



Superfluidity and Superconductivity in Neutron Stars

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Abstract. Neutron stars, the compact stellar remnants of core-collapse supernova explosions, are unique cosmic laboratories for exploring novel phases of matter under extreme conditions. In particular, the occurrence of superfluidity and superconductivity in neutron stars will be briefly reviewed.

Keywords. Neutron stars—superfluidity—superconductivity—dense matter.

1. Introduction

Formed in the furnace of gravitational core-collapse supernova explosions of stars with a mass between 8 and 10 times that of the Sun (Deshpande *et al.* 1995), neutron stars contain matter crushed at densities exceeding that found inside the heaviest atomic nuclei (for a general review about neutron stars, see, Srinivasan (1997) and Haensel *et al.* (2007)). A proto neutron star is initially fully fluid with a mass of about one or two solar masses, a radius of about 50 km and internal temperatures of the order of 10^{11} – 10^{12} K (for a review about neutron-star formation, see Prakash *et al.* (2001)). About one minute later, the proto-neutron star becomes transparent to neutrinos that are copiously produced in its interior, thus rapidly cools down and shrinks into an ordinary neutron star. After a few months, the surface of the star – possibly surrounded by a very thin atmospheric plasma layer of light elements – still remains liquid. However, the layers beneath crystallize thus forming a solid crust (Chamel & Haensel 2008). At this point, the core is much colder than the crust because of the cooling power of the escaping neutrinos. After several decades, the interior of the star reaches a thermal equilibrium with temperatures of about 10^8 K (except for a thin outer heat-blanketing envelope). The last cooling stage takes place after about a hundred thousand years, when heat from the interior diffuses to the surface and is dissipated in the form of radiation (for a recent review about neutron-star cooling, see Potekhin *et al.* (2015)).

With typical temperatures of order 10^7 K, the highly degenerate matter in neutron stars is expected to become cold enough for the appearance of superfluids and superconductors – frictionless quantum liquids respectively electrically neutral and charged (Leggett 2006) – made of neutrons and protons, and more speculatively of other particles such as hyperons or quarks. If these phase transitions really occur, neutron stars would not only be the largest superfluid and superconducting systems known in the Universe (Srinivasan 1997; Chamel & Haensel 2008; Sauls 1989; Sedrakian & Clark 2006; Page *et al.* 2014; Graber *et al.* 2017), but also the hottest ones with critical temperatures of the order of 10^{10} K as compared to a mere 203 K for the world record achieved in 2014 in terrestrial laboratories and consisting of hydrogen-sulphide compound under high pressure (Drozdov *et al.* 2015).

After describing the main properties of terrestrial superfluids and superconductors, an overview of the theoretical developments in the modelling of superfluid and superconducting neutron stars will be given. Finally, the possible observational manifestations of these phases will be briefly discussed.

2. Terrestrial superfluids and superconductors

2.1 Historical milestones

Superconductivity and superfluidity were known long before the discovery of pulsars in August 1967. Heike

Table 1. Properties of various superfluid and superconducting systems in order of their critical temperature T_c . Adapted from Table 1.1 in Leggett (2006).

System	Density (cm^{-3})	T_c (K)
Neutron stars	$\sim 10^{39}$	$\sim 10^{10}$
Cuprates and other exotics	$\sim 10^{21}$	1–165
Electrons in ordinary metals	$\sim 10^{23}$	1–25
Helium-4	$\sim 10^{22}$	2.17
Helium-3	$\sim 10^{22}$	2.491×10^{-3}
Fermi alkali gases	$\sim 10^{12}$	$\sim 10^{-6}$
Bose alkali gases	$\sim 10^{15}$	$\sim 10^{-7}$ – 10^{-5}

Kamerlingh Onnes and his collaborators were the first to liquefy helium in 1908, thus allowing them to explore the properties of materials at lower temperatures than could be reached before. On 8th April 1911, they observed that the electric resistance of mercury dropped to almost zero at temperature $T_c \simeq 4.2$ K (for an historical account of this discovery, see e.g., van Delft & Kes (2010)). Two years later, lead and tin were found to be also superconducting. In 1914, Onnes showed that superconductivity is destroyed if the magnetic field exceeds some critical value. He later designed an experiment to measure the decay time of a magnetically induced electric current in a superconducting lead ring, and did not notice any change after an hour. Superconducting currents can actually be sustained for more than hundred thousand years (File & Mills 1963). Keesom & Kok (1932) found that the heat capacity of tin exhibits a discontinuity as it becomes superconducting thus demonstrating that this phase transition is of second order. A year later, Meissner & Ochsenfeld (1933) made the remarkable observation that when a superconducting material initially placed in a magnetic field is cooled below the critical temperature, the magnetic flux is expelled from the sample. This showed that superconductivity represents a new thermodynamical equilibrium state of matter. Rjabinin & Shubnikov (1935a, b) at the Kharkov Institute of Science and Technology in Ukraine discovered that some so-called ‘hard’ or type II superconductors (as opposed to ‘soft’ or type I superconductors) exhibit two critical fields, between which the magnetic flux partially penetrates the material. Various superconducting materials were discovered in the following decades.

During the 1930s, several research groups in Leiden, Toronto, Moscow, Oxford and Cambridge (United Kingdom), found that below $T_\lambda \simeq 2.17$ K, helium-4 (referred to as helium II) does not behave like an ordinary liquid (for a review of the historical context,

see Balibar (2007, 2014)). In particular, helium II does not boil, as was actually first noticed by Kamerlingh Onnes and his collaborators the same day they discovered superconductivity (van Delft & Kes 2010). Helium II can flow without resistance through very narrow slits and capillaries, almost independently of the pressure drop. The term ‘superfluid’ was coined by Pyotr Kapitsa in 1938 by analogy with superconductors (Kapitsa 1938). Helium II also flows up over the sides of a beaker and drip off the bottom (for ordinary liquids, the so-called Rollin film is clamped by viscosity). The existence of persistent currents in helium II was experimentally established at the end of the 1950s and the beginning of 1960s (Reppy & Depatie 1964). The analog of the Meissner–Ochsenfeld phenomenon, which was predicted by Fritz London, was first observed by George Hess and William Fairbank at Stanford in June 1967 (Hess & Fairbank 1967): the angular momentum of helium-4 in a slowly rotating container was found to be reduced as the liquid was cooled below the critical temperature T_λ .

