



## Dense matter in strong gravitational field of neutron star

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**Abstract.** Mass, radius and moment of inertia are direct probes of compositions and Equation of State (EoS) of dense matter in neutron star interior. These are computed for novel phases of dense matter involving hyperons and antikaon condensate and their observable consequences are discussed in this article. Furthermore, the relationship between moment of inertia and quadrupole moment is also explored.

**Keywords.** Neutron stars—equation of state.

### 1. Introduction

The discovery of double pulsar system PSR J0737-3039 is a significant milestone in the study of neutron stars. This discovery paved the way for measuring several post-Keplerian parameters accurately. Consequently, masses of both pulsars in the double pulsar binary were determined with high precision (Burgay *et al.* 2003). The measurement of Shapiro delay parameters led to the first  $\sim 2M_{\text{solar}}$  neutron star (Demorest 2010). With the advent of the Square Kilometre Array (SKA) telescope and its tremendous timing precision, there will be substantial increase in the number of relativistic neutron star binaries.

The spin-orbit coupling which is also known as the Lense–Thirring (LT) effect, in the double pulsar system PSR J0737-3039 is a very promising tool to estimate the moment of inertia of pulsar A. It contributes to the total advance of periastron. If this contribution is comparable to the second Post-Newtonian (2PN) correction, it would have measurable effect (Damour & Schaefer 1988). The LT effect might be extracted from the total advance of periastron using timing measurements. Consequently, it would be possible to estimate the moment of inertia of Pulsar A in the double pulsar system. The moment of inertia along with the accurately measured mass of pulsar A would determine the radius of pulsar A accurately and put constraint on the EoS (Lattimer & Schutz 2005).

Neutron stars are the densest observable compact astrophysical objects and unique laboratories for cold and highly dense matter. The baryon density in the core of a neutron star could exceed by several times normal nuclear matter density ( $2.7 \times 10^{14} \text{ g/cm}^3$ ). Novel phases with large strangeness fraction such as hyperon matter (Glendenning 1992, 1996; Chatterjee & Vidana 2016; Weissenborn *et al.* 2012), Bose–Einstein condensates of strange mesons (Kaplan & Nelson 1986; Pal *et al.* 2000; Banik & Bandyopadhyay 2001a; Knorren *et al.* 1995) and quark matter (Farhi & Jaffe 1984) could possibly exist in neutron star interior due to large baryon chemical potential. The theoretically calculated neutron star mass has to be compatible with the observation of most massive ( $2.01 \pm 0.04M_{\text{solar}}$ ) neutron star (Antoniadis *et al.* 2013).

In this article, we compute mass, radius, moment of inertia and quadrupole moment of slowly rotating neutron stars using different compositions and EoS of matter. In section 2, we discuss strange matter including hyperons and antikaon condensate that might appear in the high density regime of neutron stars. Findings of our calculation are reported in section 3. We conclude in section 4.

### 2. Equation of state for strange matter

We adopt two charge neutral and  $\beta$ -equilibrated strange matter equations of state in this work. We also perform the calculation using the nucleons only EoS of

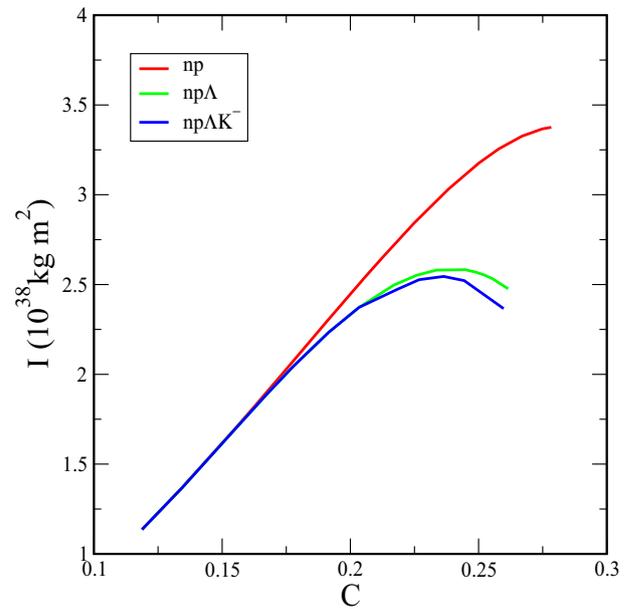
Hempel & Schaffner-Bielich (2010). The matter made of light and heavy nuclei together with unbound but interacting nucleons at low temperature and below the saturation density ( $\sim 2.7 \times 10^{14}$  g/cm<sup>3</sup>) is described within the framework of nuclear statistical equilibrium model (Hempel & Schaffner-Bielich 2010; Banik *et al.* 2014). Furthermore, baryon–baryon interaction in the medium is mediated by exchanges of scalar  $\sigma$ , vector  $\omega$ ,  $\rho$  and  $\phi$  mesons. For this, we use a relativistic mean field model with density dependent couplings (Banik *et al.* 2014; Typel *et al.* 2010). The density-dependent parameter set for nucleon–meson couplings known as the DD2 set is taken from Typel *et al.* (2010).

