J. Astrophys. Astr. (December 2017) 38:64
https://doi.org/10.1007/s12036-017-9480-7

Jeans Instability of the Self-Gravitating Viscoelastic Ferromagnetic Cylinder with Axial Nonuniform Rotation and Magnetic Field

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MS received 15 March 2017; accepted 24 May 2017; published online 27 November 2017

Abstract. The effects of nonuniform rotation and magnetic field on the instability of a self gravitating infinitely extending axisymmetric cylinder of viscoelastic ferromagnetic medium have been studied using the Generalised Hydrodynamic (GH) model. The non-uniform magnetic field and rotation are acting along the axial direction of the cylinder and the propagation of the wave is considered along the radial direction, while the ferrofluid magnetization is taken collinear with the magnetic field. A general dispersion relation representing magnetization, magnetic permeability and viscoelastic relaxation time parameters is obtained using the normal mode analysis method in the linearized perturbation equation system. Jeans criteria which represent the onset of instability of self gravitating medium are obtained under the limits; when the medium behaves like a viscous liquid (strongly coupled limit) and a Newtonian liquid (weakly coupled limit). The effects of various parameters on the Jeans instability criteria and on the growth rate of self gravitating viscoelastic ferromagnetic medium have been discussed. It is found that the magnetic polarizability due to ferromagnetization of medium marginalizes the effect of non-uniform magnetic field on the Jeans instability, whereas the viscoelasticity of the medium has the usual stabilizing effect on the instability of the system. Further, it is found that the cylindrical geometry is more stable than the Cartesian one. The variation of growth rate against the wave number and radial distance has been depicted graphically.

Keywords. Gravitational instability—viscoelastic medium—non uniform magnetic field—non-uniform rotation—ferromagnetic medium—strongly/weakly coupling limit—magnetic polarizability.

1. Introduction

The study of gravitational instability of interstellar medium plays a crucial role in understanding the star formation and its evolution, gravitational collapse, formation and evolution of interstellar molecular clouds and galactic structure. The gravitational instability occurs when an object's self gravity exceeds the opposing forces such as internal gas pressure or material rigidity, and the object collapses. Sir James Jeans was the first who gave the criterion for the onset of gravitational instability in 1902 and now known as Jeans criterion. Jeans (1929) considered an infinite homogeneous self gravitating medium and derived the criterion in terms of wave number. The self gravitational instability was later investigated by many authors including Chandrasekhar (1961), Miyama *et al.* (1984), Larson (1985) and Binney & Tremaine (1987) under the effects of uniform rotation (*Coriolis force*) and magnetic field (*Lorentz force*) and observed that the Jeans criterion remains unaffected in the presence of these effects. Further, Argal *et al.* (2014) revealed in the study of Jeans instability of rotating self-gravitating viscoelastic fluid that the rotation plays a significant role in structure processes of compact systems such as neutron stars, white dwarf stars and supernovae. The instability problems in astrophysical domain are generally investigated by considering the uniform character of magnetic field and rotation. Larson (2003) in the study related to the star formation revealed that this idealization of uniform character of magnetic field and rotation in theoretical studies is valid only for laboratory purposes, however in the interstellar interior and atmosphere, the magnetic field and rotation may be non-uniform/variable and may alter the nature of the instability. He also observed that the magnetic field is an important component in supporting molecular clouds against gravity. The effect of non-uniform rotation on the onset of self-gravitational instability has been investigated by Bel & Schatzman (1958), Anand & Kushwaha (1962), Simon (1962) and recently Dhiman & Dadwal (2010, 2011) have studied the combined effects of non-uniform rotation and magnetic field on the self-gravitating gaseous medium and observed that the Jeans criteria gets modified due to the non-uniform rotation.

Janaki *et al.* (2011) in the study of Jeans instability in a viscoelastic fluid supposed that the transitions between elastic solid state and viscous liquid state give the characteristics of viscoelastic plasma in which both properties exist together. Kaw & Sen (1998), Janaki *et al.* (2011), Rosenberg & Shukla (2011) and Prajapati and Chhajlani (2013) have also studied the behavior of viscoelastic fluid by considering the different problems in the domain of Generalized Hydrodynamic model (GH) as proposed by Frenkel (1946). In GH model, the generalized hydrodynamical equations of momentum transfer for viscoelastic medium incorporates the Frenkel's term or viscoelastic operator $\left(1 + \tau \frac{\partial}{\partial t}\right)$ (cf. Janaki *et al.* 2011) which accounts for the relaxation effects arising out of growing correlations among the particles. Further, we note that the GH model is identical to the well-known MHD model with this difference in the equation of momentum transfer.

Recently, Dhiman and Sharma (2014) studied the onset of gravitational instability of a magnetized viscoelastic medium in the longitudinal and transverse modes of wave propagation under the strongly and weakly coupling limits in the presence of rotation. They observed that the instability criteria get modified due to the viscoelastic effects under the strongly coupling limit, whereas rotation has stabilizing effect on the growth rate of instability. Sharma *et al.* (2015) in the study of Jeans instability of rotating viscoelastic fluid in the presence of uniform magnetic fluid, reported that the strongly coupled plasmas has importance in the interior of large planets, white dwarfs, neutron star, highly compressed solids, semiconductor devices, electrical discharge, nuclear matter, dusty and laser-plasma.

Jones & Spitzer (1967) gave a model for the existence of gas-dust interstellar medium with a highly pronounced property of magnetic polarizability. They reported that, the existence of such a type of medium may be due to a super paramagnetic dispersion of the fine ferromagnetic grains suspended in a gaseous cloud of molecular hydrogen. Ray & Banerji (1980) in the study of perfectly conducting ferrofluid in general relativity reported that the pulsars are the astronomical objects which are formed after a supernova explosion and occurs in massive stars collapsing at the end of their thermonuclear evolution. They also opined that the pulsars were identified, in the year 1968, as highly compact neutron stars in rotation formed by the gravitational collapse of ordinary stars at the end of their thermonuclear evolution, possibly after a supernova explosion. Ray & Banerji (1980) further reported that iron has the largest binding energy per nucleon, it is the end product of thermonuclear evolution. If super massive objects exist as suggested by some astrophysicists (cf. Baade & Zwicky 1934), then they are expected to contain a large proportion of iron at the end and the interior of evolved objects will therefore contain a large proportion of ferrous material and may play an important role in gravitational collapse.