At the time the first observed pulsars were identified as neutron stars, several materials had thus been found to be superconducting, while helium-4 was the unique superfluid known. The superfluidity of helium-3 was established by Osheroff *et al.* (1972). No other superfluids were discovered during the next two decades until the production of ultracold dilute gases of bosonic atoms in 1995 (Anderson *et al.* 1995; Davis *et al.* 1995), and of fermionic atoms in 2003 (Regal *et al.* 2004). The main properties of some known superfluids and superconductors are summarized in Table 1.

2.2 Quantum liquids

Superconductivity and superfluidity are among the most spectacular macroscopic manifestations of quantum

mechanics. Satyendra Nath Bose and Albert Einstein predicted in 1924–1925 that at low enough temperatures an ideal gas of bosons condense into a macroscopic quantum state (Bose 1924; Einstein 1925). The association between Bose–Einstein condensation (BEC) and superfluidity was first advanced by London (1938). The only known superfluid at the time was helium-4, which is a boson. The condensate can behave coherently on a very large scale and can thus flow without any resistance. It was a key idea for developing the microscopic theory of superfluidity and superconductivity. Soon afterwards, Tisza (1938) postulated that a superfluid such as He II contains two distinct dynamical components: the condensate, which carries no entropy, coexists with a normal viscous fluid. This model explained all phenomena observed at the time and predicted thermomechanical effects like ‘temperature waves’. Although Landau (1941) incorrectly believed that superfluidity is not related to BEC, he developed the two-fluid model and showed, in particular, that the normal fluid consists of ‘quasiparticles’, which are not real particles but complex many-body motions. This two-fluid picture was later adapted to superconductors (Gorter 1955).

According to the microscopic theory of superconductivity by John Bardeen, Leon Cooper and Robert Schrieffer (BCS) published in 1957 (Bardeen *et al.* 1957), the dynamical distortions of the crystal lattice (phonons) in a solid can induce an attractive effective interaction between electrons of opposite spins. Roughly speaking, electrons can thus form pairs and undergo a BEC below some critical temperature. A superconductor can thus be viewed as a charged superfluid. This picture however should not be taken too far. Indeed, electron pairs are very loosely bound and overlap. Their size $\xi \sim \hbar v_F / (k_B T_c)$ (usually referred to as the coherence length), with v_F the Fermi velocity, k_B Boltzmann’s constant, and T_c the critical temperature, is typically much larger than the lattice spacing. Moreover, electron pairs disappear at temperatures $T > T_c$. The BEC and the BCS transition are now understood as two different limits of the same phenomenon. The pairing mechanism suggested that fermionic atoms could also become superfluid, as was later confirmed by the discovery of superfluid helium-3. Since 2003, various other fermionic superfluids have been found, as mentioned in the previous section.

As first discussed by Onsager (1949) and Feynman (1955), the quantum nature of a superfluid is embedded in the quantisation of the flow

$$\oint \mathbf{p} \cdot d\boldsymbol{\ell} = Nh, \quad (1)$$

where \mathbf{p} is the momentum per superfluid particle, h denotes the Planck’s constant, N is any integer, and the integral is taken over any closed path. It can be immediately recognized that this condition is the Bohr–Sommerfeld quantisation rule. The flow quantisation follows from the fact that a superfluid is a macroscopic quantum system whose momentum is thus given by $p = h/\lambda$, where λ is the de Broglie wavelength. Requiring the length of any closed path to be an integral multiple of the de Broglie wavelength leads to equation (1). The physical origin of this condition has been usually obscured by the introduction of the ‘superfluid velocity’ $\mathbf{V}_s = \mathbf{p}/m$, where m is the mass of the superfluid particles.

In a rotating superfluid, the flow quantisation condition (1) leads to the appearance of N quantised vortices. In a region free of vortices, the superflow is characterized by the irrotationality condition

$$\nabla \times \mathbf{p} = 0. \quad (2)$$

Inside a vortex, the superfluidity is destroyed. Because superfluid vortices are essentially of quantum nature, their internal structure cannot be described by a purely hydrodynamic approach. However, vortices can be approximately treated as structureless topological defects at length scales much larger than the vortex core size. As shown by Tkachenko (1966), quantised vortices tend to arrange themselves on a regular triangular array, with a spacing given by

$$d_v = \sqrt{\frac{h}{\sqrt{3}m\Omega}}, \quad (3)$$

where Ω is the angular frequency. Vortex arrays have been observed in superfluid helium (Yarmchuk *et al.* 1979) and more recently in atomic Bose–Einstein condensates (Abo-Shaeer *et al.* 2001; Zwierlein *et al.* 2005). At length scales much larger than the intervortex spacing d_v , the superfluid flow mimics rigid body rotation such that

$$\nabla \times \mathbf{p} = mn_v \boldsymbol{\kappa}, \quad (4)$$

where n_v is the surface density of vortices given by

$$n_v = \frac{m\Omega}{\pi\hbar}, \quad (5)$$

and the vector $\boldsymbol{\kappa}$, whose norm is equal to h/m , is aligned with the average angular velocity. Landau’s original two-fluid model was further improved in the 1960s by Hall & Vinen (1956), Hall (1960), and independently by Bekarevich & Khalatnikov (1961) to account for the presence of quantised vortices within a coarse-grained average hydrodynamic description.

