Application of CORSIKA Simulation Code to Study Lateral and Longitudinal Distribution of Fluorescence Light in Cosmic Ray Extensive Air Showers

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Abstract. In this paper, we used CORSIKA code to understand the characteristics of cosmic ray induced showers at extremely high energy as a function of energy, detector distance to shower axis, number, and density of secondary charged particles and the nature particle producing the shower. Based on the standard properties of the atmosphere, lateral and longitudinal development of the shower for photons and electrons has been investigated. Fluorescent light has been collected by the detector for protons, helium, oxygen, silicon, calcium and iron primary cosmic rays in different energies. So we have obtained a number of electrons per unit area, distance to the shower axis, shape function of particles density, percentage of fluorescent light, lateral distribution of energy dissipated in the atmosphere and visual field angle of detector as well as size of the shower image. We have also shown that location of highest percentage of fluorescence light is directly proportional to atomic number of elements. Also we have shown when the distance from shower axis increases and the shape function of particles density decreases severely. At the first stages of development, shower axis distance from detector is high and visual field angle is small; then with shower moving toward the Earth, angle increases. Overall, in higher energies, the fluorescent light method has more efficiency. The paper provides standard calibration lines for high energy showers which can be used to determine the nature of the particles.

Keywords. Cosmic ray—fluorescence light—extensive air shower—Monte Carlo method—CORSIKA.

1. Introduction

One of the important aspects of cosmic ray experiments is determining energy of primary particles. But at very high energies, due to very low flux of radiation, direct detection method is not possible. Instead, in these energy ranges indirect detection methods should be used; such as the study of particles and light in Extensive Air Showers (EAS) induced by the interaction of primary particles with the Earth’s atmosphere.

One of these methods is the detection of fluorescent light induced by secondary cosmic ray particles when passing through the Earth’s atmosphere and interacting with nitrogen in the air. Fluorescent method is based on collecting emitted light from excited molecules by charged particles. These methods are used by the Fly’s Eye array.

For primary particles with Ultra High Energy (UHE), enough light will be produced by a large number of secondaries in shower (EAS) so that the light can be seen by detectors from a distance of several kilometers (G’ora et al. 2001). Since the fluorescent light entirely depends on the number of particles in shower, this method provides a good standard for measuring primary energy.

Air shower detection by fluorescence method (air glow) with sufficient sensitivity, in good climatic condition and favorable background light conditions, can be useful to follow large showers that pass through its vision field.

The air fluorescence method offers several important advantages when compared with other particle detector type shower arrays:

(1) Instead of sampling the showers only at a few points in an array on the ground at a particular stage of development and requiring that the shower axis be within the detector array for reliable data interpretation, the atmospheric fluorescence method allows observing the shower from the outside, at some distance and angle.
Fluorescence techniques are capable of revealing almost the entire history of each of the recorded events, unlike particle detector arrays. It records the longitudinal profile of the showers as well.

This method does not subject to large observational fluctuations because of the abundant number of photons that are detected, even from distant events, as compared to the small number of particles that are usually recorded by the detectors of an array per shower.

Like particle detector arrays, it is capable of determining the direction of incidence of the primary and the point of impact of the axis on the ground.

There are, however, several disadvantages and limitations to this method that restrict its application, but fluorescence method is now being used very successfully.

To determine the type, direction and energy of primary particles, energy of secondary particles are used. At very high particle energies sometimes the number of secondaries reach millions of particles at the observation level. So using the equations is difficult, and even in some cases, impossible. To solve this problem, we considered proper approximations to describe the air showers, which implies to simulate them using the Monte Carlo method.

The four main components of light can be distinguished as: (i) fluorescence light, with isotropic emission; (ii) direct Cherenkov radiation, emitted primarily in the forward direction; (iii) Rayleigh-scattered Cherenkov light; (iv) Mie-scattered Cherenkov light. The relative contributions of these components depend on the geometry of the shower with respect to the detector, but in most cases the fluorescent light dominates the recorded signal. Assuming only minor effects on the shower width by absorption and scattering processes during the fluorescence light propagation from the shower to the detector, the light fraction \( F(\alpha) \) which is the total light recorded by fluorescent detector, is mainly determined by the corresponding light fraction \( F(r) \) (light fraction) which is the portion of particles or the proportional part of the fluorescent light that is located at a distance of 0 to \( r \), emitted around the shower axis where \( f(r) \) is the lateral distribution of fluorescent light emitted, in the other words, it is the shape function or lateral distribution of photons while \( r \) is the distance from the shower axis.

