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Behaviour of physical parameters in extended gravity with hyperbolic function

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Abstract. In this paper, we have constructed the cosmological model of the universe in $f(R, T)$ theory of gravity in a Bianchi type VI_h universe for the functional $f(R, T)$ in the form $f(R, T) = \mu R + \mu T$, where R and T are respectively Ricci scalar and trace of energy momentum tensor and μ is a constant. We have made use of the hyperbolic scale factor to find the physical parameters and metric potentials defined in the space-time. The physical parameters are constrained from different representative values to build up a realistic cosmological model aligned with the observational behaviour. The state finder diagnostic pair is found to be in the acceptable range. The energy conditions of the model are also studied.

Keywords. Hyperbolic scale factor—modified gravity—equation of state parameter—effective cosmological constant.

1. Introduction

Several attempts were made and proposed to generalize Einstein's field equations in last few decades. One such attempt is the most recent $f(T)$ gravity theory, which is the generalized version of teleparallel gravity. In the formulation of this $f(T)$ gravity theory, the Weitzenbock connection is used instead of Levi-Civita connection. The significance is that without involving the Dark Energy (DE), the cosmic acceleration of the universe can been defined. In this theory, researchers have done a considerable amount of work (Cai *et al.* 2016; Paliathanasis *et al.* 2016; Mirza & Oboudiat 2017; Khurshudyan *et al.* 2017) to obtain a reason for this cosmic acceleration. A lot of research has been done to address instability problems with the help of General Relativity (GR). But in the presence of dark matter, addressing this instability problem using the cosmological and astrophysical phenomena is not helpful. Particularly, the recent results on expanding universe obtained through observations has put the theoretical cosmology into crisis. Since on large scale, there is some limitations in GR, astrophysicists have turned their attention to modified theories. Among different modified gravity theories, $f(R)$ theory has been well accepted by researchers to address the problem of instability. This theory actually involves a standard Einstein–Hilbert action that includes an arbitrary function R, which is known as Ricci scalar. The importance of this theory is that the cosmic expansion issue can be dealt without involving the dark energy component. The significance of the additional term added to the Einstein–Hilbert Lagrangian is being prominent to address this issue of cosmic expansion (Vollick 2003; Carroll *et al.* 2004; Poplawski 2006; Zaregonbadi *et al.* 2016; Tripathy & Mishra 2016; Cosmai *et al.* 2016).

It is observed that in recent years, the hot big bang cosmological models have gained a lot of attention because of its simple theoretical structure and its ability to handle the complex observational problems. These types of cosmological models were successful in explaining the observed light element abundances and prediction of relic Cosmic Microwave Background (CMB). With this, the cosmological theories have come up with a number of tested predictions such as relic non baryonic dark matter candidates, extra dimensions, inflation, etc. The recent observational data provided by supernovae cosmology project group and the high

z-supernovae group not only bridge the gap between observation and theory of the study but also provide a new avenue to deal with the physics involved in the study of early universe (Riess *et al.* 1998; Schmidt *et al.* 1998; Perlmutter *et al.* 1999). In these observations, sufficient evidences were provided to predict that the universe is expanding in an accelerated manner. It is also predicted that an unknown form of energy called DE is responsible for such an expansion. Hence, in order to address the cosmic speed up phenomena, the study on modified theories of gravity becomes important (Capozziello 2002).