Laroze *et al.* (2013) defined the ferrofluids as the magnetic stable colloidal suspensions of magnetic nanoparticles dispersed in a carrier liquid. They reported that in the absence of an external magnetic field, the magnetic moments of the particle are randomly orientated and there is no net macroscopic magnetization. In an external magnetic field, however, the particle's magnetic moments easily orient and a large (induced) magnetization is present. They investigated the Bénard–Marangoni instability in a viscoelastic ferrofluid and emphasize the effects of the viscoelasticity and the Kelvin force on the instability thresholds. The various aspects of ferrohydrodynamics have been discussed in detail by Rosensweig (1985) in his book.

Mamun and Shukla (2002) also supported the existence of ferromagnetic dust particles in a magnetically supported dark interstellar self-gravitating interstellar molecular cloud. They theoretically studied the propagation characteristics of magnetized waves, as well as the instabilities of sound waves in a self-gravitating dark interstellar molecular cloud containing ferromagnetic dust grains and baryonic gas clouds. In addition to the usual Jeans instability, they also observed that the sound waves suffer a new type of instability, which is due to the combined effects of the baryonic gas dynamics and self-gravitational field in both weakly and highly collisional regimes.

Odenbach (2003) studied the magnetoviscous and viscoelastic effects in ferrofluids and discussed various effects due to the influence of magnetic fields on the rotation of single magnetic nanoparticles as well as cooperative phenomena and their importance for viscous effects in ferrofluids. Mathew *et al.* (2013) studied the gravitational instability in a ferromagnetic fluid saturated porous medium with non-classical heat conduction and investigated the influence of porous, magnetic and non-magnetic parameters on the onset of ferroconvection. They found that the Bénard problem for a Maxwell–Cattaneo ferromagnetic fluid is always less stable than the classical ferroconvection problem. Ram & Sharma (2014) also studied the effect of rotation and MFD viscosity on ferrofluid flow with rotating disk and calculated the displacement thickness of the boundary layer, angle of rotation and an expression for total volume flowing outward the z-axis.

Cylindrical structures of astronomical objects are of great importance in many ways as reported by Dhiman & Sharma (2016) in their work related to the viscoelastic effects on the onset of gravitational instability in the simultaneous presence of non-uniform rotation and magnetic field in cylindrical geometry under the strongly and weakly coupling limits. They observed that the viscoelasticity of the medium modifies the instability criterion under the strongly coupling limit. Dhiman & Dadwal (2012) have studied the effect of ferromagnetic dust cloud on the gravitational instability of a gaseous axisymmetric cylinder in the presence of non-uniform rotation and magnetic field and observed that the effect of non-uniform magnetic field on the gravitational instability is marginalized by the magnetic polarizability of ferrofluid.

On the basis of the above discussions and motivated by various studies related to the possibility of presence of ferromagnetic medium with strongly coupled effects in some astrophysical objects; (in atmosphere of neutron star and supernova) our aim here is to investigate the Jeans instability of an axisymmetric cylinder of a ferromagnetic viscoelastic medium in the presence of non-uniform rotation and magnetic field. Further, the considered physical model is justified as the neutron star is supposed to be composed of viscoelastic fluid with magnetic field $H_0 \sim 1013\,\mathrm{G}$ and rotation $\Omega \sim 100\,\mathrm{s}^{-1}$ (cf. Cumming *et al.* 2004; Rudiger *et al.* 2009; Potekhin & Chabrier 2000) shall be useful to study the instability in the self-gravitating neutron star incorporating effects of both elastic, viscosity and ferromagnetism.

In the present paper, the self-gravitational instability of an infinitely extending axisymmetric cylinder of viscoelastic ferromagnetic medium permeated with non-uniform magnetic field and rotation is studied under both the strong and weakly coupling limits. The physical problem is mathematically formulated using the GH model. The non-uniform magnetic field and rotation are considered to act along the axial direction of the cylinder. The dispersion relation is obtained by using the normal mode analysis and further analyzed under the strongly and weakly coupling limits. The effects of wave number and radial distance on the growth rate of Jeans instability under both strongly and weakly coupling limits have been calculated and the obtained values are depicted graphically. The present paper thus extends to the analysis of Janaki *et al.* (2011), Dhiman & Dadwal (2012) and Dhiman & Sharma (2016) to investigate the effects of viscoelasticity and magnetic polarizability on the onset of gravitational instability of ferromagnetic medium.

2. Mathematical formulation of the problem

Consider an infinitely extending homogeneous, self gravitating, infinitely, electrically conducting viscoelastic ferromagnetic medium permeated with non-uniform magnetic field and rotation.

We shall study the onset of gravitational instability of this physical system by considering the GH model for viscoelastic fluid and the set of ferrohydrodynamic equations given by Shliomis (1972, 2001, 2002) for ferromagnetic medium. Further, it is assumed that the fluid and the ferrous particles have the same velocity while writing the modified momentum transfer equation.

Rosensweig (1985) in his study of ferrohyrodynamics showed that due to the presence of ferrofluid, an additional force (magnetic force) comes into play in the momentum transfer equation, which is given by

$$f_m = -\nabla\left[\mu_0\int_0^H\left(\frac{\partial \vec{M}\upsilon}{\partial \upsilon}\right)_{H,T}\mathrm{d}H + \frac{1}{2}\mu_0\vec{H}^2\right] + \vec{B}\,\nabla\vec{H}$$

which upon utilizing the relation; $\vec{B} = \mu_0(\vec{H} + \vec{M})$ yields

$$f_m = -\nabla\left[\mu_0\int_0^H\left(\frac{\partial \vec{M}\upsilon}{\partial \upsilon}\right)_{H,T}\mathrm{d}H\right] + \mu_0\vec{M}\cdot\nabla\vec{H}.$$

Also, we can have $\mu_0\vec{M}\,\nabla\,\vec{H} = \mu_0\left(\frac{\vec{M}}{\vec{H}}\right)\frac{1}{2}\nabla\,(H\cdot H) = \mu_0\,(\vec{M}\cdot\nabla)\vec{H}$, where μ_0 is the permeability of free space and $\mu_0 = 4\pi\times 10^{-7}$ Henry m^{-1}.