Since this work is intended as a general study, the resulting photon distribution after light propagation is assumed to be recorded by an ideal detector and simulation is done for standard atmosphere default by CORSIKA.

About 25% of the emission occurs at short wavelength that is very poorly transmitted by the atmosphere. Hardly any of the light is found at wavelengths longer than 428 nm, hence, the signal-to-noise ratio can be improved by using a filter that transmits only in the deep blue and ultraviolet. Taking into account the transmission of a normal atmosphere as well as the energy loss of fast electrons in air, the effective yield at the detector is approximately one to two photons per meter of track length for a large shower that strikes at a distance of about 10 km. The background light from the dark (moonless) night sky is at best about \( 10^5 \) ph m\(^{-2} \) \( \mu \)s\(^{-1} \) sr\(^{-1} \).

### 2. Longitudinal development of fluorescent light

To study the longitudinal development of extensive air showers, one can measure the fluorescence radiation by air molecules when the charged particles are passing through. Shower particles excite nitrogen molecules and ions in the atmosphere and as a result, 10–50 nanoseconds after the excitation; they export fluorescence photons in average of 4–5 photons per meter for each electron (Shellard & Diazy 1999).

Approximately 80% of the emitted fluorescence light is in the range of 300–450 nm (three main lines are 337.1, 357.7 and 391.4 nm) for which the atmosphere is quite transparent (Shellard & Diazy 1999). This fluorescent light will be detected by optical detectors installed on the telescope when the light source moves along the axis of shower.

As the number of fluorescent photons generated at each depth of \( X \) is proportional to the number of electrons in that depth, with longitudinal study of the development of these photons, one can obtain the number of electrons in that depth and one may use their number to measure the primary energy (Song et al. 2000).

#### 2.1 Fluorescence yield

If we define fluorescence yield \( Y \) as the number of generated photons per electron per unit of length, then we will have

\[
Y = \frac{N_\gamma}{N_{el}}.
\]

So the intensity of observed light is directly proportional to the number of charged particles in each spot of longitudinal development of extensive air shower, and the number of photons per unit solid angle per
unit length can be obtained from the following equation (G’ora et al. 2004):

$$\frac{d^2N_e}{d\Omega d\lambda} \approx \frac{YN_e}{4\pi}\left[\text{photon}\right].$$  \hspace{1cm} (2)

The total number of particles is given by the Gaisser–Hillas equation (Gaisser & Hillas 1977):

$$N_e(X) = N_{\max} \left(\frac{X - X_0}{X_{\max} - X_0}\right)^{(X_{\max} - X_0)/\lambda} \exp\left(\frac{X_{\max} - X}{\lambda}\right),$$ \hspace{1cm} (3)

in which $X$ is the inclined atmospheric depth, $X_0$ is the depth of first interaction, $X_{\max}$ is the shower maximum i.e. the depth at which number of particles in shower reaches a maximum value, $\lambda$ is the hadronic interaction length in the air (generally equals 70 g/cm$^2$) and $N_{\max}$ is the maximum number of particles in the shower that is given by the following equation (Baltrusaitis et al. 1985):

$$N_{\max} = 0.7597\left(\frac{E_0 [\text{GeV}]}{10^9}\right)^{1.010} \ast 10^9,$$ \hspace{1cm} (4)

where $E$ is the primary energy of the shower.

### 2.2 Constant yield

Measurements have shown that changes in fluorescence yield ($Y$) as a function of height for electrons with constant energy, is quite small. For example, for an electron with energy of 80 MeV, the measured fluorescence yields over 20 km effective height in the atmosphere which will change lower than 12% around the mean value of 4.8 photons/m (G’ora et al. 2004).

These slight changes motivate us to use a constant that is the average fluorescence yield of the shower particles. On the other hand, since the fluorescence is caused by ionization and excitation of molecules of limited air, it is expected that the fluorescence yield depends on the ionization density in the particle path.