In an effort to address the cosmic speed up issue, Harko *et al.* (2011) introduced a modified gravity theory known as $f(R,T)$ gravity. Several studies were made in this theory addressing different contexts such as energy conditions (Alvarenga *et al.* 2013; Kiani & Nozari 2014), wormhole solution (Azizi 2013; Moraes *et al.* 2017), anisotropy cosmology (Sharif & Zubair 2014; Mishra *et al.* 2016a, b), higher dimensions (Troisi 2017) and non-interacting Chaplygin gas (Shabani 2016; Shabani & Farhoudi 2014). Sharma and Singh (2014) have studied the string cosmological model with magnetic field in Bianchi Type II space-time. With a rescaled functional of $f(R,T)$ gravity, extensive investigations were carried out in Bianchi type VI$_h$ space-time to understand the dynamical behaviour of the aniotropic universe (Mishra *et al.* 2018a, b). Zubair *et al.* (2017) have investigated the anisotropy source with the dynamical analysis of cylindrically symmetric space-time whereas Mishra and Vadrevu (2017a) have constructed a cylindrically symmetric model with the exact solution. Aktas and Aygun (2017) have shown that magnetized field vanishes in FRW universe for $f(R,T)$ gravity. Many more Bianchi type cosmological models have been developed in recent past (Shamir 2015; Zubair & Hassan 2016; Mishra *et al.* 2016a, b; Chaubey & Shukla 2017; Mishra *et al.* 2018c). It can be noted that few works were done on the third model of Harko *et al.* (2011). Houndjo (2012) has developed a cosmological reconstruction of $f(R,T) = f_1(R) + Tf_2(R)$ and discussed a transition of a matter-dominated phase to an accelerated phase. Santos (2013) demonstrated that Godel solution occurs in $f(R,T)$ theory and suggested a path to understand the smallness of the cosmological constant. Shamir (2017) has investigated exact solution of LRS Bianchi I space time in $f(R,T)$ gravity using the third model of Harko *et al.* (2011).

In connection to the Hilbert Einstein type variational principle, Harko *et al.* (2011) derived the field equations for $f(R,T)$ gravity, whose action can be described as

$$S = \int \left(\frac{1}{16\pi} f(R,T)\sqrt{(-g)} + \mathcal{L}_m\sqrt{(-g)}\right) d^4x. \quad (1)$$

Now, keeping in mind the cosmic expansion issue, we consider the functional $f(R,T)$ in the form $f(R,T) = f(R) + f(T)$ and subsequently the $f(R,T)$ field equations can be derived as (Harko *et al.* 2011; Mishra *et al.* 2016a, b)

$$f_R(R)R_{ij} - \frac{1}{2}f(R)g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right) f_R(R) = [8\pi + f_T(T)]\,T_{ij} + \left[pf_T(T) + \frac{1}{2}f(T)\right] g_{ij}, \quad (2)$$

where $f_R = \frac{\partial f(R)}{\partial R}$ and $f_T = \frac{\partial f(T)}{\partial T}$ are the partial differentiations of $f(R,T)$ with respect to their respective variable. If we consider $f(R)$ and $f(T)$ in linear form respectively as $f(R) = \mu R$ and $f(T) = \mu T$, μ being a constant scaling factor, (2) can be reduced to

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \mu}{\mu}\right) T_{ij} + \Lambda(T)g_{ij}, \quad (3)$$

where $\Lambda(T) = p + \frac{1}{2}T$ is an effective cosmological constant that depends on time. Its evolutionary behaviour would be picked up through the matter field components.

We are aware that the scale factor $\mu \neq 0$, as the model diverges for this value. Also, it is certain that one can not recover the corresponding field equations of GR by manually putting a value of μ. However, we may obtain viable models by rescaling the GR equations through this parameter μ. One significant feature of this model is that, we obtain a field equation that appears just like that of GR with a time varying cosmological constant and redefined Einstein constant κ. The burden of late time cosmic acceleration is shared jointly by the terms R and T. Motivated with this development of $f(R,T)$ gravity, in this paper, we have investigated the physical behaviours of the cosmological model obtained with Bianchi type VI$_h$ space-time in the presence of hyperbolic scale factor. The basic framework of $f(R,T)$ gravity along with Bianchi type VI$_h$ space-time in terms of the hyperbolic scale factor for the value of the exponent $h = -1$ has been presented in section 2. In section 3, the derivation and analysis of a general mathematical scheme for the equation of state (EoS) parameter and effective cosmological constant and other physical parameters have been derived. The physical properties of the model along with state finder diagnostic pair and energy conditions are also discussed in section 4. Finally the conclusion is given in section 5. It can be noted that throughout the paper, the adopted physical quantities are expressed in Planckian unit system ($c = G = h = 1$). Also, we have considered, 1 unit of cosmic time = 1 billion years, where c, G and h are the generic constants in the Einstein field equation of general relativity.