The quantisation condition (1) also applies to superconductors. But in this case, the momentum (in CGS units) is given by $\mathbf{p} \equiv m\mathbf{v} + (q/c)\mathbf{A}$ where m , q , and v are the mass, electric charge and velocity of superconducting particles respectively, and \mathbf{A} is the electromagnetic potential vector. Introducing the density n of superconducting particles and the ‘super-current’ $\mathcal{J} = nq\mathbf{v}$, the situation $N = 0$ as described by equation (2) leads to London’s equation

$$\nabla \times \mathcal{J} = -\frac{c}{4\pi\lambda_L^2}\mathbf{B}, \quad (6)$$

where $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic field induction, and $\lambda_L = \sqrt{mc^2/(4\pi nq^2)}$ is the London penetration depth. Situations with $N > 0$ are encountered in type II superconductors for which $\lambda_L \gtrsim \xi$. Considering a closed contour outside a sample of such a superconductor for which $\mathcal{J} = 0$ and integrating the momentum \mathbf{p} along this contour, leads to the quantisation of the total magnetic flux Φ into fluxoids (also referred to as flux tubes or fluxons)

$$\Phi = \oint \mathbf{A} \cdot d\boldsymbol{\ell} = N\Phi_0, \quad (7)$$

where $\Phi_0 = hc/|q|$ is the flux quantum. The magnetic flux quantization, first envisioned by London, was experimentally confirmed in 1961 by Bascom Deaver and William Fairbank at Stanford University (Deaver & Fairbank 1961), and independently Robert Doll and Martin Nábauer at the Low Temperature Institute in Hersching (Doll & Nábauer 1961). As predicted by Abrikosov (1957), these fluxoids tend to arrange themselves into a triangular lattice with a spacing given by

$$d_\Phi = \sqrt{\frac{2hc}{\sqrt{3}|q|B}}. \quad (8)$$

Averaging at length scales much larger than d_Φ , the surface density of fluxoids is given by

$$n_\Phi = \frac{B}{\Phi_0} = \frac{|q|B}{hc}, \quad (9)$$

where B denotes the average magnetic field strength. The size of a fluxoid (within which the superconductivity is destroyed) is of the order of the coherence length ξ . The magnetic field carried by a fluxoid extends over a larger distance of the order of the London penetration length λ_L . The nucleation of a single fluxoid thus occurs at a critical field $H_{c1} \sim \Phi_0/(\pi\lambda_L^2)$, and superconductivity is destroyed at the critical field $H_{c2} \sim \Phi_0/(\pi\xi^2)$ at which point the cores of the fluxoids touch.

3. Superstars

3.1 Prelude: internal constitution of a neutron star

A few meters below the surface of a neutron star, matter is so compressed by the tremendous gravitational pressure that atomic nuclei, which are supposedly arranged on a regular crystal lattice, are fully ionised and thus coexist with a quantum gas of electrons. With increasing depth, nuclei become progressively more neutron-rich. Only in the first few hundred metres below the surface can the composition be completely determined by experimentally measured masses of atomic nuclei (Wolf *et al.* 2013). In the deeper layers recourse must be made to theoretical models (Pearson *et al.* 2011; Kreim *et al.* 2013; Chamel *et al.* 2015; Sharma *et al.* 2015, Utama *et al.* (2016), Chamel *et al.* 2017). At densities of a few 10^{11} g cm⁻³, neutrons start to ‘drip’ out of nuclei (see Chamel *et al.* (2015) for a recent discussion). This marks the transition to the inner crust, an inhomogeneous assembly of neutron-proton clusters immersed in an ocean of unbound neutrons and highly degenerate electrons. According to various calculations, the crust dissolves into a uniform mixture of neutrons, protons and electrons when the density reaches about half the density $\sim 2.7 \times 10^{14}$ g cm⁻³ found inside heavy atomic nuclei (see Chamel & Haensel (2008) for a review about neutron-star crusts). Near the crust-core interface, nuclear clusters with very unusual shapes such as elongated rods or slabs may exist (see section 3.3 of Chamel & Haensel (2008), see also Watanabe & Maruyama (2012)). These so-called ‘nuclear pastas’ could account for half of the crustal mass, and play a crucial role for the dynamical evolution of the star and its cooling (Pons *et al.* 2013; Horowitz *et al.* 2015). The composition of the innermost part of neutron-star cores remains highly uncertain: apart from nucleons and leptons, it may also contain hyperons, meson condensates, and deconfined quarks (Haensel *et al.* (2007); see also Sedrakian (2010), Chatterjee & Vidaña (2016)).

3.2 Superfluid and superconducting phase transitions in dense matter

Only one year after the publication of the BCS theory of superconductivity, Bogoliubov (1958) was the first to consider the possibility of superfluid nuclear matter. Migdal (1959) speculated that the interior of a neutron star might contain a neutron superfluid, and its critical temperature was estimated by Ginzburg & Kirzhnits (1964) using the BCS theory. Proton superconductivity in neutron stars was studied by Wolf (1966). The

possibility of anisotropic neutron superfluidity was explored by Hoffberg *et al.* (1970), and independently by Tamagaki (1970).

Neutrons and protons are fermions, and due to the Pauli exclusion principle, they generally tend to avoid themselves. This individualistic behaviour, together with the strong repulsive nucleon–nucleon interaction at short distance, provide the necessary pressure to counterbalance the huge gravitational pull in a neutron star, thereby preventing it from collapsing. However at low enough temperatures, nucleons may form pairs (Broglia & Zelevinsky 2013) similarly as electrons in ordinary superconductors as described by the BCS theory¹. These bosonic pairs can therefore condense, analogous to superfluid helium-3. While helium-3 becomes a superfluid only below 1 mK, nuclear superfluidity could be sustainable even at a temperature of several billions degrees in a neutron star due to the enormous pressure involved. The nuclear pairing phenomenon is also supported by the properties of atomic nuclei (Dean & Hjorth-Jensen 2003).

