p = \frac{2\alpha - 3(\alpha + \beta t)^2}{t^2} - \frac{1}{18(kt^\alpha e^{\beta t})^6}\left[(X_1^2 + X_2^2 + X_3^2) - 9\phi_0\omega\right] + \frac{m^2}{l_1^2(kt^\alpha e^{\beta t})^2}\exp\left[\frac{2X_1}{3k^3}F(t)\right], \tag{47}$$

$$\rho = \frac{3(\alpha + \beta t)^2}{t^2} - \frac{1}{18(kt^\alpha e^{\beta t})^6}\left[(X_1^2 + X_2^2 + X_3^2) - 9\phi_0\omega\right] - \frac{3m^2}{l_1^2(kt^\alpha e^{\beta t})^2}\exp\left[\frac{2X_1}{3k^3}F(t)\right] \tag{48}$$

It has been discovered that the aforementioned solutions fulfill the energy conservation equation (16) identically and hence constitute exact solutions of the Einstein's modified field equations (21–26). From Eq. (48), we observe that the energy density ρ is always positive and decreasing function of time.

Figure 2 plots the isotropic pressure p with time t. We observe from this figure that p was positive in early phase of the universe whereas it is negative at present epoch due to dominance of dark energy. It is a prediction that this negative pressure at present epoch may be a cause of expansion of the universe.

Figure 3 depicts the variation of ρ with t showing it a positive decreasing function of time and approaching to a very small positive value at $t \to \infty$. It is clear from the corresponding graphs that p and ρ approach -1 and 1 respectively. Combining this with the fact that q approaches -1, it means that the model approaches well-known de Sitter stage.

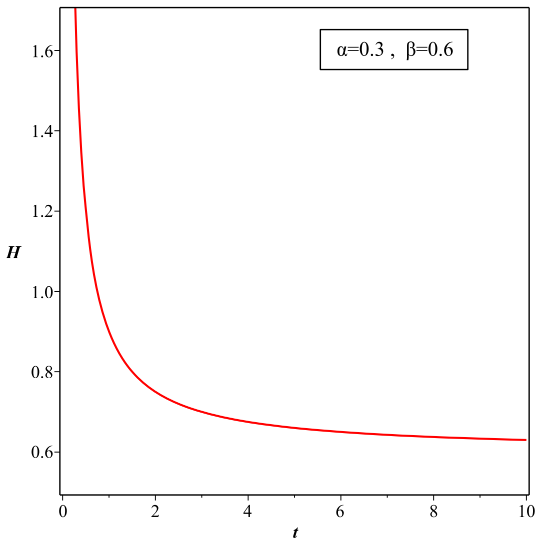

Figure 4. The plot of Hubble parameter H versus t.

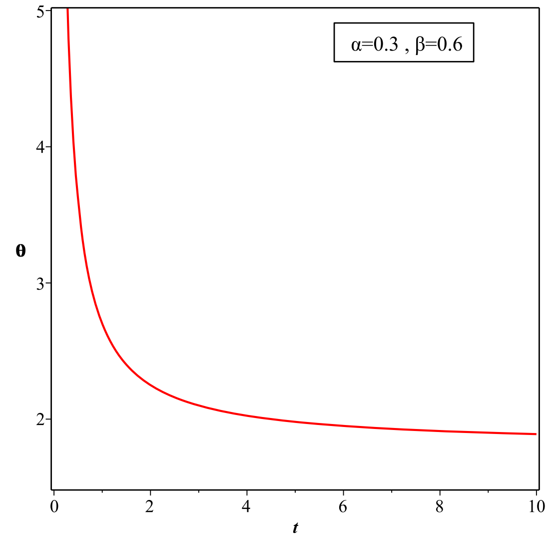

Figure 5. The plot of expansion scalar θ versus t.