2. Basic framework with scale factor

The space-time for Bianchi type VI_h universe can be represented as

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2x}dy^2 - C^2e^{2hx}dz^2, \tag{4}$$

where the metric potentials $A = A(t)$, $B = B(t)$ and $C = C(t)$. We have considered the energy momentum tensor as a composition of two fluids such as the usual perfect fluid and the anisotropic fluid, which can be defined as

$$T_{ij} = (p+\rho)u_iu_j - pg_{ij} + \xi x_ix_j, \tag{5}$$

where $u^iu_i = -x^ix_i = 1$ and $u^ix_i = 0$. In a comoving co-ordinate system, u^i is the four-velocity vector of the fluid. x^i represents the direction of anisotropic fluid in the x-direction and is orthogonal to u^i. p and ρ respectively denotes the pressure and energy density. The energy density is composed of the perfect fluid and anisotropic fluid ξ. Now, the field equations (3) for Bianchi type VI_h space-time (4) with the energy momentum tensor (5) can be obtained as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{h}{A^2} = -\eta p + \eta\xi + \frac{\rho}{2}, \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2} = -\eta p - \frac{\xi}{2} + \frac{\rho}{2}, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\eta p - \frac{\xi}{2} + \frac{\rho}{2}, \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1+h+h^2}{A^2} = -\frac{\xi}{2} - \frac{p}{2} + \eta\rho, \tag{9}$$

$$\frac{\dot{B}}{B} + h\frac{\dot{C}}{C} - (1+h)\frac{\dot{A}}{A} = 0, \tag{10}$$

where $\eta = (16\pi + 3\mu)/2\mu$. A dot over a field variable denotes time differentiation. An interesting part in the space time is the constant exponent h, which takes integral values $-1, 0, 1$. These three integral values decide the behaviour of the model. In a recent paper, Tripathy *et al.* (2015) have calculated the energy and momentum anisotropic $B\mathrm{VI}_h$ universes and shown that the energy and momentum of such universe vanish for $h = -1$. If we assume that the metric should envisage an isolated universe with null total energy and momentum, then it is certain that the choice $h = -1$ is preferable to any other value of the exponent. In view of this, we assume this value of $h = -1$ and study the dynamics of the anisotropic universe in the presence of anisotropic energy sources. The directional Hubble rates may be considered as $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$. With $h = -1$, it is straightforward to get $H_y = H_z$ from (10) by suitably absorbing the integration constant. The mean Hubble parameter becomes, $H = \frac{\dot{a}}{a} = \frac{1}{3}(H_x + H_y + H_z) = \frac{1}{3}(H_x + 2H_z)$. Here, a is the scale factor of the universe. The set of field equations (6)–(10) can be expressed in terms of the scale factor as

$$\frac{6}{(k+2)}\frac{\ddot{a}}{a} - \frac{3(2k-5)}{(k+2)^2}\frac{\dot{a}^2}{a^2} + a^{-\left(\frac{6k}{k+2}\right)} = -\eta p + \eta\xi + \frac{\rho}{2}, \tag{11}$$

$$\frac{3(k+1)}{(k+2)}\frac{\ddot{a}}{a} + \frac{3(2k+1)^2}{(k+2)^2}\frac{\dot{a}^2}{a^2} - a^{-\left(\frac{6k}{k+2}\right)} = -\eta p - \frac{\xi}{2} + \frac{\rho}{2}, \tag{12}$$

$$\frac{9(2k+1)}{(k+2)^2}\frac{\dot{a}^2}{a^2} - a^{-\left(\frac{6k}{k+2}\right)} = -\frac{p}{2} - \frac{\xi}{2} + \eta\rho. \tag{13}$$