Most particles in the shower which were involved in energy deposition in air have a kinetic energy lower than MeV to several hundred MeV and the energy is in the range where the ionization density depends significantly on particle density.

The fluorescence yield per unit length is almost independent (about $\pm 10\%$) of pressure (or height) and temperature. Air fluorescence decay time is generally less than 10 ns.

In the spectral region between 300 nm to 430 nm, the total measured yield divided by energy loss (in MeV) in air at 1 atmospheric pressure is about 15 photons. In this wavelength range, values of 4.4 photons per meter will be obtained by dividing the length over the minimum particle ionization at sea level (4.4 ph m$^{-1}$e$^{-1}$) (Grieder 2010).

The fluorescence yield is primarily a function of the electron energy, temperature and atmospheric pressure. But in most calculations it will be considered constant and equal to 4.02 photons per meter.

### 3. Lateral development of fluorescent light

For primary particles with very high energy, sufficient fluorescent light will be produced in shower process, due to the large number of secondary particles. So the shower can be recorded from a distance of several kilometers by a suitable optical detector.

Since shower particles are close to shower axis, large percentage of the fluorescent light is at a distance of less than several ten meters and the observed fluorescent light will be seen as a spot, so studying fluorescent light as a point will be sufficient.

With an optical imaging system for recording the light emitted by a shower, the width of the shower is defined as a minimum angular diameter $\alpha$ of the image to include a specific part of $F(\alpha)$ of the total light recorded by the fluorescence detector.

If $\rho_N(X, r)$ be the number of electrons per square meter in depth $X$ and distance $r$ from the shower axis, then the shape function $f(r)$ will be defined as $f(r) = \frac{\rho_N(X,r)}{N_{\max}(X)}$. It is called shape function because if fluorescence yield is considered constant, then this function gives us a lateral distribution of photons. So in this case, $F(r)$ (light fraction) is the portion of particles or the proportional part of the fluorescent light that is located at a distance of 0 to $r$:

$$F(r) = \int_0^r f(r) 2\pi r \, dr.$$  \hspace{1cm} (5)

The final image of the shower will be made by the photons that reach the detector at the same time (G’ora et al. 2001). These photons that make an instantaneous image of the shower come from the process of development of the shower. If $\frac{dE}{dX}$ is energy deposit per unit length that electron leaves when it passes through the atmosphere, and if this value has been estimated by its mean value $\left(\frac{dE}{dX}\right)$, then in case of the NKG approximation, lateral distribution of energy in the atmosphere, $\rho_E(X, r)$ can be obtained from the following equation (G’ora et al. 2004):

$$\rho_E(X, r) = \left(\frac{dE}{dX}\right)N_{\max}\rho_N(X, r).$$ \hspace{1cm} (6)
3.1 Angular size of shower image

For pure electromagnetic showers, density of electron at the shower \( \rho_N(X, R) \) is given by the NKG formula (G’ora et al. 2004; Kamata et al. 1958):

\[
\rho_N(X, r) = \frac{N_e(X)}{r_M^2} \left( \frac{r}{r_M} \right)^{s-2} \left( 1 + \frac{r}{r_M} \right)^{(s-4.5)} \\
\times \frac{\Gamma(4.5-s)}{2\pi \Gamma(s)(4.5-2s)}, \quad (7)
\]

i.e., \( s = \frac{3X/X_v}{X/X_v + 2 \ln(E_0/e_0)} \) and \( X = \frac{X_v}{\cos \theta} \), where \( X_v = \int_h^{\infty} \rho(h)dh \) is the critical depth in atmosphere, \( \theta \) is the zenith angle of shower, \( E_0 \) is the primary energy of the shower, \( e_0 \) is the critical energy and \( X_l \) is the radiation length in the air. \( \rho(h) \) is the air density at an altitude \( h \) and \( r_M \) is Moliere radius. The age parameter \( s \), specifies the current stage of shower evolution \( (s = 1, \text{ and means shower is at its maximum}) \).