Because the nuclear interactions are spin-dependent and include non-central tensor components (angular momentum-dependent), different kinds of nucleon–nucleon pairs could form at low enough temperatures. The most attractive pairing channels² are 1S_0 at low densities and the coupled 3PF_2 channel at higher densities (Gezerlis *et al.* 2014). In principle, different types of pairs may coexist. However, one or the other are usually found to be energetically favored (Lombardo & Schulze 2001). Let us mention that nucleons may also form quartets such as α -particles, which can themselves condense at low enough temperatures (Schuck 2014). Most microscopic calculations have been carried out in pure neutron matter using diagrammatic, variational, and more recently, Monte Carlo methods (see Gezerlis *et al.* 2014; Lombardo & Schulze 2001 for a review). At concentrations below $\sim 0.16 \text{ fm}^{-3}$, as encountered in the inner crust and in the outer core of a neutron star, neutrons are expected to become superfluid by forming 1S_0 pairs, with critical temperatures of about 10^{10} K at most (Gezerlis *et al.* 2014; Cao *et al.*

2006; Maurizio *et al.* 2014; Ding *et al.* 2016). At neutron concentrations above $\sim 0.16 \text{ fm}^{-3}$, pairing in the coupled 3PF_2 channel becomes favored but the maximum critical temperature remains very uncertain, predictions ranging from $\sim 10^8 \text{ K}$ to $\sim 10^9 \text{ K}$ (Maurizio *et al.* 2014; Ding *et al.* 2016; Baldo *et al.* 1998; Dong *et al.* 2013). This lack of knowledge of neutron superfluid properties mainly stems from the highly nonlinear character of the pairing phenomenon, as well as from the fact that the nuclear interactions are not known from first principles (see Machleidt (2017) for a recent review).

Another complication arises from the fact that neutron stars are not only made of neutrons. The presence of nuclear clusters in the crust of a neutron star may change substantially the neutron superfluid properties. Unfortunately, microscopic calculations of inhomogeneous crustal matter employing realistic nuclear interactions are not feasible. State-of-the-art calculations are based on the nuclear energy density functional theory, which allows for a consistent and unified description of atomic nuclei, infinite homogeneous nuclear matter and neutron stars (see Chamel *et al.* (2013) and references therein). The main limitation of this approach is that the *exact* form of the energy density functional is not known. In practice, phenomenological functionals fitted to selected nuclear data must therefore be employed. The superfluid in neutron-star crusts, which bears similarities with terrestrial multiband superconductors, was first studied within the band theory of solids in Chamel *et al.* (2010). However, this approach is computationally very expensive, and has been so far limited to the deepest layers of the crust. For this reason, most calculations of neutron superfluidity in neutron-star crusts (Margueron & Sandulescu 2012) have been performed using an approximation introduced by Wigner & Seitz (1933) in the context of electrons in metals: the Wigner–Seitz or Voronoi cell of the lattice (a truncated octahedron in case of a body-centred cubic lattice) is replaced by a sphere of equal volume. However, this approximation can only be reliably applied in the shallowest region of the crust due to the appearance of spurious shell effects (Chamel *et al.* 2007). Such calculations have shown that the phase diagram of the neutron superfluid in the crust is more complicated than that in pure neutron matter; in particular, the formation of neutron pairs can be enhanced with increasing temperature (Margueron & Khan 2012; Pastore *et al.* 2013; Pastore 2015). Microscopic calculations in pure neutron matter at densities above the crust-core boundary are not directly applicable to neutron stars due to the presence of protons, leptons, and possibly other particles in neutron-star cores. Few microscopic calculations have been performed so far in beta-stable matter

¹The high temperatures $\sim 10^7 \text{ K}$ prevailing in neutron star interiors prevent the formation of electron pairs recalling that the highest critical temperatures of terrestrial superconductors do not exceed $\sim 200 \text{ K}$. In particular, iron expected to be present in the outermost layers of a neutron star was found to be superconducting in 2001, but with a critical temperature $T_c \simeq 2 \text{ K}$ (Shimizu *et al.* 2001). See also Ginzburg (1969).

²A given channel is denoted by $^{2S+1}L_J$, where J is the total angular momentum, L is the orbital angular momentum, and S the spin of nucleon pair.

(Zhou *et al.* 2004). Because the proton concentration in the outer core of a neutron star is very low, protons are expected to become superconducting in the 1S_0 channel. However, the corresponding critical temperatures are very poorly known due to the strong influence of the surrounding neutrons (Baldo & Schulze 2007). Neutron–proton pairing could also in principle occur, but is usually disfavored by the very low proton content of neutron stars (Stein *et al.* 2014). Other more speculative possibilities include hyperon–hyperon and hyperon–nucleon pairing (Chatterjee & Vidaña (2016) and references therein). The core of a neutron star might also contain quarks in various color superconducting phases (Alford *et al.* 2008).

According to cooling simulations, the temperature in a neutron star is predicted to drop below the estimated critical temperatures of nuclear superfluid phases after $\sim 10\text{--}10^2$ years. The interior of a neutron star is thus thought to contain at least three different kinds of superfluids and superconductors (Page *et al.* 2014): (i) an isotropic neutron superfluid (with 1S_0 pairing) permeating the inner region of the crust and the outer core, (ii) an anisotropic neutron superfluid (with 3PF_2 pairing) in the outer core, and (iii) an isotropic proton superconductor (with 1S_0 pairing) in the outer core. The neutron superfluids in the crust and in the outer core are not expected to be separated by a normal region.