It has been long argued that besides nucleons, additional degrees of freedom would be populated at high densities in neutron stars. The appearance of hyperons, Bose–Einstein condensate of  $K^-$  mesons or quarks might soften the EoS and reduce the maximum mass. Current astrophysical constraint demands that the theoretically calculated maximum mass neutron star should satisfy  $2M_{\text{solar}}$  benchmark. Here we consider two different compositions of matter with strangeness fraction. In one case, the constituents of high density matter are neutrons, protons and  $\Lambda$  hyperons (denoted by  $n$ ,  $p$ ,  $\Lambda$ ). In the other case, the Bose–Einstein condensate of  $K^-$  mesons along with  $np\Lambda$  is denoted by  $np\Lambda K^-$  (Char & Banik 2014). Meson- $\Lambda$  hyperons and meson- $K^-$  couplings are obtained respectively, from Banik *et al.* (2014) and Char & Banik (2014). An antikaon potential depth of  $-120$  MeV at normal nuclear matter density is used here.

In the following section, we describe the role of compositions and EoS on the mass, radius, moments of inertia ( $I$ ) and quadrupole moment ( $Q$ ) of slowly rotating neutron stars.

### 3. Results and discussion

It is noted that  $\Lambda$  hyperons are populated first at baryon density  $n_b = 2.2n_0$  where the saturation density is  $n_0 = 0.149 \text{ fm}^{-3}$ . The appearance of  $\Lambda$  hyperons delays the onset of antikaon condensation to higher density  $n_b = 3.69n_0$ . It is noted that  $\Lambda$  hyperons and  $K^-$  condensate make the EoS softer compared with that of nucleons-only matter. Next the structures of (non)rotating neutron stars are computed using the  $np$ ,  $np\Lambda$  and  $np\Lambda K^-$  EoS described above. Maximum masses of non-rotating neutron stars corresponding to  $np$ ,  $np\Lambda$ ,  $np\Lambda K^-$  matter are  $2.42$ ,  $2.1$  and  $2.09M_{\text{solar}}$ , respectively. This shows that maximum neutron star masses are compatible with  $2M_{\text{solar}}$  in all three cases.