In view of above discussion, it is to note that due to the presence of ferrofluid the momentum transfer equation given by Janaki *et al.* (2011) using the GH model gets modified. Therefore, the equation of continuity, modified momentum transfer equation, Poisson equation and ferrofluid magnetization that govern the above physical model are now given by (cf. Janaki *et al.* 2011; Rosenberg & Shukla 2011; Shliomis 1972, 2001, 2002)

$$\frac{\partial\rho}{\partial t} + (\vec{u}\cdot\ \mathrm{grad}\ )\,\rho + \rho\,(\nabla\cdot\vec{u}) = 0, \tag{1}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho\left(\frac{\partial\vec{u}}{\partial t}+(\vec{u}\cdot\text{ grad})\,\vec{u}\right)-\rho\text{ grad }\phi\right.$$
$$\left.+\text{ grad }p^{*}+\mu_0(\vec{M}\cdot\nabla)\vec{H}-2\rho(\vec{u}\times\vec{\Omega})\right]$$
$$=\mu\nabla^2\vec{u}+\left(\xi+\frac{\mu}{3}\right)\nabla(\nabla\cdot\vec{u}), \tag{2}$$
$$\nabla^2\phi=-4\pi G(\rho-\rho_0), \tag{3}$$
$$\frac{\mathrm{d}\vec{M}}{\mathrm{d}t}=\frac{1}{2}(\nabla\times\vec{u})\times\vec{M}-\alpha(\vec{M}-\vec{M}_0)$$
$$-\beta\vec{M}\times(\vec{M}\times\vec{H}). \tag{4}$$

In Chu formation of electrodynamics (cf. Penfield & Haus 1967), which gives the relation amongst the magnetic field $\vec{H}$, ferrofluid magnetization $\vec{M}$ and magnetic induction $\vec{B}$,

$$\vec{B}=\mu_0(\vec{H}+\vec{M}). \tag{5}$$

In addition to the above equations, we have the following set of magnetic induction equations derived from Maxwell's equations.

$$\nabla\times\vec{H}=0, \tag{6}$$
$$\nabla\cdot\vec{B}=0, \tag{7}$$

we have

$$\nabla\cdot\vec{H}=-\nabla\cdot\vec{M}. \tag{8}$$

In the above equations, $\vec{u}, \vec{H}, \vec{M}, \vec{B}, \vec{\Omega}$ and $\vec{r}$ respectively represent the velocity, magnetic field, ferrofluid magnetization, magnetic induction, rotation and position vectors respectively; $\tau, \rho,$ and ϕ denote the viscoelastic relaxation time, density of fluid and gravitational potential; μ, ξ, μ_0, G and c_s respectively denote the coefficient of viscosity, coefficient of bulk viscosity, magnetic permeability, the universal gravitational constant and the speed of sound in isothermal medium. Further, $\alpha = \tau_{\mathrm{B}} = \frac{3V\eta}{K_{bT}}$ is the Brownian time (magnetization relaxation time) of rotational particle diffusion, $\beta = \frac{1}{6\eta\varphi}$, where η is the dynamic viscosity of the carrier fluid, $\varphi = nV$ is the volume fraction of magnetic grains in the liquid. Here, n is the number density and V the volume of a single particle.

Also, in equation (2),

$$p^{*}=p+\mu_0\int_0^H\left(\upsilon\frac{\partial\vec{M}}{\partial\upsilon}\right)_{H,T}\mathrm{d}M+\mu_0\int_0^H\vec{M}\,\mathrm{d}H,$$

where $p^{*} = p + p_s + p_m$ is the composite pressure and p, p_s and p_m are the hydrostatic pressure, magnetostrictive and fluid magnetic pressure, respectively.

Moreover, in deriving equation (2) we have used the Maxwell constitutive relation given by

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho\left(\frac{\partial\vec{u}}{\partial t}+(\vec{u}\,.\,\text{grad}\,)\,\vec{u}\right)\right.$$
$$\left.-\rho\text{ grad }\phi+\text{ grad }p\right]=\text{ grad }\sigma_{ij},$$

where σ_{ij} represents the strain tensor.

Following Spiegel and Thiffeault (2003), equation (3) representing the gravitational potential above is considered so as to avoid Jeans Swindle. If we consider $\vec{M}$ collinear with $\vec{H}$ at any moment and determined by its instaneous value, and consequently in the phenomenological magnetization equation given by Shliomis (2002), both the magnetization relaxation time (α) and $\vec{M}\times\vec{H}$ vanishes. Also, $(\vec{M}\cdot\nabla)\vec{H}$ vanishes due to collinearty of $\vec{M}$ and $\vec{H}$. Hence, initially when $\vec{u}=0, \rho=\rho_0, p=p_0, \phi=\phi_0, \frac{\partial}{\partial t}=0$ and $\vec{M}, \vec{H}$ are collinear equations (1)–(4) are identically satisfied, thus avoiding the Jeans Swindle in the present case.

In the present paper, we shall consider an infinitely extending axisymmetric cylinder of a homogeneous, self gravitating, infinitely electrically conducting viscoelastic ferromagnetic medium permeated with non-uniform magnetic field and rotation and investigate the onset of instability of this otherwise, stable configuration. In order to study the gravitational instability of axis-ymmetric configurations, the system is described by means of cylindrical coordinates. The cylinder is assumed to be rotating about its axis (z-axis) with non-uniform angular velocity ω. The propagation of wave is taken along the radial direction r of the cylinder, hence $\partial/\partial r$ is the only non zero component of the gradient.