3. Derivation and analysis of parameters

In the above set of field equations, a linear anisotropic relation among the directional Hubble rates in the form $H_x = kH_z$, where k is a constant, is assumed to obtain a mathematical scheme for the cosmological parameters. So, an algebraic manipulation of the field equations (11)–(13) yields

$$p = \frac{2}{(1-4\eta^2)(k+2)^2}[[3(k^2+k-2) + 6\eta(k^2+3k+2)]\dot{H} + [9(k^2-k-3) + 18\eta(k^2+k+1)]H^2] - 2\frac{(e^{\int H\cdot dt})^{-\frac{6k}{k+2}}}{1-2\eta}, \tag{14}$$

$$\rho = \frac{2}{(1-4\eta^2)(k+2)^2}[6(k+2)\dot{H} + [27 - 18\eta(2k+1)]H^2] + 2\frac{(e^{\int H\cdot dt})^{-\frac{6k}{k+2}}}{1-2\eta}, \tag{15}$$

$$\xi = -\frac{2}{(1-2\eta)(k+2)}[3(k-1)(\dot{H} + 3H^2)] + 2\frac{(e^{\int H\cdot dt})^{-\frac{6k}{k+2}}}{1-2\eta}. \tag{16}$$

Subsequently, the EoS parameter $\omega = \frac{p}{\rho}$ and the effective cosmological constant Λ are obtained as

$$\omega = -1 + (1+2\eta)\left[\frac{3(k^2+3k+2)\dot{H}+9(k^2-k)H^2}{6(k+2)\dot{H}+27H^2+(k+2)^2(e^{\int H\cdot dt})^{-\frac{6k}{k+2}}-2\eta[9(2k+1)H^2-(k+2)^2(e^{\int H\cdot dt})^{-\frac{6k}{k+2}}]}\right], \tag{17}$$

$$\Lambda = \frac{6}{(1+2\eta)(k+2)}[\dot{H}+3H^2]. \tag{18}$$

The dynamical features of the model are decided by the physical quantities given in equations (14)–(18). However, these quantities depend on the Hubble parameter H. If we choose a Hubble parameter, the background cosmology and the associated dynamics of the model can be studied. Therefore, here, we have assumed the Hubble parameter as a hyperbolic function in the form of $H = \frac{\dot{a}}{a} = \beta \tanh \beta t$ such that scale factor a can be obtained as $a = \alpha \cosh \beta t$. Both α and β are assumed to be constants (Mishra *et al.* 2017b). Now the set of equations (14)–(18) reduces to

$$p = 6\beta^2\left[\frac{(k^2+k-2)+(2k^2-4k-7)\tanh^2\beta t+2\eta[(k^2+3k+2)+(2k^2+1)\tanh^2\beta t]}{(1-4\eta^2)(k+2)^2}\right] - \frac{2(\alpha\cosh\beta t)^{-\frac{6k}{k+2}}}{1-2\eta}, \tag{19}$$

$$\rho = 6\beta^2\left[\frac{2(k+2)-(2k-5)\tanh^2\beta t-6\eta(2k+1)\tanh^2\beta t}{(1-4\eta^2)(k+2)^2}\right] + \frac{2(\alpha\cosh\beta t)^{-\frac{6k}{k+2}}}{1-2\eta}, \tag{20}$$

$$\xi = -6\beta^2\left[\frac{(k-1)(1+2\tanh^2\beta t)}{(1-2\eta)(k+2)}\right] + \frac{2(\alpha\cosh\beta t)^{-\frac{6k}{k+2}}}{1-2\eta}. \tag{21}$$