The expressions for physical parameters such as spatial volume (V), directional Hubble parameters ($H_i, i = 1, 2, 3$), Hubble parameter (H), scalar of expansion (θ), shear scalar (σ) and the anisotropy parameter (A_m) for model (43) are, respectively, given by :

$$V = (kt^{\alpha}e^{\beta t})^3, \tag{49}$$

$$H_i = \frac{X_i}{3(kt^{\alpha}e^{\beta t})^3} + \beta + \frac{\alpha}{t}, \tag{50}$$

$$\theta = 3H = 3\left(\beta + \frac{\alpha}{t}\right). \tag{51}$$

$$\sigma^2 = \frac{X_1^2 + X_2^2 + X_3^2}{18(kt^{\alpha}e^{\beta t})^6}, \tag{52}$$

$$A_m = \frac{(X_1^2 + X_2^2 + X_3^2)t^2}{27(kt^{\alpha}e^{\beta t})^6(\alpha + \beta t)^2}. \tag{53}$$

It is clear from Equations (49) and (51) that the spatial volume is zero at $t = 0$ and the expansion scalar is infinity, which demonstrate that the universe begins developing with zero volume at $t = 0$ which is big bang scenario. We see from Equations (40)−(42) that the spatial scale factors are zero at the underlying age $t = 0$ and thus the model has a point type singularity (MacCallum 1971). Thereafter, we observe a legitimate volume increments with time.

Figures 4 and 5 show the variation of Hubble parameter (H) and expansion scalar (θ) with time respectively. The graphs show that the Hubble constant is definitely not constant, though it does tend to a constant value at late time. The same behavior of θ is also observed.

Figure 6 corresponds to the Equation (52) and portrays the variation of shear scalar (σ^2) with time t.

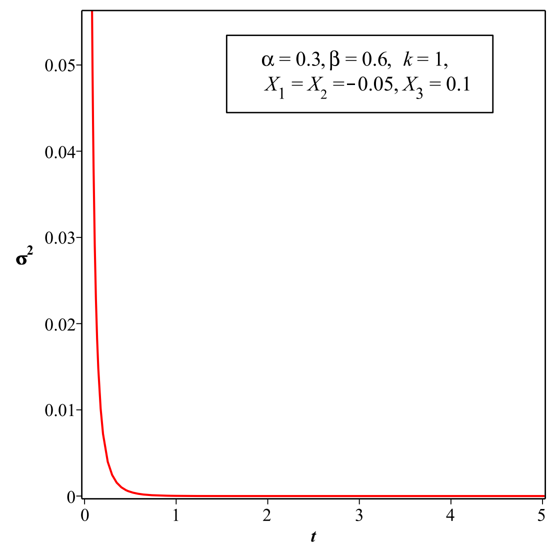

Figure 6. The plot of shear scalar σ^2 versus t.

From the figure, we see that σ^2 decreases with time and $\sigma^2 \to 0$ as $t \to \infty$. It can also be observed that $\lim_{t\to 0}\left(\frac{\rho}{\theta^2}\right)$ is constant. Accordingly the model of the universe goes up homogeneity and matter is progressively irrelevant close to the origin. This is in great concurrence with the outcome given by Collins (1977).

The behavior of the mean anisotropic parameter (53) relies upon $X_1^2 + X_2^2 + X_3^2$ and furthermore when $t \to \infty$, $A_m \to 0$. Therefore, our model has changed from beginning anisotropy to isotropy at present age which is in good agreement with current observations. Figure 7 shows the behavior of anisotropic parameter (A_m) with time t. From the figure, we see that A_m diminishes with time and tends to zero as $t \to \infty$. Accordingly, the observed isotropy of the universe can be accomplished

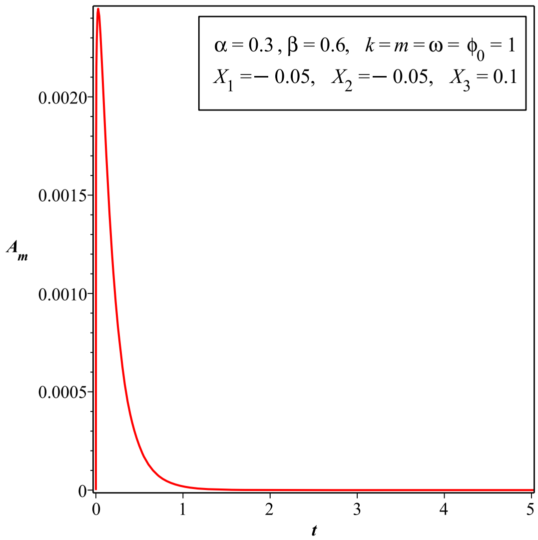

Figure 7. The graph of anisotropic parameter A_m versus t.