The Moliere radius is a natural horizontal scale that is caused by multiple scattering, and determines the lateral distribution of the shower. Since the electron’s radiation length in the air depends on temperature and pressure, Moliere radius varies along the shower. Particle distribution in a shower at a certain depth relatively depends on the variation \( r_M \) along the shower instead of value of \( r_M \) in a certain depth. To consider it in a certain atmospheric depth, \( r_M \) value at twice the length of electron’s radiation length, is calculated as follows (G’ora et al. 2004):

\[
r_M[m] = 272.5 \frac{T[K]}{P[mb] - 37.94 \cos \theta}^{1.25588}, \quad (8)
\]

where \( T \) is the temperature and \( P \) is the atmospheric pressure at height \( h \).

As HiRes Group, we use a constant value for the fluorescence yield \( Y = 4.02 \text{ photon/m} \). Spatial distribution of the light emitted will be defined by NKG formula. As a result, fluorescence distribution \( F(r) \) is determined by the normalized incomplete beta function, analytically:

\[
F(r) = I_x(a, b) = \frac{1}{B(a, b)} \int_0^x u^{a-1}(1 - u)^{b-1}du, \quad (9)
\]

i.e., \( x = 1/(1 + r_M/r) \), \( a = s, b = 4.5 - 2s \) and \( B(a, b) \) is the Euler beta function. Using expansion of \( I_x(a, b) \) series (Abramowitz & Stegun 1965), we can determine the distribution of fluorescent light by the following equation:

\[
F(r) = \left( 1 + \frac{r}{r_M} \right)^{4.5-s} \frac{1}{sB(s, 4.5 - 2s)} \times \left( 1 + \sum_{n=0}^{\infty} B(s+1, n+1) \left( \frac{1}{1 + \frac{r}{r_M}} \right)^{n+1} \right). \quad (10)
\]

For \( s = 1 \)(maximum of shower), the result is reduced to the following formula:

\[
F(r) = 1 - \left( 1 + \frac{r}{r_M} \right)^{-2.5}. \quad (11)
\]

Using the above reversed equation, if at a moment, detector distance to shower is \( R \), for \( \alpha \) angle which is a specific portion of the total fluorescent light \( (F(r)) \), we have

\[
\tan \left( \frac{\alpha}{2} \right) = \frac{r}{R}, \quad \alpha = 2 \tan^{-1} \left( \frac{r}{R} \right) = 2 \tan^{-1} \left( \frac{r}{r_M} \right) \left( 1 - F(r) \right)^{-0.4} - 1). \quad (12)
\]

This angle gives a visual field which corresponds to the percentage of light located at a distance of 0 to \( r \).

4. Details of simulated samples

4.1 Simulations for protons with energy 10 eV

At this point, using CORSIKA (Heck et al. 1998), initially primary beam of protons was simulated and lateral and longitudinal distribution of fluorescence light induced by excitation of nitrogen molecules in the atmosphere has been studied and obtained, and then the angular size of shower image for simulated data was calculated. The height is CORSIKA default for KASKADE, 110 m above sea level.

First \( \rho_N(X, r) \) (i.e. the number of electrons per unit area in the depth of \( X) \) and the distance to the shower axis, \( r \), have been obtained from simulations and are shown in Fig. 1(a). Then to obtain the shape function of particles density, \( f(r) \) (i.e. lateral distribution of photons), \( \rho_N(X, r) \) curve data have been divided to the number of electrons at the observation level. As a result, the value of \( N_e = 75 \times 10^8 \) is obtained by simulation; results can be seen in Fig. 1(b). To obtain the percentage of particles (fluorescence light) located at a distance of \( 0 \) to \( r \), \( F(r) \), first we fitted the lateral distribution curve with a function of \( \frac{b}{e^{vy}c} \) and reached
Figure 1. Simulation results for protons with energy 10 eV. (a) Number of electrons per unit area in the depth \((X)\) and distance to the shower axis \((r)\) \((\rho_N(X, r))\). (b) Percentage of particles (fluorescence light) located at a distance of 0 to \(r(F(r))\). (c) Shape function of particles density \(f(r)\) (lateral distribution of photons). (d) Lateral distribution of energy dissipated in the atmosphere \(\rho_E(X, r)\), and (e) Detector visual field angle for collecting 90% of fluorescent light for proton in 10 eV.