Subsequently, the EoS parameter $\omega = \frac{p}{\rho}$ and the effective cosmological constant Λ with respect to hyperbolic scale factor can obtained as

$$\omega = -1 + (1+2\eta)\left[\frac{(k^2+3k+2)+2(k^2-3k-1)\tanh^2\beta t}{[2(k+2)+[(5-2k)-6\eta(2k+1)]\tanh^2\beta t]+(\frac{1+2\eta}{3})\left[\left(\frac{k+2}{\beta}\right)^2(\alpha\cosh\beta t)^{-\frac{6k}{k+2}}\right]}\right], \tag{22}$$

$$\Lambda = \frac{6\beta^2}{(1+2\eta)(k+2)}[1+2\tanh^2\beta t]. \tag{23}$$

4. Physical properties of the model

The graphical representations of the energy density (20) and the EoS parameter (22) have been represented in Figures 1 and 2 respectively. It can be observed that the energy density gradually decreases from early universe to future universe and at late universe it remains fixed. The EoS parameter starts from high negative value at the early phase and maintain the same evolutionary behaviour at late phase satisfying the recent observational range according to Planck's data (Ade *et al.* 2016).

The graphical representation of the Hubble parameter is shown in Fig. 3. The parameter is governed by the constant β and the cosmic time. Since we have already assumed a positive constant β to obtain a model that fits observationally, the parameter is now totally controlled by the cosmic time. It can be observed that the Hubble parameter starts increasing from low positive value to high value as time increases. The deceleration parameter q can be defined as $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\cosh^2\beta t$. In Fig. 4, we have shown the evolution of the deceleration parameter as a function of redshift. It can be observed that the deceleration

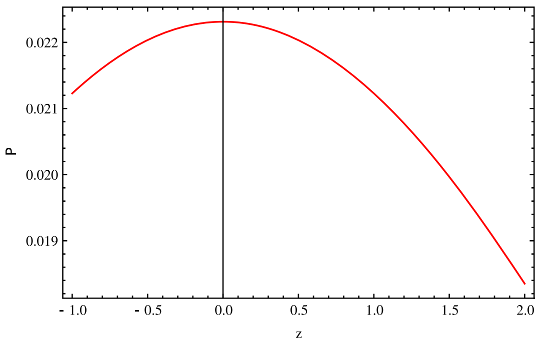

Figure 1. Energy density ρ vs. redshift z ($\alpha = 0.58, \beta = 0.22, k = 0.98, \mu = -0.19$).

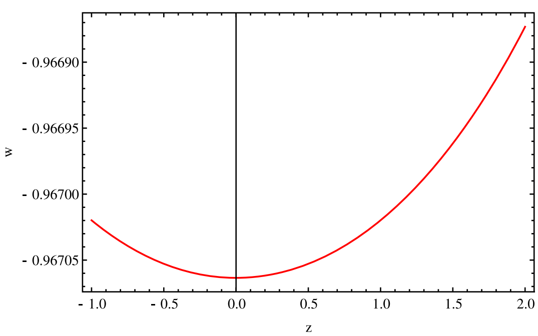

Figure 2. EoS parameter ω vs. redshift z ($\alpha = 0.58, \beta = 0.22, k = 0.98, \mu = -0.19$).

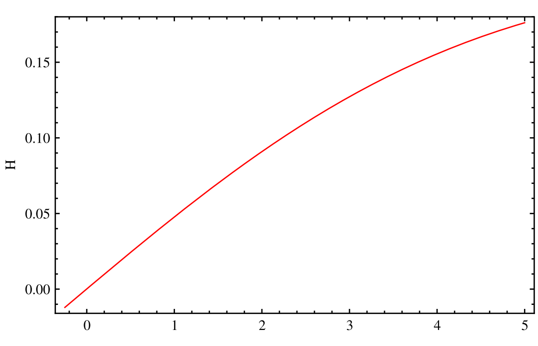

Figure 3. Hubble parameter H vs. redshift z ($\alpha = 0.58, \beta = 0.22, k = 0.98, \mu = -0.19$).

parameter decreases from -1 to a high negative value. Since the scale factor is hyperbolic and can never be negative, it confirms that the deceleration parameter will always remain in the negative domain.