Now, proceeding as in Janaki *et al.* (2011), Dhiman & Dadwal (2012) and Dhiman & Sharma (2016), the system of generalized basic hydrodynamic equations (1)–(4), (6) and (8) that governs this physical configuration representing the equation of continuity, equations of motion, Poisson equation, ferrofluid magnetization and magnetic induction under these assumptions in cylindrical coordinates are respectively given by

$$\frac{\partial\rho}{\partial t}+(\vec{u}\cdot\text{ grad})\,\rho+\rho\,(\nabla\cdot\vec{u})=0, \tag{9}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho\left(\frac{\partial u_r}{\partial t}+(\vec{u}\cdot\text{ grad})\,u_r-\frac{u_\theta^2}{r}\right)\right.$$
$$-\mu_0\left(\frac{\partial H_r}{\partial r}M_r-\frac{M_\theta H_\theta}{r}\right)-\rho\frac{\partial\phi}{\partial r}+c_s^2\frac{\partial\rho}{\partial r}$$
$$\left.-2\rho\,(u_\theta\omega_z-u_z\omega_\theta)\right]$$

$$= \mu \left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right) + \left(\xi + \frac{\mu}{3} \right) \left[\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right) \right], \quad (10)$$

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \left[\rho \left(\frac{\partial u_\theta}{\partial t} + (\vec{u}.\ \text{grad}\ ) u_\theta - \frac{u_r u_\theta}{r} \right) - \frac{\mu_0}{2} \left(\frac{\partial H_\theta}{\partial r} M_r + H_r \frac{\partial M_\theta}{\partial r} + M_\theta \frac{\partial H_r}{\partial r} - H_\theta \frac{\partial M_r}{\partial r} + \frac{2 M_\theta H_r}{r} \right) - \frac{1}{r} \frac{\rho \partial \phi}{\partial \theta} + c_s^2 \frac{1}{r} \frac{\partial \rho}{\partial \theta} - 2 \left(u_z \omega_r - u_r \omega_z \right) \right] = \mu \left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} \right), \quad (11)$$

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \left[\rho \left(\frac{\partial u_z}{\partial t} + (\vec{u} \cdot \text{grad})\, u_z \right) - \frac{\mu_0}{2} \left(\frac{\partial H_z}{\partial r} M_r + H_r \frac{\partial M_z}{\partial r} + M_z \frac{\partial H_r}{\partial r} - H_z \frac{\partial M_r}{\partial r} + \frac{M_z H_r - M_r H_z}{r} \right) - \frac{\partial \phi}{\partial z} + c_s^2 \frac{\partial \rho}{\partial z} - 2 \rho \left(u_r \omega_\theta - u_\theta \omega_z \right) \right] = \mu \left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} \right), \quad (12)$$

$$\frac{1}{r} \left(\frac{\partial}{\partial r} \left(r \frac{\partial \phi_0}{\partial r} \right) \right) = -4\pi G \left(\rho - \rho_0 \right) \quad (13)$$

$$\frac{\partial M_r}{\partial t} + u_r \frac{\partial M_r}{\partial r} + \frac{1}{2} \left(M_\theta \left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \right) + M_z \frac{\partial u_z}{\partial r} \right) + \beta \left(M_\theta \left[M_r H_\theta - H_r M_\theta \right] - M_z \left[M_z H_r - H_z M_r \right] \right) = 0, \quad (14)$$

$$\frac{\partial M_\theta}{\partial t} + u_r \frac{\partial M_\theta}{\partial r} - \frac{1}{2} \left(M_r \left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \right) \right) + \beta \left(M_z \left[M_\theta H_z - H_\theta M_z \right] - M_r \left[M_r H_\theta - H_r M_\theta \right] \right) = 0, \quad (15)$$

$$\frac{\partial M_z}{\partial t} + u_r \frac{\partial M_z}{\partial r} - \frac{1}{2} \left(M_r \frac{\partial u_z}{\partial r} \right) + \beta \left(M_r \left[M_z H_r - H_z M_r \right] - M_\theta \left[M_\theta H_z - H_\theta M_z \right] \right) = 0, \quad (16)$$

$$\frac{\partial H_z}{\partial r} = 0, \quad (17)$$

$$\left(\frac{\partial H_\theta}{\partial r} + \frac{H_\theta}{r} \right) = 0, \quad (18)$$

$$\left(\frac{\partial H_r}{\partial r} + \frac{H_r}{r} \right) = - \left(\frac{\partial M_r}{\partial r} + \frac{M_r}{r} \right). \quad (19)$$

In view of the above physical configuration, the equilibrium (basic) state under discussion is clearly characterized as

$$\vec{u} = (0, r\omega, 0);\ \vec{H} = (0, 0, H_z);\ \vec{M} = (0, 0, M_z); \quad \vec{\omega} = (0, 0, \omega);\ \phi = \phi_0 \ \text{ and } \ \rho = \rho_0. \quad (20)$$

Using the above basic state described in equation (20) in equations (9)–(19), we have the following equations;

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \left[\rho \left(r\omega^2 - \frac{\partial \phi}{\partial r} - 2 u_\theta \omega \right) + c_s^2 \frac{\partial \rho}{\partial r} \right] = 0, \quad (21)$$

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \rho \left(\frac{\partial r\omega}{\partial t} \right) = \mu \left(\frac{\partial^2 r\omega}{\partial r^2} + \frac{1}{r} \frac{\partial r\omega}{\partial r} - \frac{r\omega}{r^2} \right), \quad (22)$$

$$\left(1 + \tau \frac{\partial}{\partial t}\right) (-2\rho u_\theta \omega) = 0, \quad (23)$$

$$\frac{1}{r} \left(\frac{\partial}{\partial r} \left(r \frac{\partial \phi_0}{\partial r} \right) \right) = 0, \quad (24)$$

$$\frac{\partial H_z}{\partial r} = 0. \quad (25)$$

From equation (25), we get

$$H_z = \text{constant}. \quad (26)$$

In order to investigate the gravitational instability of the considered system, let the infinitesimally small perturbations are added to the initial state of the system represented in equation (20). So, we have the following perturbed quantities:

$$\vec{u} = (0 + u_r, r\omega + u_\theta, 0 + u_z)\,; \quad \vec{H} = (0 + h_r, 0 + h_\theta, H_z + h_z)\,; \quad \vec{M} = (0 + m_r, 0 + m_\theta,\ M_z + m_z)\,, \quad \phi = \phi_0 + \delta\phi \ \text{ and } \ \rho = \rho_0 + \delta\rho. \quad (27)$$

Here, (u_r, u_θ, u_z), (h_r, h_θ, h_z), (m_r, m_θ, m_z), $\delta\phi$ and $\delta\rho$ are the perturbations in the basic velocity, magnetic field, magnetization, gravitational potential and density, respectively.