Figure 2. Simulation results for proton primaries with other energies. (a) Number of electrons per unit area in the depth \((X)\) and distance to the shower axis \((r)\) \((\rho_N(X, r))\). (b) Shape function of particles density \(f(r)\) (lateral distribution of photons). (c) Percentage of particles (fluorescence light) located at a distance of 0 to \(r(F(r))\). (d) Lateral distribution of energy dissipated in the atmosphere \(\rho_E(X, r)\). (e) Detector visual field angle for collecting 90% of fluorescent light for proton in \(10^{14}\) eV, \(10^{15}\) eV and \(10^{16}\) eV.
the fit parameters $a = 110/7, b = 1070$ and $c = 92/3$. Then we got integral from this function to achieve $F(r)$. The results are shown in Fig. 1(c). To obtain the lateral distribution of energy dissipated in the atmosphere $\rho_E(X, r)$, which is a measure of the fluorescence photon’s intensity, we have considered $\frac{dE}{dx}$ equal to 2 MeV/(gr/cm²) because of constant fluorescence yield (4.02 photons per meter) (G’ora et al. 2004). The results for the lateral distribution of energy left in the atmosphere $\rho_E(X, r)$, are shown in Fig. 1(d). Finally visual field angle of detector for different distances to shower axis were calculated for our data and are shown in Fig. 1(e).

The results are derived from CORSIKA with vertical radiation angle at a height 110 m above sea level. In the CORSIKA simulations performed for this analysis, High-energy interactions are calculated by the QGSJET02 model and low-energy interactions are calculated by the GHEISHA2002 model. Since each simulation lasts about more than 10 days for each shower, only 2 showers were simulated for any energy.

4.2 Simulations for proton primaries with other energies

At this point, using CORSIKA and the same approach, primary beam of protons with energies $10^{14}$, $10^{15}$ and $10^{16}$ eV were simulated to investigate the effects of primary energy. The height is equal to 180 m above sea level. First $\rho_N(X, r)$ have been calculated and are shown in Fig. 2(a). Then to obtain $f(r)$, $\rho_N(X, r)$ curve data have been divided to the number of electrons in the observation level that had been obtained $N_e = 5617.97, N_e = 105814.88, N_e = 1549639.16$ for energies $10^{14}$, $10^{15}$ and $10^{16}$ eV by simulation. The results can be seen in Fig. 2(b). Then we fitted the lateral distribution curve with function $b / e^{ax}$ and we reached the fit parameters $a_{14} = 15110, b_{14} = 2059, c_{14} = -125.5, a_{15} = 177500, b_{15} = 2299, c_{14} = -125.5, a_{16} = 31.86, b_{16} = 1537$ and $c_{16} = -124.2$. Then we got an integral from this function to achieve $F(r)$ (Fig. 2(c)). The results for the lateral distribution of energy left in the atmosphere $\rho_E(X, r)$ is shown in Fig. 2(d). Finally visual field angle of detector for different distances to...
shower axis were calculated for our data and are shown in Fig. 2(e).

4.3 Simulations for other cosmic ray species

In the next step, using CORSIKA and the same approach, simulation was done for primary beam of oxygen, calcium, proton, silicon, helium and iron with energies $10^{14}$ and $10^{15}$ eV. The height is equal to 180 m above sea level.

First $\rho_N(X, r)$ have been calculated and are shown in Figures 3(a) and 4(a). Then to obtain the shape function of particles density $f(r)$ (lateral distribution of photons), $\rho_N(X, r)$ curve data have been divided to the number of electrons in the observation level that had been obtained as $N_e$ in Table 1. The results can be seen in Figures 3(b) and 4(b). Then we got an integral from this function to achieve $F(r)$ (Figures 3(c) and 4(c)). The results for $\rho_E(X, r)$ are shown in Figures 3(d) and 4(d). Visual field angle of detector for different distances to shower axis were calculated for our data and are shown in Figures 3(e) and 4(e). All elements with the same energy were

Table 1. Number of electrons in the observation level ($N_e$) and fit parameters for different species in $10^{14}$ eV and $10^{15}$ eV.