3.3 Role of a high magnetic field

Most neutron stars that have been discovered so far are radio pulsars with typical surface magnetic fields of order 10^{12} G (as compared to $\sim 10^{-1}$ G for the Earth’s magnetic field), but various other kinds of neutron stars have been revealed with the development of the X-ray and gamma-ray astronomy (Harding 2013). In particular, a small class of very highly magnetised neutron stars thus dubbed *magnetars* by Thompson & Duncan (1992) (see Woods & Thompson (2006) for a review) have been identified in the form of soft-gamma ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs). Tremendous magnetic fields up to about 2×10^{15} G have been measured at the surface of these stars from both spin-down and spectroscopic studies (Olausen & Kaspi 2014; Tiengo *et al.* 2013; An *et al.* 2014), and various observations suggest the existence of even higher internal fields (Stella *et al.* 2005; Kaminker *et al.* 2007; Watts & Strohmayer 2007; Samuelsson & Andersson 2007; Vietri *et al.* 2007; Rea *et al.* 2010; Makishima *et al.* 2014). Although only 23 such stars are currently known (Olausen & Kaspi 2014), recent observations

indicate that ordinary pulsars can also be endowed with very high magnetic fields of order 10^{14} G (Ng & Kaspi 2011). According to numerical simulations, neutron stars may potentially be endowed with internal magnetic fields as high as 10^{18} G (see Pili *et al.* 2014; Chatterjee *et al.* 2015 and references therein).

The presence of a high magnetic field in the interior of a neutron star may have a large impact on the superfluid and superconducting phase transitions. Proton superconductivity is predicted to disappear at a critical field of order $10^{16}\text{--}10^{17}$ G (Baym *et al.* 1969a). Because spins tend to be aligned in a magnetic field, the formation of neutron pairs in the 1S_0 channel is disfavored in a highly magnetised environment, as briefly mentioned by Kirzhnits (1970). It has been recently shown that 1S_0 pairing in pure neutron matter is destroyed if the magnetic field strength exceeds $\sim 10^{17}$ G (Stein *et al.* 2016). Moreover, the magnetic field may also shift the onset of the neutron-drip transition in dense matter to higher or lower densities due to Landau quantisation of electron motion, thus changing the spatial extent of the superfluid region in magnetar crusts (Chamel *et al.* 2015; Fantina *et al.* 2016; Basilico *et al.* 2015; Chamel *et al.* 2016).

3.4 Dynamics of superfluid and superconducting neutron stars

The minimal model of superfluid neutron stars consists of at least two distinct interpenetrating dynamical components (Baym *et al.* 1969): (i) a plasma of electrically charged particles (electrons, nuclei in the crust and protons in the core) that are essentially locked together by the interior magnetic field, and (ii) a neutron superfluid. Whether protons in the core are superconducting or not, they co-move with the other electrically charged particles (Sauls 1989).

The traditional heuristic approach to superfluid hydrodynamics blurring the distinction between velocity and momentum makes it difficult to adapt and extend Landau’s original two-fluid model to the relativistic context, as required for a realistic description of neutron stars (Carter & Khalatnikov 1994). In particular, in superfluid mixtures such as helium-3 and helium-4 (Andreev & Bashkin 1976), or neutrons and protons in the core of neutron stars (Sedrakyan & Shakhbasyan 1980; Vardanyan & Sedrakyan 1981), the different superfluids are generally mutually coupled by entrainment effects whereby the true velocity \mathbf{v}_X and the momentum \mathbf{p}_X of a fluid X are not aligned:

$$\mathbf{p}_X = \sum_Y \mathcal{K}^{XY} \mathbf{v}_Y, \quad (10)$$

where \mathcal{K}^{XY} is a symmetric matrix determined by the interactions between the constituent particles. In the two-fluid model, entrainment can be equivalently formulated in terms of ‘effective masses’. Considering the neutron–proton mixture in the core of neutron stars, the neutron momentum can thus be expressed as $\mathbf{p}_n = m_n^* \mathbf{v}_n$ in the proton rest frame ($\mathbf{v}_p = 0$), with $m_n^* = \mathcal{K}^{nn}$. Alternatively, a different kind of effective mass can be introduced, namely $m_n^\ddagger = \mathcal{K}^{nn} - \mathcal{K}^{np} \mathcal{K}^{pp} / \mathcal{K}^{pp}$, such that $\mathbf{p}_n = m_n^\ddagger \mathbf{v}_n$ in the proton momentum rest frame ($\mathbf{p}_p = 0$). These effective masses should not be confused with those introduced in microscopic many-body theories (Chamel & Haensel 2006). Because of the strong interactions between neutrons and protons, entrainment effects in neutron-star cores cannot be ignored (see Chamel & Haensel 2006; Gusakov & Haensel 2005; Chamel 2008; Kheto & Bandyopadhyay 2014; Sourie *et al.* 2016 for recent estimates). In the neutron-rich core of neutron stars, we typically have $m_n^* \sim m_n^\ddagger \sim m_n$, and $m_p^* \sim m_p^\ddagger \sim (0.5\text{--}1)m_p$, where m_n and m_p denote the ‘bare’ neutron and proton masses respectively. As shown by Carter (1975), at the global scale of the star, general relativity induces additional couplings between the fluids due to Lense–Thirring effects, which tend to counteract entrainment. As recently found in Sourie *et al.* (2017), frame-dragging effects can be as important as entrainment.

An elegant variational formalism to derive the hydrodynamic equations of any relativistic (super)fluid mixtures was developed by Carter and collaborators (Carter 1989, 2001; Gourgoulhon 2006; Andersson & Comer 2007). This formalism relies on an action principle in which the basic variables are the number densities and currents of the different fluids. The equations of motion can be derived by considering variations of the fluid particle trajectories. Dissipative processes (e.g. viscosity in non-superfluid constituents, superfluid vortex drag, ordinary resistivity between non-superfluid constituents, nuclear reactions) can be treated within the same framework. The convective formalism developed by Carter was later adapted to the comparatively more intricate Newtonian theory within a 4-dimensionally covariant framework (see Carter & Chamel 2004, 2005a, b; see also Prix 2004, 2005; Andersson & Comer 2006 and references therein for a review of other approaches using a 3+1 spacetime decomposition). This fully covariant approach not only facilitates the comparison with the relativistic theory (Carter *et al.* 2006; Chamel 2008), but more importantly

lead to the discovery of new conservation laws in superfluid systems such as the conservation of generalised helicity currents.