Using these perturbed quantities defined in equation (27) in equations (9)–(19), using the equations (21)–(25) and then linearizing the resulting equations by neglecting the second and higher order perturbed quantities, we get the following linearized perturbation equations:

$$\frac{\partial \delta\rho}{\partial t} + \rho_0 \frac{\partial u_r}{\partial r} + \rho_0 \frac{u_r}{r} = 0, \quad (28)$$

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \left[\rho \left(\frac{\partial u_r}{\partial t} - 2 u_\theta \omega - \frac{\partial \delta\phi}{\partial r} \right) + c_s^2 \frac{\partial \delta\rho}{\partial r} \right] = \left(\xi + \frac{4\mu}{3} \right) \left[\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right) \right], \quad (29)$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial r\omega}{\partial r}+u_r\omega\right)\right]$$
$$=\mu\left(\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r^2}\right), \quad (30)$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\rho\frac{\partial u_z}{\partial t}-\frac{\mu_0}{2}\left(\frac{\partial M_z}{\partial r}h_r+M_z\frac{1}{r}\frac{\partial(rh_r)}{\partial r}\right.\right.$$
$$\left.\left.+H_z\frac{1}{r}\frac{\partial(rm_r)}{\partial r}\right)\right]=\mu\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right), \quad (31)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\delta\phi}{\partial r}\right)=-4\pi G\delta\rho, \quad (32)$$

$$\frac{\partial h_z}{\partial r}=0, \quad (33)$$

$$\left(\frac{\partial h_\theta}{\partial r}+\frac{h_\theta}{r}\right)=0, \quad (34)$$

$$\frac{1}{r}\frac{\partial(rm_r)}{\partial r}=-\frac{1}{r}\frac{\partial(rh_r)}{\partial r}, \quad (35)$$

$$\frac{\partial m_r}{\partial t}+\frac{1}{2}\left(m_\theta\left(\frac{\partial(r\omega)}{\partial r}+\omega\right)+M_z\frac{\partial u_z}{\partial r}\right)$$
$$+\beta\left(-M_z\left[M_z h_r-H_z m_r\right]\right)=0, \quad (36)$$

$$\frac{\partial m_\theta}{\partial t}-\frac{1}{2}m_r\left(\frac{\partial r\omega}{\partial r}+\omega\right)$$
$$+\beta\left(M_z\left[m_\theta H_z-h_\theta M_z\right]\right)=0, \quad (37)$$

$$\frac{\partial m_z}{\partial t}+u_r\frac{\partial M_z}{\partial r}=0. \quad (38)$$

3. Gravitational instability

In order to investigate the stability of the above stationary state, we shall apply the normal mode method and thus considering the dependence of the perturbations on r and t of the form;

$$\psi^*(r)\exp(\sigma t). \quad (39)$$

Here σ is the frequency of the perturbation.

Using the above dependence (39) of the perturbations representing one of the perturbed quantities; $\psi^*(r)\equiv(u_r, u_\theta, u_z, m_r, m_\theta, m_z, \omega_r, \omega_\theta, \omega_z, h_r, h_\theta, h_z, \delta\rho, \delta\phi)$, we have

$$\frac{\partial}{\partial t}\equiv\sigma \quad\text{and}\quad \frac{\partial}{\partial r}\psi^*(r)\equiv\frac{\mathrm{d}}{\mathrm{d}r}\psi^*(r).$$

So, in view of the above dependence, the linearized perturbation equations (28)–(38) have the following forms after simplifications;

$$\sigma\delta\rho+\frac{\rho_0}{r}\frac{\mathrm{d}(ru_r)}{\mathrm{d}r}=0, \quad (40)$$

$$(1+\tau\sigma)\left[\sigma u_r-2u_\theta\omega-\frac{\mathrm{d}\delta\phi}{\mathrm{d}r}+\frac{c_s^2}{\rho}\frac{\mathrm{d}\delta\rho}{\mathrm{d}r}\right]$$
$$=\left(\xi+\frac{4\mu}{3}\right)\left[\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}\right)\right], \quad (41)$$

$$(1+\tau\sigma)\left[\sigma u_\theta+u_r\frac{\mathrm{d}(r\omega)}{\mathrm{d}r}+u_r\omega\right]$$
$$=\nu\left(\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r^2}\right), \quad (42)$$

$$(1+\tau\sigma)\left[\sigma u_z-\frac{\mu_0}{2\rho}\left(\frac{\mathrm{d}M_z}{\mathrm{d}r}h_r+M_z\frac{1}{r}\frac{\mathrm{d}(rh_r)}{\mathrm{d}r}\right.\right.$$
$$\left.\left.+H_z\frac{1}{r}\frac{\mathrm{d}(rm_r)}{\mathrm{d}r}\right)\right]=\nu\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right), \quad (43)$$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\delta\phi}{\mathrm{d}r}\right)=-4\pi G\delta\rho, \quad (44)$$

$$\frac{\mathrm{d}h_z}{\mathrm{d}r}=0, \quad (45)$$

$$\frac{1}{r}\frac{\mathrm{d}(rh_\theta)}{\mathrm{d}r}=0, \quad (46)$$

$$\frac{1}{r}\frac{\mathrm{d}(rm_r)}{\mathrm{d}r}=-\frac{1}{r}\frac{\mathrm{d}(rh_r)}{\mathrm{d}r}, \quad (47)$$

$$\sigma m_r+\frac{1}{2}\left(m_\theta\left(\frac{\mathrm{d}(r\omega)}{\mathrm{d}r}+\omega\right)+M_z\frac{\mathrm{d}u_z}{\mathrm{d}r}\right)$$
$$+\beta\left(-M_z\left[M_z h_r-H_z m_r\right]\right)=0, \quad (48)$$

$$\sigma m_\theta-\frac{1}{2}\left(m_r\left(\frac{\mathrm{d}r\omega}{\mathrm{d}r}+\omega\right)\right)$$
$$+\beta\left(M_z\left[m_\theta H_z-h_\theta M_z\right]\right)=0, \quad (49)$$

$$\sigma m_z+u_r\frac{\mathrm{d}M_z}{\mathrm{d}r}=0. \quad (50)$$

We shall now investigate the local stability of the above system in the neighborhood of $r=r_0$, since the above equations involve variable coefficients. So, following the analysis adopted in Dhiman & Dadwal (2012), let us assume that the perturbations have the following periodic form in the neighborhood of $r=r_0$,