<table>
<thead>
<tr>
<th>Species</th>
<th>Oxygen</th>
<th>Calcium</th>
<th>Proton</th>
<th>Silicon</th>
<th>Helium</th>
<th>Iron</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{e14}$</td>
<td>59820.45</td>
<td>44194.89</td>
<td>105814.9</td>
<td>49550.26</td>
<td>87389.11</td>
<td>39103.26</td>
</tr>
<tr>
<td>$a_{14}$</td>
<td>$1.088 \times 10^{-7}$</td>
<td>$1.551 \times 10^{-7}$</td>
<td>$5.199 \times 10^{-7}$</td>
<td>$1.06 \times 10^{-21}$</td>
<td>$6.548 \times 10^{-7}$</td>
<td>$1.075 \times 10^{-31}$</td>
</tr>
<tr>
<td>$b_{14}$</td>
<td>708</td>
<td>668.1</td>
<td>335.5</td>
<td>75.72</td>
<td>310.7</td>
<td>814</td>
</tr>
<tr>
<td>$c_{14}$</td>
<td>91.76</td>
<td>95.71</td>
<td>48.34</td>
<td>$-47.19$</td>
<td>46.68</td>
<td>18.13</td>
</tr>
<tr>
<td>$N_{e15}$</td>
<td>2556.77</td>
<td>1945.6</td>
<td>5617.97</td>
<td>2294.47</td>
<td>4611.45</td>
<td>1660.68</td>
</tr>
<tr>
<td>$a_{15}$</td>
<td>$1.346 \times 10^{-6}$</td>
<td>$6.896 \times 10^{-7}$</td>
<td>$1.52 \times 10^{-6}$</td>
<td>$1.566 \times 10^{-19}$</td>
<td>$7.181 \times 10^{-7}$</td>
<td>$2.208 \times 10^{-48}$</td>
</tr>
<tr>
<td>$b_{15}$</td>
<td>220.8</td>
<td>316.2</td>
<td>198.5</td>
<td>1704</td>
<td>277.3</td>
<td>801.5</td>
</tr>
<tr>
<td>$c_{15}$</td>
<td>36</td>
<td>49.34</td>
<td>30.43</td>
<td>65.49</td>
<td>39.63</td>
<td>10.97</td>
</tr>
</tbody>
</table>
plotted on one graph to compare the composition of the various elements with each other.

5. Conclusions

We obtained the number of electrons per unit area in the depth (X) and distance to the shower axis (r), \( \rho_N(X, r) \). The shape function of particle density was \( f(r) \) with the percentage of fluorescence light \( F(r) \). Lateral distribution of energy dissipated in the atmosphere is \( \rho_E(X, r) \) and visual field angle of detector is \( \alpha \). These parameters were calculated for primary beams of oxygen, calcium, proton, silicon, helium and iron, in different energies and in this way we were able to calculate lateral and longitudinal profiles of fluorescent light.

In this method, the image size of shower, \( \alpha \), is related to the width of shape function of particle density \( f(r) \), and it can be calculated in the maximum of shower using the above equations for fixed Moliere radius, fixed fluorescence distribution \( F(r) \) and fixed detector distance to shower \( (R) \). In simulation results for protons with energy 10 eV, as seen in Fig. 1(c), about 90% of fluorescent light in 10 eV was located at a distance of 50 m from the shower axis. In simulations results for proton primaries with other energies, as seen in Fig. 2(c), about 90% of fluorescent light in 10\(^{14} \) eV and 10\(^{15} \) eV were located respectively at distances of 22, 17 and 15 m from the shower axis.

In simulation results for other cosmic ray species, as seen in Figures 3(c) and 4(c), about 90% of fluorescent light in 10\(^{14} \) eV were located at distances of proton: 21 m, helium: 24 m, oxygen: 28 m, silicon: 30 m, calcium: 37 m and iron: 38 m from the shower axis. And in 10\(^{15} \) eV, they were located at distances of proton: 29 m, helium: 35 m, oxygen: 53 m, silicon: 66 m, calcium: 68 m and iron: 76 m from the shower axis. And as the atomic number of these elements are protons: 1, helium: 2, oxygen: 8, silicon: 14, calcium: 20 and iron: 26; it can be concluded that 90% of the fluorescent light for elements with lower atomic number, has been located at lesser distances from the axis of the shower.

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