Some kinematical parameters of the universe such as the volume expansion V, scalar expansion θ, shear scalar σ and average anisotropy parameter $\mathcal{A}$ can be defined as

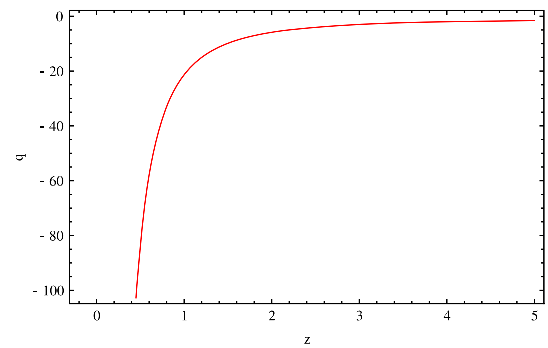

Figure 4. Deceleration parameter q vs. redshift z ($\alpha = 0.58, \beta = 0.22, k = 0.98, \mu = -0.19$).

$$V = a^3 = \alpha^3 \cosh^3 \beta t,$$
$$\theta = \Sigma H_i = 3\beta \tanh \beta t,$$
$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\Sigma H_i^2 - \frac{1}{3}\theta^2\right)$$
$$= \frac{9}{4}\left[\frac{k(k-2)}{(k+2)^2}\right]\beta^2 \tanh^2 \beta t,$$
$$\mathcal{A} = \frac{1}{3}\Sigma\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2}{3}\left(\frac{k-1}{k-2}\right),$$

where $\Delta H_i = H_i - H$ with $i = x, y, z$. $\mathcal{A}$ is a measure of deviation from isotropic expansion. The anisotropy of the expansion results in the isotropic expansion of the universe for $\mathcal{A} = 0$. This can be achieved when $k = 1$, which yields $H_x = H_y = H_z$. Since the representative value of α is considered to be positive and the scale factor is hyperbolic, it can be observed that the volume scale factor becomes positive throughout the evolution and increases gradually with increase in time. The scalar expansion increases with increase in time. The shear scalar vanishes for two representative values of the anisotropy parameter $k = 0, 2$. However, it increases everywhere except in the range $k = (0, 2)$.

The evolutionary behaviour of the energy density of the anisotropic fluid source, ξ, is shown in Fig. 5. Its evolutionary behaviour is studied by assuming a fixed anisotropy and an accelerating universe with the value of a deceleration parameter. The magnitude of ξ is more at an initial epoch compared to late phase. At a late cosmic phase, ξ becomes negligible. It is also observed that ξ increases with increase in the value of μ. The scaling constant also decides the rate of decrement in the energy density of the anisotropic fluid source. Higher the value of μ, completely decides the behaviour of the anisotropy fluid source ξ.

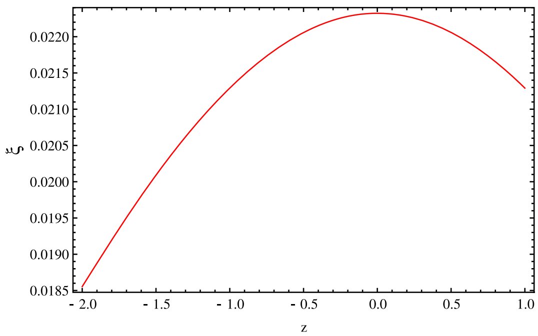

Figure 5. Anisotropic fluid ξ vs. redshift z ($\alpha = 0.58$, $\beta = 0.22$, $k = 0.98$, $\mu = -0.19$).