As pointed out by Ginzburg & Kirzhnits (1964), the interior of a rotating neutron star is expected to be threaded by a very large number of neutron superfluid vortices (for a discussion of the vortex structure in $^1\text{S}_0$ and $^3\text{P}_F2$ neutron superfluids, see Sauls 1989). Introducing the spin period P in units of 10 ms, $P_{10} \equiv P/(10 \text{ ms})$, the surface density of vortices (5) is of the order

$$n_v \sim 6 \times 10^5 P_{10}^{-1} \text{ cm}^{-2}. \quad (11)$$

The average intervortex spacing (3) of order

$$d_v \sim n_v^{-1/2} \sim 10^{-3} \sqrt{P_{10}} \text{ cm}, \quad (12)$$

is much larger than the size of the vortex core (Yu & Bulgac 2003). Neutron superfluid vortices can pin to nuclear inhomogeneities in the crust. However, the pinning strength remains uncertain (see Wlazłowski *et al.* 2016 and references therein; see also section 8.3.5 of Chamel & Haensel (2008)). Protons in the core of a neutron star are expected to become superconducting at low enough temperatures. Contrary to superfluid neutrons, superconducting protons do not form vortices. As shown by Baym *et al.* (1969a, b), the expulsion of the magnetic flux accompanying the transition takes place on a very long time scale $\sim 10^6$ years due to the very high electrical conductivity of the dense stellar matter. The superconducting transition thus occurs at constant magnetic flux. The proton superconductor is usually thought to be of type II (Baym *et al.* 1969a) (see also Charbonneau & Zhitnitsky 2007; Alford *et al.* 2008 and references therein), in which case, the magnetic flux penetrates the neutron star core by forming fluxoids, with a surface density (9) of order

$$n_\Phi \sim 5 \times 10^{18} B_{12} \text{ cm}^{-2}, \quad (13)$$

where the magnetic field strength B is expressed as $B_{12} \equiv B/(10^{12} \text{ G})$. This surface density corresponds to a spacing (8) of order

$$d_\Phi \sim n_\Phi^{-1/2} \sim 5 \times 10^{-10} \sqrt{B_{12}^{-1}} \text{ cm}. \quad (14)$$

Since the magnetic flux is frozen in the stellar core, fluxoids can form even if the magnetic field is lower than the critical field $H_{c1} \sim 10^{15} \text{ G}$ (Baym *et al.* 1969a). Proton superconductivity is destroyed at the higher critical field $H_{c2} \sim 10^{16} \text{ G}$ (Baym *et al.* 1969a). Due to entrainment effects, neutron superfluid vortices carry a magnetic flux as well, given by (Sedrakyan & Shakhbasyan 1980; Alpar *et al.* 1984)

$$\Phi = \Phi_0 \left(\frac{m_p^\#}{m_p} - 1 \right), \quad (15)$$

where $\Phi_0 = hc/(2e)$. Electrons scattering off the magnetic field of the vortex lines leads to a strong frictional coupling between the core neutron superfluid and the electrically charged particles (Alpar *et al.* 1984). Neutron superfluid vortices could also interact with proton fluxoids (Sauls 1989; Muslimov & Tsygan 1985; Mendell 1991; Chau *et al.* 1992), and this may have important implications for the evolution of the star (Srinivasan 1997; Sauls 1989; Srinivasan *et al.* 1990; Ruderman 1995; Ruderman *et al.* 1998; Bhat-tacharya 2002). For typical neutron star parameters ($P = 10$ ms, $B = 10^{12}$ G, radius $R = 10$ km), the numbers of neutron superfluid vortices and proton fluxoids are of the order $n_v \pi R^2 \sim 10^{18}$ and $n_\phi \pi R^2 \sim 10^{30}$, respectively. Such large numbers justify a smooth-averaged hydrodynamical description of neutron stars. However, this averaging still requires the understanding of the underlying vortex dynamics (Graber *et al.* 2017). A more elaborate treatment accounting for the macroscopic anisotropy induced by the underlying presence of vortices and/or flux tubes was developed by Carter based on a Kalb-Ramond type formulation (Carter 2000) (see also Gusakov & Dommès (2016) and references therein). In recent years, simulations of large collections ($\sim 10^2$ – 10^4) of vortices have been carried out, thus providing some insight on collective behaviors, such as vortex avalanches (Warszawski & Melatos 2013). However, these simulations have been restricted so far to Bose condensates. The extent to which the results can be extrapolated to neutron stars remains to be determined. Such large-scale simulations also require microscopic parameters determined by the local dynamics of individual vortices (Bulgac *et al.* 2013).

The variational formulation of multifluid hydrodynamics was extended for studying the magnetoelastohydrodynamics of neutron star crusts, allowing for a consistent treatment of the elasticity of the crust, superfluidity and the presence of a strong magnetic field, both within the Newtonian theory (Carter *et al.* 2006; Carter & Chachoua 2006) and in the fully relativistic context (Carter & Samuelsson 2006). In particular, these formulations can account for the entrainment of the neutron superfluid by the crustal lattice (Carter *et al.* 2006), a non-dissipative effect arising from Bragg scattering of unbound neutrons first studied in Carter *et al.* (2005), Chamel (2005, 2006) using the band theory of solids. More recent systematic calculations based on a more realistic description of the crust have confirmed that these entrainment effects can be very strong (Chamel

2012). These results are at variance with those obtained from hydrodynamical studies (Epstein 1988; Sedrakian 1996; Magierski & Bulgac 2004a, b; Magierski 2004; Martin & Urban 2016). However, as discussed in Martin & Urban (2016), these approaches are only valid if the neutron superfluid coherence length is much smaller than the typical size of the spatial inhomogeneities, a condition that is usually not fulfilled in most region of the inner crust. The neglect of neutron pairing in the quantum calculations of Chamel (2012) has been recently questioned (Gezerlis *et al.* 2014; Martin & Urban 2016). Although detailed numerical calculations are still lacking, the analytical study of Carter *et al.* (2005) suggested that neutron pairing is unlikely to have a large impact on the entrainment coupling.

