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Review

Supernova neutrinos: Flavor conversion independent of their mass

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Abstract. In extremely dense neutrino environments like in supernova core, the neutrino-neutrino refraction may give rise to self-induced flavor conversion. These neutrino flavor oscillations are well understood from the idea of the exponentially growing modes of the interacting oscillators in the flavor space. Until recently, the growth rates of these modes were found to be of the order of the vacuum oscillation frequency $\Delta m^2/2E$ [$\mathcal{O}(1\ \mathrm{km}^{-1})$] and were considered slow growing. However, in the last couple of years it was found that if the system was allowed to have different zenith-angle distributions for the emitted ν_e and $\bar{\nu}_e$ beams then the fastest growing modes of the interacting oscillators grew at the order of $\mu = \sqrt{2}G_{\mathrm{F}}n_\nu$, a typical ν–ν interaction energy [$\mathcal{O}(10^5\ \mathrm{km}^{-1})$]. Thus the growth rates are very large in comparison to the so-called 'slow oscillations' and can result in neutrino flavor conversion on a much faster scale. In fact, the point that the growth rates are no longer dependent on the vacuum oscillation frequency $\Delta m^2/2E$, makes these 'fast flavor conversions' independent of Δm^2 (thus mass) and energy. This is a surprising result as neutrino flavor conversions are considered to be the ultimate proof of massive neutrinos. However, the importance of this effect in the realistic astrophysical scenarios still remains to be understood.

Keywords. Supernovae—dense neutrinos—flavor conversion.

1. Introduction

Almost 99% of the energy liberated in core collapse supernova (SN) explosion or in neutron-star mergers are released in the form of neutrinos and antineutrinos of all flavors. However, the fluxes and spectra differ strongly between ν_e, $\bar{\nu}_e$ and the other heavy lepton (μ, τ) flavor neutrinos (referred as ν_x). Flavor evolution of these neutrinos can influence the energy deposition away from the decoupling sphere (neutrino sphere), nucleosynthesis, and detection of the neutrino signal from the next galactic supernova or the diffuse neutrino flux from all past core-collapse events (Mirizzi *et al.* 2015). However, all these would require a proper understanding of flavor evolution in neutrino dense environments like SN. This has still remained elusive because of complications arising from the nonlinear nature of collective flavor oscillations. Recently another new idea (Sawyer 2015) got added to this list of complications, i.e., the surprising fact that under certain conditions the collective flavor conversion may not depend on neutrino mixing parameters.

Collective neutrino oscillations are understood in the form of two generic phenomena. The synchronization, i.e., the different modes (different vacuum oscillation frequencies $\omega = \Delta m^2/2E$) of the neutrino mean field oscillate together with a single mean frequency. The other phenomenon is that of the self-induced flavor conversion, connected to the collective modes growing exponential. The growth rates in the linear regime and the overall evolution were thought to be dependent on $\Delta m^2/2E$. However, it has been found that under certain circumstances, i.e., given an appropriate seed the effect can occur even for unmixed neutrinos. Thus flavor conversion can happen even in the absence of neutrino mass (Chakraborty *et al.* 2016).

Collective flavor oscillations effectively represents a flavor reshuffling among different modes but any change of flavor in the overall ensemble. For example, in a dense neutrino gas, the ν_e and $\bar{\nu}_e$ can convert to ν_μ and $\bar{\nu}_\mu$ without change of total lepton number, such pair processes can happen in the form of non-forward scattering (proportional to G_{F}^2), and can also take place at the refractive level (proportional to G_{F}). In general, the conversion rate was actually found to be of the order of

$\Delta m^2/2E$, i.e., ω driven. The other possible frequency for such a system is the neutrino–neutrino interaction $\mu = \sqrt{2}G_F n_\nu$ and for a 'dense' neutrino gas $\mu \gg \omega$. However, this frequency μ remains dominant only when the neutrino and anti-neutrino angle distributions are different and that would translate into 'fast' oscillations. On the other hand, with similar angle distributions the conversion rates are still driven by ω, corresponding to slow conversion. Thus, fast conversions can occur even without any vacuum frequency ω, i.e, even in the absence of neutrino masses (Dasgupta *et al.* 2016; Izaguirre *et al.* 2016; Capozzi *et al.* 2017; Dasgupta 2017).