$$\exp(-ikr),$$

where k is the wave number. For this type of dependence, we have

$$\mathrm{d}/\mathrm{d}r\equiv -ik.$$

Now, using this dependence in equations (40)–(50) and then substituting the values of $\delta\rho$, F, T and u_r as defined by Dhiman & Sharma (2016) in equations (60)–(64) in the resulting equations, we have the following equations, obtained after some simplifications;

$$(1+\tau\sigma)\left[\left(\sigma^2-4\pi G\rho+c_s^2k^2\right)T-2\omega u_\theta\right]$$
$$=-\frac{\left(\xi+\frac{4\mu}{3}\right)\sigma}{\rho}\left(k^2+\frac{1}{r^2}\right)T, \quad (51)$$

$$(1+\tau\sigma)\left(\sigma u_\theta + \frac{\sigma}{\rho}\frac{T}{r}F\right) = -\nu\left(k^2+\frac{1}{r^2}\right)u_\theta, \tag{52}$$

$$(1+\tau\sigma)\left[\sigma u_z - \frac{\mu_0}{2\rho}\left(2\iota k m_r M_z + \iota k H_z m_r\right)\right] = -\nu\left(k^2+\frac{1}{r^2}\right)u_z, \tag{53}$$

$$\sigma m_r - \frac{\iota k M_z u_z}{2} - \frac{1}{2}m_\theta F + \beta\left(M_z\left[M_z m_r + H_z m_r\right]\right) = 0, \tag{54}$$

$$\sigma m_\theta + \frac{1}{2}m_r F + \beta m_\theta H_z M_z = 0, \tag{55}$$

$$\sigma m_z + \frac{\iota k\sigma T M_z}{\rho r} = 0. \tag{56}$$

From equation (52), we have

$$u_\theta = \frac{(1+\tau\sigma)\,\sigma T F}{\rho r\left\{(1+\tau\sigma)\,\sigma + \nu\left(k^2+\frac{1}{r^2}\right)\right\}}. \tag{57}$$

Further, equation (51) upon using the value of u_θ from the equation (57), yields the following equation;

$$\left\{(1+\tau\sigma)\,\sigma + \nu\left(k^2+\frac{1}{r^2}\right)\right\}(1+\tau\sigma)\left[\left(\sigma^2 - 4\pi G\rho + c_s^2k^2\right)T\right] + (1+\tau\sigma)^2\,2\omega\sigma F T + \left\{(1+\tau\sigma)\,\sigma + \nu\left(k^2+\frac{1}{r^2}\right)\right\}\frac{\left(\xi+\frac{4\mu}{3}\right)\sigma}{\rho} \times\left(k^2+\frac{1}{r^2}\right)T = 0. \tag{58}$$

Hence, equations (53)–(56) and (58) yields the following dispersion relation in view of the existence of non-trivial solution of these equations;

$$\left\{(1+\tau\sigma)\left(\sigma^3+Q\sigma\right)A + \frac{\left(\xi+\frac{4\mu}{3}\right)\sigma^2}{\rho}\left(k^2+\frac{1}{r^2}\right)^2 A + (1+\tau\sigma)^2\,2\,\omega\,\sigma^2 F\right\} \times\left[\left\{(1+\tau\sigma)\,\sigma + \nu\left(k^2+\frac{1}{r^2}\right)\right\} \times\left\{(\sigma + M_z\beta B_z)(\sigma + M_z\beta H_z) + \frac{1}{4}F^2\right\} - (1+\tau\sigma)\frac{\mu_0}{2\rho}2\iota k B_z\left(\frac{\iota k}{2}M_Z\right)(\sigma + M_z\beta H_z)\right] = 0. \tag{59}$$

Here,

$$A = \left\{(1+\tau\sigma)\,\sigma + \nu\left(k^2+\frac{1}{r^2}\right)\right\},$$

$$B_z = M_z + H_z,\ Q = c_s^2k^2 - 4\pi G\rho.$$

It is to note here that dispersion relation equation (59) includes the effect of shear, bulk viscosity, magnetic permeability, non-uniform rotation, ferro fluid magnetization of the medium and magnetic field respectively represented by $\nu, \xi, \mu_0, \omega, \vec{M}$ and $\vec{H}$.

It is clear from relation (59) that, either

$$\left\{(1+\tau\sigma)\left(\sigma^3+Q\sigma\right)A + \frac{\left(\xi+\frac{4\mu}{3}\right)\sigma^2}{\rho}\left(k^2+\frac{1}{r^2}\right)^2 A + (1+\tau\sigma)^2\,2\omega\sigma^2 F\right\} = 0 \tag{60}$$

or

$$\left[\left\{(1+\tau\sigma)\,\sigma + \nu\left(k^2+\frac{1}{r^2}\right)\right\} \times\left\{(\sigma + M_z\beta B_z)(\sigma + M_z\beta H_z) + \frac{1}{4}F^2\right\} - (1+\tau\sigma)\frac{\mu_0}{2\rho}2\iota k B_z\left(\frac{\iota k}{2}M_Z\right)(\sigma + M_z\beta H_z)\right] = 0. \tag{61}$$

The relations (60) and (61) respectively represent the self-gravitating mode and non self-gravitating mode for viscoelastic ferromagnetic medium.

As discussed by Prajapati and Chhajlani (2013) and Dhiman & Sharma (2016) in their studies, under the assumption of the strongly coupling limit, the wave frequency is much greater than the inverse of the viscoelastic relaxation time and is represented as $\sigma\tau \gg 1$, and the medium behaves like a viscous liquid (SCP) or solid and hence τ can be retained in the equation of motion. However, under the weakly coupling limit, the wave frequency is much lower than the inverse of viscoelastic relaxation time τ and is represented as $\sigma\tau \ll 1$ and the medium behaves like a liquid and the effect of τ can be ignored in this limit.

We shall now discuss the gravitational instability of the system under both the strongly and weakly coupling limits and shall derive the instability criteria for the present problem from the relation (60).

