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KP Equation in a Three-Dimensional Unmagnetized Warm Dusty Plasma with Variable Dust Charge

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Abstract. In this work, we investigate the propagation of three-dimensional nonlinear dust-acoustic and dust-Coulomb waves in an unmagnetized warm dusty plasma consisting of electrons, ions, and charged dust particles. The grain charge fluctuation is incorporated through the current balance equation. Using the perturbation method, a Kadomtsev–Petviashvili (KP) equation is obtained. It has been shown that the charge fluctuation would modify the wave structures, and the waves in such systems are unstable due to high-order long wave perturbations.

Keywords. Dust charge fluctuation—unmagnetized dust plasma—KP equation—long-wave perturbations.

1. Introduction

In recent years, plasma physics has been widely studied, and different research had been presented, such as dusty plasma and dust lattice (Duan *et al.* 2007; Xie & He 2001), magnetoplasma (Ren *et al.* 2007; Garcia *et al.* 2005), and so on. Once the heavily charged dust grains are included, the dynamics of the dusty plasma become very different, and the corresponding physical effects are very distinct from those of an ordinary three-component plasma system. Using the reductive perturbation, a KdV equation of the quantum magnetohydrodynamics (QMHD) system has been derived (Feng *et al.* 2011) as well as the amplitude and width of magnetosonic soliton. In the laboratory, it had been shown (Wang *et al.* 1992) that the dust grains exist as impurities and that they significantly influence the behavior of the surrounding plasma. Since the masses of the dust grains are much greater than that of both electrons and ions in the dusty plasma, the dust grains are considered to nearly have no consequence on high frequency oscillations except on the damping factor (Rao *et al.* 1990). Such investigations were also applied to dust ion-acoustic waves (DIAWs) at high frequency (Shukla & Silin 1992). In reality, the dust grains have different size distributions both in space plasma and in the lab experiments (Duan *et al.* 2007; Meuris 1997). The presence of the charged dust grains in a plasma can modify the collective behavior of the plasma, as well as excite new modes (Nejob 1997; Xie 2001; Shukla & Vladimirov 1995; Rosenberg 1993; Tsytovich & Havnes 1993; Bingham *et al.* 1991; Xie 2000). Therefore, the dust charge fluctuation plays a significant role in the plasma collective motion. All previous investigations of charge fluctuation effects have focused on the unmagnetized one-dimensional case. The wave structure and stability in the higher-dimensional system will be modified as the anisotropy is introduced into the system. For example, at least some transverse perturbations will exist in the higher dimensional system, even if the system is unmagnetized. In the present paper, we study the three-dimensional nonlinear waves under the dust charge fluctuation effect. Using the perturbation method, a 3D Kadomtsev–Petviashvili (KP) equation in an unmagnetized dust plasma is deduced. The waves are considered to have variations in the x-direction, and weak variations in the y-direction for simplicity. The combined effects of grain charge fluctuation, the transverse perturbation on the nonlinear wave structures and stability are discussed in detail.

2. Governing equations

The ion and electron densities are governed by the Boltzmann distribution, and defined by their respective temperatures, which can be assumed to be constant,

$$n_i = n_{i0} \exp\left(-\frac{e\Phi}{kT_i}\right),$$
$$n_e = n_{e0} \exp\left(\frac{e\Phi}{kT_e}\right), \tag{1}$$

where n_{i0}, n_{e0} are the equilibrium values of electrons/ions densities respectively, k is Boltzman's constant, and Φ is the normalized electro-static potential.

When the streaming velocities of the electrons and ions are much smaller than their respective thermal velocities, the expressions for the electron and ion currents for spherical grains of radius R are given by their Maxwellian distributions

$$I_e = -\frac{R^2}{b}\frac{\omega_{pe}}{\sqrt{2\pi}\lambda_{De}}(b\psi)\exp(b\phi),$$
$$I_i = \frac{R^2}{m}\frac{\omega_{pi}}{\sqrt{2\pi}\lambda_{Di}}(1-m\psi)\exp(-m\phi), \tag{2}$$

where $b = e/kT_e$, $m = e/kT_i$, $\psi = q_d/R$, $\omega_{pe,i} = (4\pi e^2 n_{e,i0}/m_{e,i})^{1/2}$ is the plasma frequency, $\lambda_{\mathrm{De},i} = (kT_e, i/4\pi e^2 n_{e,i0})^{1/2}$ is the Debye length, and ϕ is the normalized electrostatic wave potential. Here, e, i, d, q_d, n, T are denoted to electron, ion, dust, dust charge, density, and temperature respectively. The exponential factor in equation (2) is due to the interactions of electrons/ions with the negatively charged grains. Here, the current balance equation is given as

$$I_e + I_i = 0. \tag{3}$$

Next, we write the set of normalized three-dimensional equation of continuity, equations of motion, and Poisson's equation describing the dynamics of a dust wave in such a plasma:

$$\frac{\partial n_d}{\partial t} + \frac{\partial u_d n_d}{\partial x} + \frac{\partial w_d n_d}{\partial z} = 0, \tag{4}$$

the x-component of the momentum equation

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial x} + w_d\frac{\partial u_d}{\partial z} + \frac{q_d}{m_d}\frac{\partial \phi}{\partial x} + \frac{v_{td}^2}{n_d}\frac{\partial n_d}{\partial x} = 0, \tag{5}$$

the y-component of the momentum equation

$$\frac{\partial v_d}{\partial t} + u_d\frac{\partial v_d}{\partial x} + w_d\frac{\partial v_d}{\partial z} = 0, \tag{6}$$

and the z-component of the momentum equation,

$$\frac{\partial w_d}{\partial t} + u_d\frac{\partial w_d}{\partial x} + w_d\frac{\partial w_d}{\partial z} + \frac{q_d}{m_d}\frac{\partial \phi}{\partial z} + \frac{v_{td}^2}{n_d}\frac{\partial n_d}{\partial z} = 0, \tag{7}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} + 4\pi(q_d n_d + e n_i - e n_e) = 0, \tag{8}$$

where n_d is the dust particles density, u_d, v_d, and w_d are the velocity components of the dust particles in the x-, y-, and z-direction respectively, m_d and T_d are the mass and temperature of the dust grains.

3. The derivation of the KP equation

We adopt the standard transverse perturbation method, where the independent variables are extended as

$$X = \epsilon x \;\rightarrow\; \frac{\partial}{\partial x} = \frac{\partial X}{\partial x}\frac{\partial}{\partial X} = \epsilon\frac{\partial}{\partial X};\; \frac{\partial}{\partial y} = \frac{\partial Y}{\partial y}\frac{\partial}{\partial Y} = 0,$$
$$Z = \epsilon^{1/2}(z - \lambda t) \;\rightarrow\; \frac{\partial}{\partial z} = \frac{\partial Z}{\partial z}\frac{\partial}{\partial Z} = \epsilon^{1/2}\frac{\partial}{\partial Z},$$
$$T = \epsilon^{3/2} t \;\rightarrow\; \frac{\partial}{\partial t} = \frac{\partial Z}{\partial t}\frac{\partial}{\partial Z} + \frac{\partial T}{\partial t}\frac{\partial}{\partial T}$$
$$= \epsilon^{1/2}\left(-\lambda\frac{\partial}{\partial Z}\right) + \epsilon^{3/2}\frac{\partial}{\partial T}, \tag{9}$$

where the expansion parameter ϵ (real & $\epsilon << 1$) measures the weakness of the amplitude or dispersion and λ is the phase velocity of the wave along the z-direction. The dependent variables are then expanded in terms of the expansion parameter ϵ as

$$n_d = n_{d0} + \epsilon n_{d1} + \epsilon^2 n_{d2} + \cdots,$$
$$u_d = \epsilon^{3/2} u_{d1} + \epsilon^{5/2} u_{d2} + \cdots,$$
$$v_d = \epsilon^{3/2} v_{d1} + \epsilon^{5/2} v_{d2} + \cdots,$$
$$w_d = \epsilon w_{d1} + \epsilon^2 w_{d2} + \cdots,$$
$$q_d = q_{d0} + \epsilon q_{d1} + \epsilon^2 q_{d2} + \cdots,$$
$$\phi = \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots,$$
$$\psi = \psi_0 + \epsilon\psi_1 + \epsilon^2\psi_2 + \cdots. \tag{10}$$

Here, n_{d0} and q_{d0} are the unperturbed number density and charge number of the dust particles respectively. Substituting equations (9) and (10) into equations (4) to (8), and collecting terms with same powers of ϵ, from the coefficient of lowest order (powers of $\epsilon^{3/2}$), we get

$$n_{d1} = \frac{n_{d0}}{\lambda} w_{d1},$$

where $\omega_{d1} = \lambda q_{d0}/(m_d(\lambda^2 - V_{td}^2))\phi_1$, and for higher order of ϵ (up to the second order), we get

$$q_{d1} = -R\mu\phi_1,$$
$$q_{d2} = C_1\phi_2 + C_2\phi_1^2.$$

Here $\mu = \omega_2/\omega_1$, $C_1 = -R\mu$, and $C_2 = -(bR/2)H$, where

$$\omega_1 = L_i + L_e \exp(b\psi_0),$$
$$\omega_2 = L_i(1 - m\psi_0) + L_e \exp(b\psi_0),$$
$$L_{e,i} = R\left(\frac{\omega_{pe,i}}{\sqrt{2\pi}\lambda_{De,i}}\right),$$
$$H = \left(\frac{\mu^2\omega_3}{\omega_1} + \frac{\omega_4}{\omega_1} - 2\mu\frac{\omega_5}{\omega_1}\right),\ \omega_3 = L_e,$$
$$\omega_4 = \omega_2 - L_i(1 - m\psi_0)(1 + T_e/T_i),$$