Therefore, self-induced flavor conversion (in the sense of flavor reshuffling among modes) can occur even without flavor mixing, the pre-condition is that there should exist fluctuations in flavor space which acts as seeds to the unstable modes. Even quantum fluctuations of the mean-field quantities could act as seeds. In fact, the ordinary neutrino oscillations are driven by their masses and mixing parameters, thus the fluctuations in flavor space should always exist and can seed the self-induced flavor conversion. In the next subsections, we describe the basic mathematical formulation of the problem.

1.1 *Defining the model: two-bulb supernova*

Our mathematical formulation is based on the model of Sawyer (2015), mimicking the typical supernova emission features. Here, neutrinos are emerging from a spherical surface, defined as the 'neutrino bulb', and angular characteristic is considered to be blackbody-like. Thus to a distant observer the zenith-angle distribution will be uniform in the variable $\sin^2\theta$ up to a maximum. This maximum is described by the angular size of the neutrino bulb at the observation point. The flavors ν_e and $\bar{\nu}_e$ decouple at different radius, i.e., they are emitted from different neutrino surfaces giving rise to this two-bulb emission model. The ν_e and $\bar{\nu}_e$ zenith-angle distributions depend on this two-bulb emission model and in the supernova environment, the ν_e flux exceeds $\bar{\nu}_e$.

This supernova-motivated model can be formulated in terms of the transverse velocities (Chakraborty *et al.* 2015) and is very similar to the colliding-beam examples but with different velocity distributions for different flavors. In our description, we consider a stationary two-flavor neutrino flux where the flavor evolution is only in the radial direction and no small-scale effects in the transverse direction. The neutrino field beyond the emitting surface is described by the azimuth (φ) and the zenith-angle ($u \propto \sin^2\theta$). The occupied zenith angle range is normalized to some chosen radius. Thus the u-range is independent of the test radius where we perform the stability analysis. The emission spectrum $g(\omega, u)$ has continuous labels ω and u and we assume axial symmetry of emission. Thus $g(\omega, u)$ is independent of φ.

1.2 *Stability equations*

The eigenvalue equations used for the case of axially symmetric neutrino emission were developed by Raffelt *et al.* 2013. The eigenvalue equations are expressed in terms of the integrals

$$I_n = \mu \int d\omega\, du\, \frac{u^n g(\omega, u)}{\omega + u\bar{\lambda} - \Omega} \tag{1}$$

for $n = 0$, 1 and 2. There is no φ dependence as the emission is assumed to be axially symmetric. Here, the effective multi-angle matter effect is described by $\bar{\lambda} = \lambda + \epsilon\mu$, where $\lambda = \sqrt{2}G_F n_e$ with

$$\epsilon = \int d\omega\, du\, g(\omega, u)\,. \tag{2}$$

The spectrum $g(\omega, u)$ is normalized with $\int du\, d\omega\, \mathrm{sign}(\omega)\, g(\omega, u) = 2$, which also defines μ. Here μ has the meaning of a typical neutrino–neutrino interaction. The following eigenvalue equations give the solutions for the eigenvalue Ω,

$$(I_1 - 1)^2 - I_0 I_2 = 0 \quad \text{and} \quad I_1 + 1 = 0\,. \tag{3}$$

The solutions of the first equation correspond to axial symmetry conservation, whereas the solutions of the second one break the axial symmetry spontaneously. The instabilities are found in the limit $\omega = 0$. We also assume that ν_x and $\bar{\nu}_x$ have the same emission characteristics and would drop out from $g(\omega, u)$. The ω-integrated zenith-angle distribution for neutrinos (positive ω) is $h_{\nu_e}(u) = \int_0^\infty d\omega\, g(\omega, u)$ and for anti-neutrinos it is (negative ω) $h_{\bar{\nu}_e}(u) = -\int_{-\infty}^0 d\omega\, g(\omega, u)$. Thus we have

$$\int du\, [h_{\nu_e}(u) + h_{\bar{\nu}_e}(u)] = 2, \tag{4}$$

$$\int du\, [h_{\nu_e}(u) - h_{\bar{\nu}_e}(u)] = \epsilon\,. \tag{5}$$

After the ω integration, the above integrals become

$$I_n = \int du\, \frac{u^n}{u\,(\epsilon + m) - w}\, [h_{\nu_e}(u) - h_{\bar{\nu}_e}(u)]. \tag{6}$$

Here $w = \Omega/\mu$ is the normalized eigenvalue and $m = \lambda/\mu$ describes the matter effect.