Under the *strongly coupling limit* ($\sigma\tau \gg 1$), the dispersion relation (60) reduces to

$$\sigma^4 + \sigma^2\left(2\omega F + Q + \frac{\left(\nu_c^2+\nu\right)}{\tau}\left(k^2+\frac{1}{r^2}\right)\right) - \left\{Q + \nu_c^2\left(k^2+\frac{1}{r^2}\right)\right\}\frac{\nu}{\tau}\left(k^2+\frac{1}{r^2}\right) = 0. \tag{62}$$

The constant term of the above equation yields the criterion for the onset of gravitational instability, as

$$\frac{\nu}{\tau}\left(k^2+\frac{1}{r^2}\right)\left(Q+v_c^2\left(k^2+\frac{1}{r^2}\right)\right)=0. \tag{63}$$

After simplification of equation (63), we get the following instability criterion:

$$c_s^2k^2+v_c^2\left(k^2+\frac{1}{r^2}\right)<4\pi G\rho_0. \tag{64}$$

The same criterion obtained by Dhiman & Sharma (2016) for viscoelastic medium.

Further, when the effect of viscoelasticity of the medium is absent, the relation (62) yields the same instability criterion obtained by Dhiman & Dadwal (2012) for non-viscous gaseous medium, which is given by

$$c_s^2k^2+2\omega\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\omega\right)<4\pi G\rho_0.$$

Thus, from the above inequality, we can conclude that the viscoelasticity of the medium eliminates the effect of non-uniform rotation in the self gravitating infinitely extending axisymmetric cylinder.

Under *weakly coupling limit* ($\sigma\tau\ll 1$), the dispersion relation (60) reduces to the following form;

$$\sigma^3+\sigma^2\left(k^2+\frac{1}{r^2}\right)\left(\nu+\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho}\right)+\sigma\left(2\omega F+\nu\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho}\left(k^2+\frac{1}{r^2}\right)^2-Q\right)-Q\nu\left(k^2+\frac{1}{r^2}\right)=0. \tag{65}$$

The constant term of the above equation yields the following instability criterion;

$$Q\nu\left(k^2+\frac{1}{r^2}\right)=0 \tag{66}$$

which is the same criterion as obtained by Dhiman & Sharma (2016) for the non-ferromagnetic viscoelastic medium under the weakly coupling limit.

4. Growth rate of instability

We shall now analyze the effects of wave number and radial distance on the growth rate of self-gravitational instability of viscoelastic ferromagnetic cylinder with axial non-uniform rotation and magnetic field under both the strongly and weakly coupling limits.

In order to study the effects of these parameters on the growth rate, under the strongly and weakly coupling limits respectively, writing equations (62) and (65) in the following dimensionless forms;

$$\gamma^4+\gamma^2\left[-4\omega^{*2}+k^{*2}-1+\frac{\xi^*+\nu^*}{\tau^*}\left(\kappa^{*2}+\frac{1}{r^{*2}}\right)\right]-\left\{k^{*2}-1+\xi^*\left(\kappa^{*2}+\frac{1}{r^{*2}}\right)\right\}\frac{\nu^*}{\tau^*}\left(\kappa^{*2}+\frac{1}{r^{*2}}\right)=0, \tag{67}$$

$$\gamma^3+\gamma^2\left(\frac{(\varsigma+\nu^*)}{r^{*2}}+k^{*2}\varsigma+k^{*2}\nu^*\right)+\gamma\left[4\omega^{*2}+k^{*2}-1+\frac{\varsigma\nu^*}{r^{*2}}\left(1+2\kappa^{*2}\right)+\varsigma\nu^*k^{*2}\right]-\left\{\left(k^{*2}-1\right)\nu^*\left(\kappa^{*2}+\frac{1}{r^{*2}}\right)\right\}=0, \tag{68}$$

where the dimensionless parameters used are (cf. Dhiman and Sharma 2014)

$$\gamma=\frac{\sigma}{\omega_j},\ k^*=\frac{kc_s}{\omega_j},\ \nu^*=\frac{\nu^*\omega_j}{c_s^2},\ \xi^*=\frac{v_c^2}{c_s^2},\ \omega^*=\frac{\omega}{\omega_j},$$

$$r^*=\frac{r\omega_j}{c_s},\ \varsigma=\frac{\omega_j}{c_s^2\rho_0}\left(\xi+\frac{4}{3}\mu\right),\ \tau^*=\tau\omega_j.$$

The values of growth rate of the self-gravitational instability for different values of wave numbers under the strongly and weakly coupling limits have been calculated from the dimensionless equations (67) and (68). The variation of growth rate with respect to these obtained values of wave number is shown graphically in Fig. 1. The fixed values of other parameters used in calculating the growth rates are as follows:

$$\omega^*=0.5,\ r^*=0.28,\ \xi^*=0.3,\ \varsigma=0.3,\ \tau^*=1.5,\ \nu^*=0.5$$

The effect of radial distance on the growth rate of Jeans instability has been calculated under the strongly and weakly coupling limits from the dimensionless equations (67) and (68) respectively. Figures 2 and 3 represent the variation of the growth rate with the normalized wave number under the strongly and weakly coupling limits respectively, for some fixed values of radial distance ($r^*=0.28,\ 2.83,\ 28.3$) (cf. Nagasawa 1987).

5. Results and conclusions

In the present analysis, we have studied the self gravitating instability of an infinitely extending axisymmetric

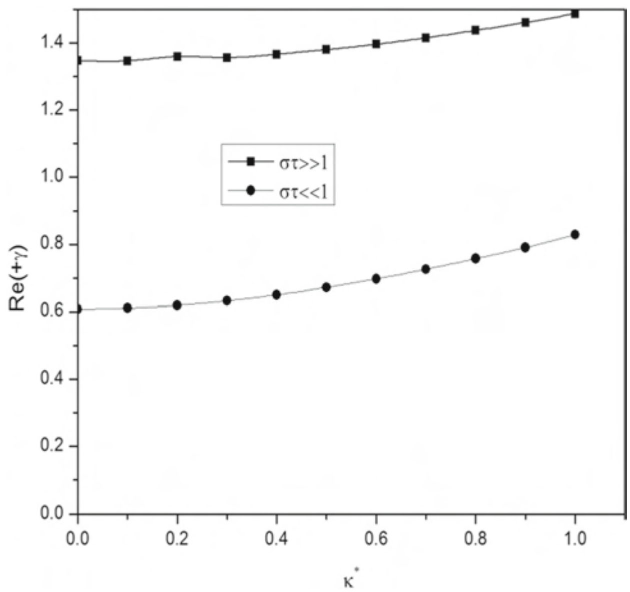

Figure 1. Variation of normalized growth rate against the normalized wave number (k^*) in strongly and weakly coupling limits.