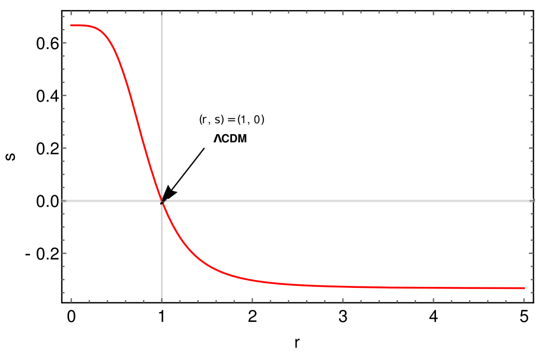

Figure 6. r vs. s ($\alpha = 0.58$, $\beta = 0.22$, $k = 0.98$, $\mu = -0.19$).

The geometrical analysis of DE models are usually performed through the state finder pair r and s given as in Fig. 6 (r and s represents green and pink line respectively). The state finder diagnostic pair can be obtained as $r = \frac{1}{\tanh^2 \beta t}$, $s = \frac{\tanh^2 \beta t - 1}{3(\tanh^2 \beta t)}$. It can be noted that when $\tanh \beta t$ approaches to 1, the state finder diagnostic pair reduces to $(1, 0)$, which further satisfies the validity of the model geometrically.

Based on Raychaudhuri equation (Hawking & Ellis 1999; Tahim *et al.* 2007), the energy conditions are essentially described by the behaviour of a congruence of time-like, space-like or light-like curves. In this work, we will consider the time-like and space-like curves for which the Raychaudhuri equation can be written respectively as

$$R_{ij}U^iU^j + \frac{\theta^2}{3} + \sigma_{ij}\sigma^{ij} - \omega_{ij}\omega^{ij} + \frac{d\theta}{d\tau} = 0, \quad (24)$$

$$R_{ij}k^ik^j + \frac{\theta^2}{2} + \sigma_{ij}\sigma^{ij} - \omega_{ij}\omega^{ij} + \frac{d\theta}{d\lambda} = 0, \quad (25)$$

where U^i and k^i are respectively time-like and space-like vectors tangent to the curves. θ as mentioned earlier is the scalar expansion which describes the expansion of volume. The positive parameters τ and λ are used to describe the curves of the congruence and the shear tensor σ_{ij} measures the distortion of the volume. ω_{ij} is the vorticity tensors that measures the rotation of the curves. The quadratic term in Raychaudhuri equation may be disregarded as the situation is of small distortions of the volume, without rotation. Then, the scalar expansion can be expressed as the function of the Ricci tensor as

$$\theta = -\tau R_{ij}U^iU^j = -\lambda R_{ij}k^ik^j. \quad (26)$$

The condition for attractive gravity is $\theta < 0$ imposing $R_{ij}U^iU^j > 0$ and $R_{ij}k^ik^j > 0$. These two conditions are called strong and null energy conditions respectively. For equivalence to GR, we can divide (2) by $f_R(R)$ to obtain

$$\begin{aligned} R_{ij} - \frac{1}{2}Rg_{ij} = \frac{1}{f_R(R)} &\left([8\pi + f_T(T)]\, T_{ij} \right. \\ &- \left(g_{ij}\Box - \nabla_i\nabla_j\right) f_R(R)\big) \\ &+ \frac{1}{2f_R(R)}\left[2pf_T(T) \right. \\ &\left. + f(T) + f(R) - Rf_R(R)\right] g_{ij}. \quad (27) \end{aligned}$$

Now, the right-hand side of equation (27) can be termed as the effective energy momentum tensor and with the assumed functional of $f(R, T)$, it reduces to the right-hand side of equation (3) and hence we obtain the pressure and energy density as in equations (14) and (15) respectively. So, the energy conditions as defined in GR can also be defined in modified theory of gravity with the new pressure and energy density term as obtained in equations (14) and (15) respectively. Energy conditions put some additional constraints on the models (Mishra *et al.* 2018b). For a perfect fluid distribution, the energy conditions can be obtained from (19)–(20). Without violating the physical parameters behaviour, fixing the relation of the parameters, we have calculated the energy conditions within our formalism of hyperbolic scale factor. Hence, the Null Energy Condition (NEC), Weak Energy Condition (WEC), Strong Energy Condition (SEC) and Dominant Energy Condition (DEC) can be expressed respectively as