The two-bulb model of neutrino emission would mean top-hat u distributions. The occupied u-range is described by the width parameter $-1 < b < +1$ in the form $u_{\nu_e} = 1 + b$ and $u_{\bar{\nu}_e} = 1 - b$. For supernova, the ν_e decouple at a larger distance, and correspond to $b > 0$. The normalized neutrino densities are described by $n_{\nu_e} = 1 + a$ and $n_{\bar{\nu}_e} = 1 - a$ where a is the 'asymmetry parameter' $-1 < a < +1$. Thus the supernova-motivated situation would correspond to the first quadrant $a, b > 0$ and thus the the zenith-angle distribution is

$$h(u) = \frac{1 \pm a}{1 \pm b} \times \begin{cases} 1 & \text{for } 0 \le u \le 1 \pm b, \\ 0 & \text{otherwise}, \end{cases} \tag{7}$$

where the upper sign refers to ν_e and the lower sign to $\bar{\nu}_e$. Therefore the final integrals are

$$I_n = \frac{1+a}{1+b} \int_0^{1+b} \mathrm{d}u \, \frac{u^n}{u\,(2a+m) - w} - \frac{1-a}{1-b} \int_0^{1-b} \mathrm{d}u \, \frac{u^n}{u\,(2a+m) - w}, \tag{8}$$

here $\epsilon = 2a$. We can find the integrals analytically, but the eigenvalues can be only found numerically.

For $b = 0$ (i.e., same zenith-angle distributions for ν_e and $\bar{\nu}_e$) but with different number density ($a \neq 0$), the integrals are

$$I_n = 2a \int_0^1 \mathrm{d}u \, \frac{u^n}{u\,(2a+m) - w}. \tag{9}$$

We found that in this case the eigenvalues are always real, i.e., no fast instability with similar angular distribution for ν_e and $\bar{\nu}_e$. In the following, we study the stability analysis of our described model. The analysis is shown for both with or without matter effect.

2. Results

2.1 *Without matter*

In the first approximation, we neglect the matter term ($m = 0$). This enables a better understanding of the simple system. The first equation (3) is the the axially symmetric solution. In this case, we do not find any instability over the parameter space $-1 < a < 1$ and $-1 < b < 1$. The solution of the other equation in (3), breaks axial symmetry and allows fast flavor conversion. The contour plot in Fig. 1 shows the imaginary part of w (in units of μ), i.e., the growth rates. These results suggest that there are no instabilities when a and b have the same sign, i.e., the first and third quadrants. The fast flavor conversion needs that the flavor (ν_e or $\bar{\nu}_e$) having broader zenith-angle distribution should have a smaller flux.

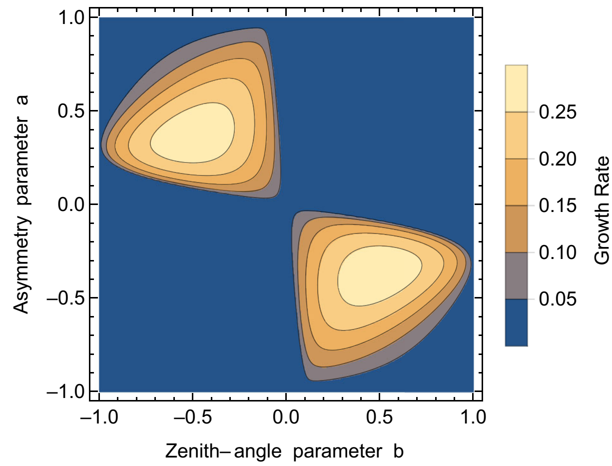

Figure 1. Contour plot of the growth rates (units of μ) for the axial-symmetry breaking case without matter. The normalized density for ν_e is $1 + a$ and $1 - a$ for $\bar{\nu}_e$. In this case, there is no instability in the SN-motivated parameters ($a > 0$ and $b > 0$), i.e., the first quadrant.