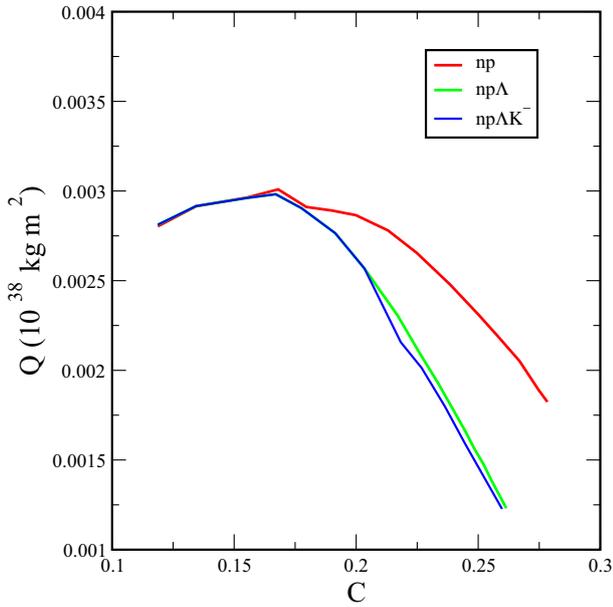


**Figure 1.** Moment of inertia as a function of compactness is shown for different compositions.

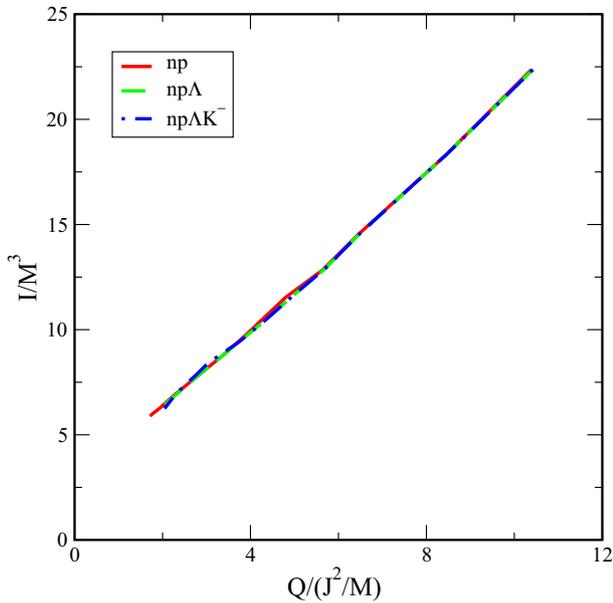
It is evident from observations that majority of neutron stars are slowly rotating. In this calculation, we compute moment of inertia and quadrupole moment of neutron stars rotating with spin-frequency 100 Hz using the LORENE library (Gourgoulhon *et al.* 2016). The theoretically estimated relationships among mass and radius or moment of inertia and compactness parameter (ratio of mass  $M$  and radius  $R$ ) have important implications because those could be directly compared with observed masses, radii and moments of inertia in future.

The moment of inertia is plotted with compactness parameter ( $C = M/R$ ) in Fig. 1 for three different EoS. The effects of compositions of matter on the values of moment of inertia are noticeable at higher compactness. The stiffer EoS of nucleons-only matter ( $np$  case) has higher moment of inertia when  $C > 0.175$  compared with others. The determination of moment of inertia of Pulsar A in the double pulsar system PSR J0737-3039 is becoming highly promising. This might lead to the estimation of radius of pulsar A, as we know the mass very accurately (Lattimer & Schutz 2005).

In Fig. 2, quadrupole moment is shown as a function compactness parameter for different compositions of matter. In all three cases, quadrupole moment decreases with increasing compactness. Quadrupole moment is the measure of how deformed a rotating neutron is. As the neutron star becomes more compact, it becomes less deformed. The deformation is significantly less for the equations of state involving  $\Lambda$  hyperons and  $K^-$  condensate compared with the nuclear matter EoS.



**Figure 2.** Quadrupole moment–compactness relationship of rotating neutron stars for different compositions and angular frequency 100 Hz.



**Figure 3.** Dimensionless moment of inertia vs. quadrupole moment is exhibited for different compositions.

Figure 3 demonstrates the relationship between dimensionless moment of inertia and quadrupole moment. Individually  $I$  or  $Q$  is sensitive to EoS, but their relationship shows universality as evident from Fig. 3. This was first noted by Yagi & Yunes (2013). The implication of  $I$ – $Q$  universal in Fig. 3 is tremendous. It would not be easy to measure the quadrupole moment of a rotating neutron star. We could extract

information about  $Q$  from the  $I - Q$  relation, if we know the moment of inertia. However, strange matter EoS adopted here does not undergo a strong first-order phase transition. This universality might be in question in case of a strong first-order phase transition.

#### 4. Conclusions

We have studied the gross properties of slowly rotating neutron stars using zero temperature strange matter EoS including  $\Lambda$  hyperons and Bose–Einstein condensates of  $K^-$  mesons. Particularly, we have explored relationships among moment of inertia and compactness parameter, quadrupole moment and compactness parameter, and dimensionless moment of inertia and quadrupole moment. It is noted that the universal relationship between dimensionless moment of inertia and quadrupole moment would be an important tool to predict the value of quadrupole moment because of the determination of moment of inertia of pulsar A in PSR J0737-3039 in future as well as in relativistic neutron star binaries in the SKA era.

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