4. Observational manifestations

4.1 Pulsar frequency glitches

Pulsars are neutron stars spinning very rapidly with extremely stable periods P ranging from milliseconds to seconds, with delays $\dot{P} \equiv dP/dt$ that in some cases do not exceed 10^{-21} , as compared to 10^{-18} for the most accurate atomic clocks (Hinkley *et al.* 2013). Nevertheless, irregularities have been detected in long-term pulsar timing observations (Lyne *et al.* 1995). In particular, some pulsars have been found to suddenly spin up. These ‘glitches’ in their rotational frequency Ω , ranging from $\Delta\Omega/\Omega \sim 10^{-9}$ to $\sim 10^{-5}$, are generally followed by a long relaxation lasting from days to years, and sometimes accompanied by an abrupt change of the spin-down rate from $|\Delta\dot{\Omega}/\dot{\Omega}| \sim 10^{-6}$ up to $\sim 10^{-2}$. At the time of this writing, 482 glitches have been detected in 168 pulsars (Espinoza *et al.* 2011). Since these phenomena have not been observed in any other celestial bodies, they must reflect specific properties of neutron stars (for a recent review, see, Haskell & Melatos (2015)). In particular, giant pulsar frequency glitches $\Delta\Omega/\Omega \sim 10^{-6}$ – 10^{-5} as detected in the emblematic Vela pulsar are usually attributed to sudden transfers of angular momentum from a more rapidly rotating superfluid component to the rest of the star whose rotation frequency is directly observed (for a short historical review of theoretical developments, see Chamel (2015) and references therein). The role of superfluidity is corroborated by the very long relaxation times (Baym *et al.* 1969a) and by experiments with superfluid helium (Tsakadze & Tsakadze 1980). The standard scenario of giant pulsar glitches is the following. The inner crust of a neutron star is permeated

by a neutron superfluid that is weakly coupled to the electrically charged particles by mutual friction forces (in a seminal work, [Alpar et al. \(1984\)](#) argued that the core neutron superfluid is strongly coupled to the core, and therefore does not participate to the glitch). The superfluid thus follows the spin-down of the star via the motion of vortices away from the rotation axis unless vortices are pinned to the crust ([Anderson & Itoh 1975](#)). In such a case, a lag between the superfluid and the rest of the star will build up, inducing a Magnus force acting on the vortices. At some point, the vortices will suddenly unpin, the superfluid will spin down and, by the conservation of angular momentum the crust will spin up. During subsequent relaxation, vortices progressively repin until the next glitch ([Pines & Alpar 1985](#)). This scenario is supported by the analysis of the glitch data, suggesting that the superfluid represents only a few per cent of the angular momentum reservoir of the star ([Alpar et al. 1993](#); [Datta & Alpar 1993](#); [Link et al. 1999](#)). On the other hand, this interpretation has been recently challenged by the 2007 glitch detected in PSR J1119–6127, and by the 2010 glitch in PSR B2334+61 ([Yuan et al. 2010](#); [Alpar 2011](#); [Akbal et al. 2015](#)). More importantly, it has also been shown that the neutron superfluid in the crust of a neutron star does not contain enough angular momentum to explain giant glitches due to the previously ignored effects of Bragg scattering ([Chamel & Carter 2006](#); [Andersson et al. 2012](#); [Chamel 2013](#); [Delsate et al. 2016](#)). This suggests that the core superfluid plays a more important role than previously thought ([Ho et al. 2015](#); [Pizzochero et al. 2017](#)). In particular, the core superfluid could be decoupled from the rest of the star due to the pinning of neutron vortices to proton fluxoids ([Ruderman et al. 1998](#); [Gügercinoğlu & Alpar 2014](#)). So far, most global numerical simulations of pulsar glitches have been performed within the Newtonian theory ([Larson & Link 2002](#); [Peralta et al. 2006](#); [Sidery et al. 2010](#); [Haskell et al. 2012](#)). However, a recent study shows that general relativity could significantly affect the dynamical evolution of neutron stars ([Sourie et al. 2017](#)).

4.2 Thermal relaxation of transiently accreting neutron stars during quiescence

In a low-mass X-ray binary, a neutron star accretes matter from a companion star during several years or decades, driving the neutron-star crust out of its thermal equilibrium with the core. After the accretion stops, the heated crust relaxes towards equilibrium (see section 12.7 of [Chamel & Haensel \(2008\)](#), see

also [Page & Reddy \(2012\)](#)). The thermal relaxation has been already monitored in a few systems (see [Waterhouse et al. \(2016\)](#) and references therein). The thermal relaxation time depends on the properties of the crust, especially the heat capacity. In turn, the onset of neutron superfluidity leads to a strong reduction of the heat capacity at temperatures $T \ll T_c$ thus delaying the thermal relaxation of the crust ([Fortin et al. 2010](#)). If neutrons were not superfluid, they could store so much heat that the thermal relaxation would last longer than what is observed ([Shternin et al. 2007](#); [Brown & Cumming 2009](#)). On the other hand, the thermal relaxation of these systems is not completely understood. For instance, additional heat sources of unknown origin are needed in order to reproduce the observations ([Waterhouse et al. 2016](#); [Brown & Cumming 2009](#); [Degenaar et al. 2013, 2014](#); [Turlione et al. 2015](#); [Degenaar et al. 2015](#); [Merritt et al. 2016](#)). These discrepancies may also originate from a lack of understanding of superfluid properties ([Turlione et al. 2015](#)). In particular, the low-energy collective excitations of the neutron superfluid were found to be strongly mixed with the vibrations of the crystal lattice, and this can change substantially the thermal properties of the crust ([Chamel et al. 2013, 2016](#)).