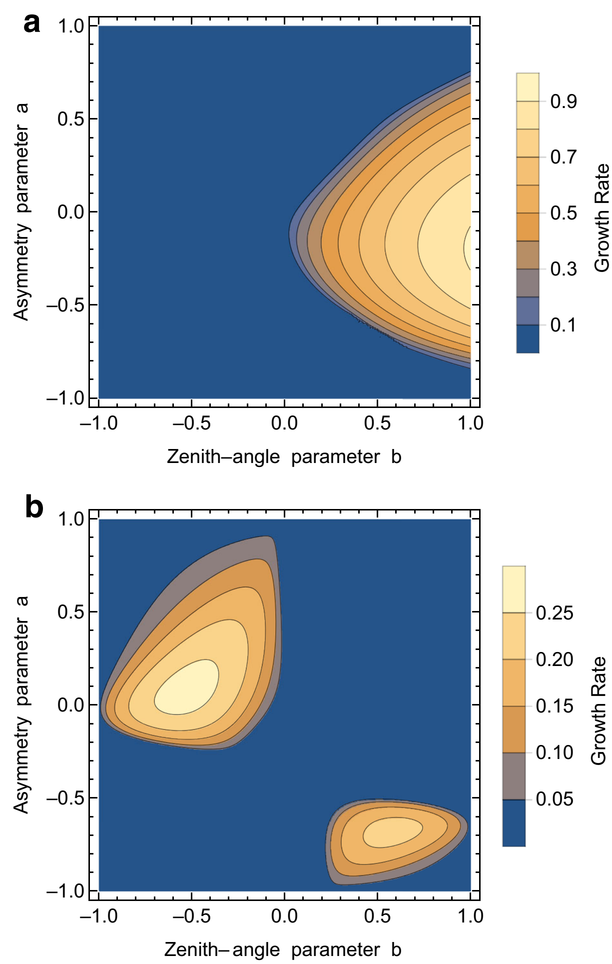

Figure 2. Growth rate (units of μ) similar to Fig. 1, but with matter $m = \lambda/\mu = 1$. (**a**) Axially symmetric. (**b**) Axial symmetry broken.

In this regard, for the neutron-star mergers, $a < 0$, as the flux is dominated by $\bar{\nu}_e$. Similarly, LESA is another interesting scenario with parameters other than the traditional supernova-motivated case.

2.2 *With matter*

The results are very different in the presence of matter. A realistic matter effect would imply λ of the order of μ (say, $m = \lambda/\mu = 1$). The growth rates are shown in Fig. 2. Here one can find fast growth rates for both the axially symmetric and symmetry breaking cases. Thus there are unstable solutions for the parameter space (first quadrant) motivated by SNe, i.e., ν_e distribution is the broader one ($b > 0$) and more ν_e than $\bar{\nu}_e$ ($a > 0$).

However, for very large matter effect ($\lambda \gg \mu$, corresponding to $m \gg 1$), the axially symmetric solution disappears, so it exists only for some range of matter density. Like during the supernova accretion phase, the fast instability would be matter suppressed similar to the 'slow' instabilities (Chakraborty *et al.* 2011a, b). Of course, our discussion was for the homogeneous case only ($\mathbf{k} = 0$) by assuming stationarity of the solution. Thus what would happen in the practical case remains to be understood.

3. Conclusion

We discussed a few examples of the interacting neutrino systems, showing the 'fast flavor conversion'. Here the unstable modes in flavor space grow with rates of the order of the neutrino–neutrino interaction energy $\mu = \sqrt{2}G_{\mathrm{F}}n_{\nu}$ instead of the smaller vacuum oscillation frequency $\omega = \Delta^2 m/2E$. The main conceptual point being that the self-induced flavor conversions are independent of flavor mixing. In the SN context, neutrino flavor conversion would have happened even if flavor mixing among neutrinos did not exist. Notice that the self-induced flavor conversion corresponds to flavor reshuffling among different modes which however may result in flavor decoherence if neighboring modes become effectively uncorrelated.

The idea of fast flavor conversion was proposed twelve years ago by Ray Sawyer (Sawyer 2005) in a three-flavor setup with only a few modes. The system predicted to flavor-equilibrate over very short distances (meters to even centimeters).

Our discussion further these studies and now we understand that the fast flavor conversions are independent of the vacuum oscillation frequencies and thus of the neutrino energy. Hence, the energy spectrum has no role, such fast flavor conversions are rather associated to nontrivial angle distributions. The initial angle distribution must not be too symmetric, a general mathematical condition on the angle distribution would need further studies.

In the supernovae or neutron–star mergers context, the most important query remains if the flavor dependent spectral characteristics of neutrinos after decoupling can be sustained and the self-induced flavor conversion together with the matter effects can lead to this fast flavor decoherence. The spatial and the temporal symmetry breaking with the fast flavor conversions can generate quick decoherence. However, the breaking of spatial homogeneity may be suppressed by the multi-angle matter effect and the breaking of stationarity depends on a narrow resonance condition.

This stability study is too simple to describe the realistic neutrino evolution near the decoupling region of a compact object. The present description of the neutrino mean field with a freely outward streaming neutrino flux is not complete. The neutrino flow is in all directions with different intensities. These toy examples keep the important questions open and are yet to provide the final conclusions for realistic flavor evolution in SN or neutron–star merger events.

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