$\omega_5 = \omega_1 - L_i(1 + T_e/T_i)$, and the dispersion relation can be expressed as

$$\lambda^2 = \frac{\lambda_D^2\omega_{pd}^2}{1 + 4\pi n_{d0}\lambda_D^2 R\mu} + V_{td}^2. \quad (11)$$

The KP equation is then derived from the above equations,

$$\frac{\partial}{\partial Z}\left[\frac{\partial\phi_1}{\partial T} + A\phi_1\frac{\partial\phi_1}{\partial Z} + B\frac{\partial^3\phi_1}{\partial Z^3}\right] + C\frac{\partial^2\phi_1}{\partial X^2} = 0, \quad (12)$$

where the coefficients A, B, and C are given by

$$A = \frac{q_{d0}}{2\lambda m_d}\left[\frac{e m_d M_{DA}^2}{q_{d0} K T_e (1 + N\mu)^2} \left\{\lambda_D^2\left(\frac{T_e}{\lambda_{Di}^2 T_i} - \frac{1}{\lambda_{De}^2}\right) - M_{DA}^2 H\right\} + E\right], \quad (13)$$

$$B = \lambda_D^2 M_{DA}^2/2\lambda(1 + N\mu)^2,\ C = \lambda/2,$$

where $M_{DA}^2 = \lambda_D^2\omega_{pd}$, $\omega_{pd}^2 = 4\pi e^2 q_d^2 n_{d0}/m_d$, $N = 4\pi n_{d0}\lambda_D^2 R$, $E = (1 + N\mu)(3\lambda^2 - V_{td}^2)/M_{DA}^2 - 3N\mu/(1 + N\mu)$. The stationary solitary wave solution of the KP equation equation (12) for $\phi = \phi_1$ can be written as (Gill *et al.* 2006)

$$\phi = \left(\frac{3U_0}{A}\operatorname{sech}^2\sqrt{\frac{U_0}{4B}}(X - U_0 t)\right),$$

where the term $3U_0/A$ is the amplitude of the soliton, and $\sqrt{4B/U_0}$ is its width. The sign of A in equation (13) is related to many dust parameters such as dust charging electrons and ions. Figure 1 clearly shows that the parameter A can be positive or negative for different plasma parameters. $A < 0$ represents the existence of compressive solitary waves, whereas $A > 0$ represents the existence of rarefractive solitary waves. This result is in agreement with Jian-Hong (2009) for the two-dimensional KP equation for dust charge fluctuations.

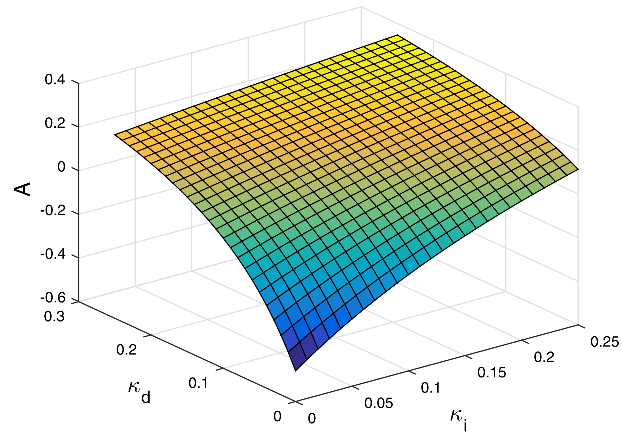

Figure 1. The parameter A from the KP equation (12) for typical dusty plasmas (Chow *et al.* 1993), where $\kappa_i = T_i/T_e$ and $\kappa_d = T_d/T_e$.

4. Conclusion

Solitons play an important role in the transportation of energy. In one-dimensional homogeneous plasma, the dynamical behavior of solitons is governed by the KdV equation (Washimi & Taniuti 1966). In this work, the transverse perturbation method is adopted to investigate the non-linear dynamical equation of dust plasma waves in an unmagnetized plasma, and a KP equation is derived describing the 3-dimensional solitary structures. Our result shows that the coefficient A of the non-linear term of the KP equation can be negative or positive, meaning that both compressive and rarefactive solitons appear in the plasma. The ion/dust particle temperature affects the sign and the magnitude of the coefficient A. $A = 0$ is the case where solitonic solutions may not be established. Whether a compressive or rarefactive solitary wave exists depends on the temperature ratio of the dust particles and ions in the dusty plasma. The coefficients A, B, and C of the KP equation are found to be modified by the effects of transverse perturbation, dust particle temperature and ion temperature. The amplitude $3U_0/A$ and the width $\sqrt{4B/U_0}$ of the soliton are strongly dependent on the dust and ion temperatures; thermal effects cause a decrease in the amplitude and an increase in the width of the solitary structures.

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