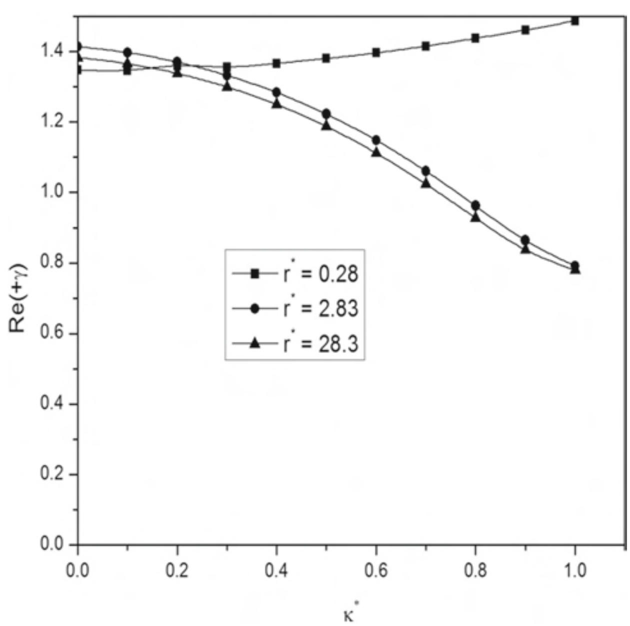

Figure 2. Variation of normalized growth rate against the normalized wave number (k^*) in strongly coupling limit for fixed values of radial distance $r^* = 0.28, 2.83, 28.3$.

cylinder of viscoelastic ferromagnetic medium in the presence of non-uniform rotation and magnetic field. A general dispersion relation, which includes the effect of ferro magnetization, magnetic permeability, viscoelastic relaxation time, non-uniform rotation and magnetic field, is obtained using the normal mode analysis method on the perturbation equations of the problem.

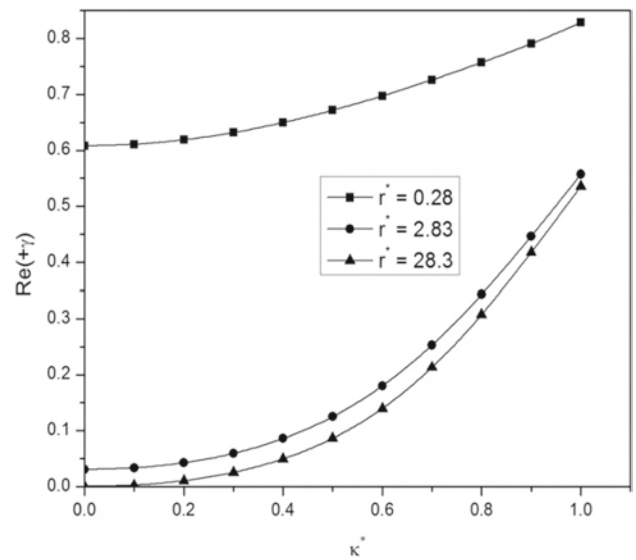

Figure 3. Variation of normalized growth rate against the normalized wave number (k^*) in weakly coupling limit for fixed values of radial distance $r^* = 0.28, 2.83, 28.3$.

The instability criteria and the effects of some physical parameters on the growth rate are discussed for this general configuration under both strongly and weakly coupling limits.

From the present investigations, we observed that the presence of ferro magnetization and non-uniform rotation and magnetic field have no effect on the gravitational instability as the Jeans criteria remains unaltered, whereas the shear and bulk viscosities have the stabilizing effect on the stability of the system. Further, it is found that the cylindrical geometry in the strongly coupling limit is more stable than the Cartesian case. Also, the criterion obtained under the weakly coupling limit are same as reported by Dhiman & Sharma (2016) in their analysis of viscoelastic axisymmetric cylinder in the presence of non-uniform magnetic field and rotation.

Further, it is clear from the above analysis that under the strongly and weakly coupling limits the magnetic field (uniform or nonuniform) has no effect on the instability of the system. This may be due to the reason that the effect of magnetic field on the gravitational instability has been marginalized by the magnetic polarizability of the ferro medium and thus instability criteria remain unchanged. Thus, we can conclude that the magnetic field applied in ferromagnetized medium has no effect on the self-gravitational instability.

Also, the effects of wave number and radial distance on the growth rate of self gravitational instability of viscoelastic ferromagnetic cylinder with axial non-uniform magnetic field and rotation have been studied. From Fig. 1, it is observed that as the wave number increases the growth rate of instability increases and takes large values under the strongly coupling limit. Further, from Figs. 2 and 3, one can observe that as the values of radial distance increases the growth rate decreases under both the strongly and weakly coupling limits, and hence have stabilizing effect on the self-gravitational instability. Thus, we can say that as the radial distance increases, the system becomes more stable.

Rosenberg & Shukla (2011), reported that the viscoelastic coefficients are the function of coupling parameter Γ_j, which characterizes the ratio of the electrostatic Coulomb interaction between neighbouring plasma particles to the thermal (kinetic) energy of the particles. Thus, the investigation about these coefficients provides the vital information about the coupling strength of the viscosity and elasticity of the material. Further, by calculating the Jeans wave number one can get information about the gravitational instability and can have an idea about the collapse of the interstellar cloud. In other words, the state in which wave number is less than the Jeans wave number, the instability occurs. Similarly, by calculating the Jeans mass we can identify that the forming star will be a neutron star or any other astrophysical object, i.e. if the Jeans Mass is smaller than the Chandrasekhar limit of 1.44 solar masses then the end product is a neutron star. The neutron stars in binaries have masses close to 1.4 times than that of the solar mass (cf. Krishan & Kushwaha 1963; Lyford *et al.* 2003; Ostriker & Hartwick 1968). Thus, these investigations may find applications in the astrophysical problems.

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