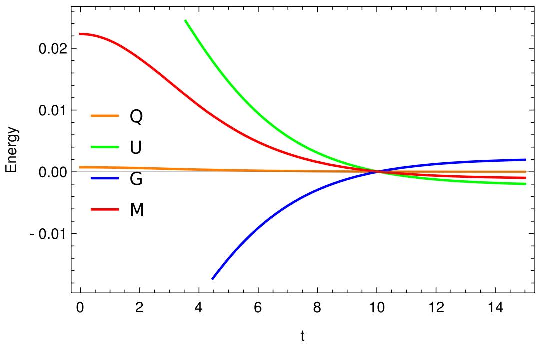

Figure 7. Energy conditions vs. time t ($\alpha = 0.58$, $\beta = 0.22$, $k = 0.98$, $\mu = -0.19$).

$$\rho + p = 6\beta^2 \left[\frac{(k^2+3k+2)(1+2\eta) + \tanh^2 \beta t \left[4\eta(k^2-3k-1) + 2(k^2-3k-6)\right]}{(1-4\eta^2)(k+2)^2} \right] \geq 0, \tag{28}$$

$$\rho + p \geq 0;\ \rho = 6\beta^2 \left[\frac{2(k+2) - (2k-5)\tanh^2 \beta t - 6\eta(2k+1)\tanh^2 \beta t}{(1-4\eta^2)(k+2)^2} \right] + \frac{2(\alpha \cosh \beta t)^{-\frac{6k}{k+2}}}{1-2\eta} \geq 0, \tag{29}$$

$$\rho + 3p = 6\beta^2 \left[\frac{(k+2)\left[(3k-1) + 6\eta(k+1)\right] + \tanh^2 \beta t \left[12\eta(k^2-k) + 2(3k^2-7k-12)\right]}{(1-4\eta^2)(k+2)^2} \right] - \frac{4(\alpha \cosh \beta t)^{-\frac{6k}{k+2}}}{1-2\eta} \geq 0, \tag{30}$$

$$\rho - p = 6\beta^2 \left[\frac{(-k^2+k+6) - 2\eta(k^2+3k+2) - \tanh^2 \beta t \left[4\eta(k^2+3k+2) + 2(k^2-k-1)\right]}{(1-4\eta^2)(k+2)^2} \right] + \frac{4(\alpha \cosh \beta t)^{-\frac{6k}{k+2}}}{1-2\eta} \geq 0. \tag{31}$$

We have also plotted the graphs of all the energy conditions together in Fig. 7, where Q, G and U respectively represent $\rho + p \geq 0$, $\rho + 3p \geq 0$ and $\rho - p \geq 0$. The NEC and DEC behaves opposite. NEC starts from high positive value and moves towards negative value at late phase whereas the DEC starts from a negative value at late phase and moves into the positive domain. This indicates the dominance of the energy density at late phase. At the same time, the DEC though starts from a positive value and remains along the baseline almost till infinite future.

5. Conclusion

The cosmological model of the universe is constructed in $f(R, T)$ gravity with the use of a hyperbolic scale factor. A more systematic mathematical formulation has been developed to express the physical, kinematical parameters as well the metric potentials involved in the study. All the physical parameters are behaving according to the observational outcomes. An accelerating model is presented here, where the physical parameters are constrained with the representative value. The interesting feature of this work is the development of the mathematical expression, where any kind of function can be used for the Hubble parameter and with the physical parameters an accelerating cosmological model can be obtained. The state finder diagnostic pair (r, s) can also be reduced to $(1, 0)$. It can be noted that for different values of scaling constant, the matter content of the gravitational theory remains the same; however the behaviour may change with a very high value. It is also worthy to note that in order to maintain a higher rate of anisotropic expansion, additional anisotropic fluid is very much required.

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