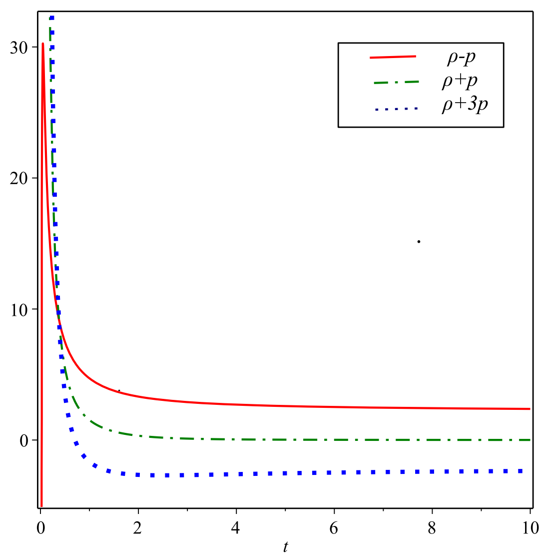

Figure 8. The graph of energy conditions versus t.

in our model at present age. As we have portrayed in introduction that the present universe is homogeneous and isotropic.

3.1 *Energy conditions*

Now we investigate the energy conditions. We know that the weak energy conditions (WEC) and dominant energy conditions (DEC) are given by (i) $\rho \geq 0$, (ii) $\rho - p \geq 0$ and (iii) $\rho + p \geq 0$. The strong energy conditions (SEC) are given by $\rho + 3p \geq 0$.

The left hand side of energy conditions have been graphed in Figure 8. From these figures, we observe that the WEC and DEC for the derived model are satisfied throughout the evolution of the universe. The SEC is satisfied in early stages of the evolution whereas it violets at present epoch because of the dominance of negative pressure. This nature of SEC is acceptable. Thus, we observe that our derived solutions are physically acceptable in concordance with the fulfillment of WEC, DEC and SEC.

4. Conclusion

The primary highlights of the proposed model are :

• The model relies on precise and new solutions of Einstein's modified field equations (due to Sáez and Ballester) for the anisotropic Bianchi type - V universe with perfect fluid and heat flow.

• The investigated model shows a transition of the universe from the early deceleration stage to the current accelerating stage (see Figure 1) which is in great concurrence with recent observations (Caldwell 2006).

• Our entire discourses have been thought by putting $\alpha = 0.3$, $\beta = 0.6$. These values gives $q_0 = -0.63$. This esteem is in great concurrence with the value acquired in observations (Aubourg *et al.* 2015; Riess *et al.* 2011; Caldwell *et al.* 2006).

• For various choices of α and β (of course within the theoretical range), we can generate a class of practical cosmological models of the universe in Bianchi compose V space-time. For instance, in the event that we set $\alpha = 0$, we get the exponential solution as acquired by Ram *et al.* (2009). For $\beta = 0$, we get power law solution. For $k = 1, \beta = \frac{1}{2}, \alpha = \frac{\kappa}{2}$, we find $a = \sqrt{t^{\kappa} e^{t}}$ which is utilized by Pradhan and Amirhashchi (2011) while investigating the accelerating dark energy models in Bianchy type-V space-time, Pradhan *et al.* (2015) in the study of Bianchi type-I in scalar tensor theory of gravitation and Jaiswal and Zia (2018) in the study of anisotropic Bianchi-V dark energy model in Brans-Dicke Theory of Gravitation.

• In our derived model, it has been found that $\lim_{t\to 0}\left(\frac{\rho}{\theta^2}\right)$ is constant. Hence, the model approaches homogeneity and matter is progressively negligible close to the origin.

• We also observed that $\frac{\sigma^2}{h_1^2}$ = constant which demonstrates that shear scalar is directly proportional to heat flow (i.e. $\sigma \propto h_1$), meaning thereby the shear increases with the increase of heat flow.

Therefore, we conclude that our derived model represents a more general model than those studied earlier. The results obtained in this model may be more useful for better understanding of the evolution of the universe as far as Bianchi type-V space-time in the framework of Sáez-Ballester scaler-tensor theory is concerned. Nevertheless, the same approach may also be used in other space-time or in the framework of other modified Einstein's theories.

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