4.3 Rapid cooling of Cassiopeia A

Cassiopeia A is the remnant of a star that exploded 330 years ago at a distance of about 11000 light years from us. It owes its name to its location in the constellation Cassiopeia. The neutron star is not only the youngest known, thermally emitting, isolated neutron star in our Galaxy, but it is also the first isolated neutron star for which cooling has been directly observed. Ten-year monitoring of this object seems to indicate that its temperature has decreased by a few per cent since its discovery in 1999 ([Heinke & Ho 2010](#)) (see also the analysis of [Elshamouty et al. \(2013\)](#), [Posselt et al. \(2013\)](#) suggesting that the temperature decline is not statistically significant). If confirmed, this cooling rate would be substantially faster than that expected from nonsuperfluid neutron-star cooling theories. It is thought that the onset of neutron superfluidity opens a new channel for neutrino emission from the continuous breaking and formation of neutron pairs. This process, which is most effective for temperatures slightly below the critical temperature of the superfluid transition, enhances the cooling of the star during several decades. As a consequence, observations of Cassiopeia A put stringent constraints on the

critical temperatures of the neutron superfluid and proton superconductor in neutron-star cores (Page *et al.* 2011; Shternin *et al.* 2011; Ho *et al.* 2015). However, this interpretation has been questioned and alternative scenarios have been proposed (Blaschke *et al.* 2013; Negreiros *et al.* 2013; Sedrakian 2013; Noda *et al.* 2013; Bonanno *et al.* 2014; Ouyed *et al.* 2015; Sedrakian 2016; Taranto *et al.* 2016), most of which still requiring superfluidity and/or superconductivity in neutron stars.

4.4 Pulsar timing noise and rotational evolution

Apart from pulsar frequency glitches, superfluidity and superconductivity may leave their imprint on other timing irregularities. In particular, pulsar timing noise (Lyne *et al.* 1995) could be the manifestation of superfluid turbulence although other mechanisms are likely to play a role (see Melatos & Link (2014) and references therein). Interpreting the long-period (~ 100 – 1000 days) oscillations in the timing residuals of some pulsars such as PSR B1828–11 (Kerr *et al.* 2016) as evidence of free precession, it has been argued that either the neutron superfluid does not coexist with the proton superconductor in the core of a neutron star, or the proton superconductor is type I so as to avoid pinning of neutron superfluid vortices to proton fluxoids (Link 2003, 2007). However, this conclusion seems premature in view of the complexity of the neutron-star dynamics (Alpar 2005; Glampedakis *et al.* 2009). Alternatively, these oscillations might be related to the propagation of Tkachenko waves in the vortex lattice (see Haskell 2011 and references therein). The presence of superfluids and superconductors in the interior of a neutron star may also be revealed from the long-term rotational evolution of pulsars by measuring the braking index $n = \Omega \ddot{\Omega} / \dot{\Omega}^2$. Deviations from the canonical value $n = 3$ as predicted by a rotating magnetic dipole model in vacuum can be explained by the decoupling of the neutron superfluid in the core of a neutron star (due to pinning to proton fluxoids for instance) (Alpar & Baykal 2006; Ho & Andersson 2012). However, a similar rotational evolution could be mimicked by other mechanisms without invoking superfluidity (see Pétri (2016) for a recent review).

4.5 Quasi-periodic oscillations in soft gamma-ray repeaters

Quasi-periodic oscillations (QPOs) in the hard X-ray emission were detected in the tails of giant flares from SGR 1806-20, SGR 1900+14 and SGR 0526-66, with

frequencies ranging from 18 Hz to 1800 Hz (see Turolla *et al.* (2015) for a recent review). As anticipated by Duncan (1998), these QPOs are thought to be the signatures of global magneto-elastic seismic vibrations of the star. If this interpretation is confirmed, the analysis of these QPOs could thus provide valuable information on the interior of a neutron star. In particular, the identification of the modes could potentially shed light on the existence of superfluid and superconducting phases (Gabler *et al.* 2013).

5. Conclusion

The existence of superfluid and superconducting phases in the dense matter constituting the interior of neutron stars has been corroborated both by theoretical developments and by astrophysical observations. In particular, neutron stars are expected to contain a 1S_0 neutron superfluid permeating the inner region of the crust and the outer core, a 3PF_2 neutron superfluid in the outer core, and a 1S_0 proton superconductor in the outer core. Still, many aspects of these phenomena need to be better understood. Due to the highly nonlinear character of the pairing mechanism giving rise to nuclear superfluidity and superconductivity, the associated critical temperatures remain very uncertain, especially for the 3PF_2 channel. The dynamics of these phase transitions as the star cools down, and the possible formation of topological defects need to be explored. Although the formalism for describing the relativistic smooth-averaged magnetoelastohydrodynamics of superfluid and superconducting systems already exists, modelling the global evolution of neutron stars in full general relativity still remains very challenging. To a large extent, the difficulty lies in the many different scales involved, from the kilometre size of the star down to the size of individual neutron vortices and proton fluxoids at the scale of tens or hundred fermis.

Studies of neutron-star dynamics using the Newtonian theory provide valuable qualitative insight, and should thus be pursued.

The presence of other particles such as hyperons or deconfined quarks in the inner core of neutron stars adds to the complexity. The occurrence of exotic superfluid and superconducting phases remains highly speculative due to the lack of knowledge of dense matter. On the other hand, astrophysical observations offer a unique opportunity to probe the phase diagram of matter under extreme conditions that are inaccessible in terrestrial laboratories.

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