Power Spectrum Density of Stochastic Oscillating Accretion Disk

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Abstract. In this paper, we employ a stochastic oscillating accretion disk model for the power spectral index and variability of BL Lac object S5 0716+714. In the model, we assume that there is a relativistic oscillation of thin accretion disks and it interacts with an external thermal bath through a friction force and a random force. We simulate the light curve and the power spectrum density (PSD) at (i) over-damped, (ii) critically damped and (iii) under-damped cases, respectively. Our results show that the simulated PSD curves depend on the intrinsic property of the accretion disk, and it could be produced in a wide interval ranging from 0.94 to 2.05 by changing the friction coefficient in a stochastic oscillating accretion disk model. We argue that accretion disk stochastic oscillating could be a possible interpretation for observed PSD variability.

Key words. Accretion–accretion disks—black hole physics—instabilities.

1. Introduction

It is well known that the compact object is often surrounded by an accretion disk. As an important theoretical model, the accretion disk can interpret a large number of astrophysical phenomena, such as the observational features of the black hole binaries and Active Galactic Nuclei (AGN) is the strong and chaotic time variation in X-rays. The disk is usually considered to be composed of massive particles that move in the gravitational field of the central object. A lot of disk models have been proposed, for example, an ensemble of oscillations and waves in geometrically thin disks have been studied extensively both within Newtonian gravity (Kato 2001) and within a relativistic framework (Zanotti *et al.* 2005; Blaes *et al.* 2006; O'Neill *et al.* 2009; Shi & Li 2010).

In recent years, Harko & Mocanu (2012) have studied in detail the vertical stochastic oscillations of the accretion disks in both the static Schwarzschild and the rotating Kerr geometries, and explicitly obtained the vertical displacements, velocities and luminosities of the stochastically perturbed disks by numerically integrating the corresponding Langevin equation.

By using the stochastic oscillations of the accretion disks, Leung *et al.* (2011a) have explained the observations of optical Intra Day Variability (IDV) of BL Lac objects. Wang *et al.* (2013) calculated the luminosity and power spectrum density (PSD) for an oscillating disk. They pointed out that the simulated PSD curves of luminosity for disk oscillation have the same profile as the observed PSD of black hole X-ray binaries, and the stochastic resonance of accretion disk vertical oscillation may be an alternative interpretation of the persistent low-frequency quasi-periodic oscillations (Wang *et al.* 2013). Soon afterward, the influences of the radial distance parameter, the mass and spin parameter of the black hole was discussed, and it was pointed out that these parameters have significant influences on the stochastic resonance in PSD curves (Wang *et al.* 2015). Hence the stochastic resonance phenomena in a black hole system can be explained by the stochastic oscillations of accretion disk model.

BL Lac S5 0714+716 is an object with confirmed and documented variability in all wavelengths and on a wide interval of timescales. Theoretical tools have managed to satisfactorily explain most of the variability behaviour. Many literatures have shown that the spectral index variability with wide ranges is observed in S5 0716+714 (Azarnia *et al.* 2005; Poon *et al.* 2009; Dexter & Agol 2011; Leung *et al.* 2011b; Mocanu & Marcu 2012). Some investigators considered that the PSD variability implies that the emission process is accompanied by a turbulence (Mocanu & Marcu 2012). Some argue that it could be interpreted as scintillation of radio waves caused by the turbulent interstellar medium of the Milky Way (Gabányi *et al.* 2007; Marchili *et al.* 2012). On the other hand, the PSD variability are interpreted as a seasonal effect (Rickett *et al.* 2001; Qian & Zhang 2001; Dennett-Thorpe & de Bruyn 2003). As an open issue, various models including the source-intrinsic (Lobanov & Zensus 1999; Hagen-Thorn *et al.* 2008; Bach *et al.* 2009) and the source-extrinsic (Jauncey & Macquart 2001; Liu *et al.* 2013) have been proposed.

Our aim here is to determine that the observed PSD variability could be reproduced or not by a stochastic oscillating accretion disk model. We organize our paper as follows: In section 2, we describe the stochastic oscillating accretion disk model briefly, and then, we solve the general Langevin type differential equation in three cases. The disk luminosity and PSD indexes are calculated in section 3. We discuss and conclude our results in section 4.

2. The stochastic oscillating accretion disk

We basically follow the method of Harko & Mocanu (2012) to reproduce the PSD variability. We now give a brief description of the model.

2.1 *The basic equation and solutions*

We consider the relativistic oscillations of thin accretion disks around compact astrophysical objects interacting with an external thermal bath through a friction force and a random force. The vertical oscillation of the unit mass particle in this stochastically perturbed disk can be described by a general Langevin type equation (Harko & Mocanu 2012),

$$\frac{d^2Z(t)}{dt^2} + c\mu\frac{dZ(t)}{dt} + (c\omega_c)^2 Z(t) = c^2\xi(t), \tag{1}$$

where $Z(t)$ is the vertical oscillating displacement of the disk in unit of cm, μ is the friction coefficient in unit of s^{-1}, ω_c is the characteristic frequency of the disk in unit of Hz, c is a normalized parameter by the normalized factor 1.0×10^{10} cm s^{-1}. The function $\xi(t)$ denotes the random force of white noise type in unit of cm s^{-2}, and the statistical characteristics show as

$$\begin{cases} \langle \xi(t) \rangle = 0 \ , \\ \langle \xi(t_1)\xi(t_2) \rangle = D\delta(t_1 - t_2) \ , \end{cases} \tag{2}$$

where $\delta(t)$ is the Dirac delta function, and D that indicates the intensity of random white noise, is the intensity factor. In our work, we adopt $D = 1$. This results in the function $\xi(t)$ producing a random white noise in the range of (0, 1).

It is clearly seen that the left-hand side of equation (1) is an harmonic oscillator equation, for simplicity, we let $\gamma = c\mu/2$, $\omega = c\omega_c$, $F(t) = c^2\xi(t)$, then, we can rewrite equation (1) as

$$\frac{d^2Z(t)}{dt^2} + 2\gamma\frac{dZ(t)}{dt} + \omega^2 Z(t) = F(t). \tag{3}$$

Using the separation of variables, we can solve equation (3) in (i) over-damped case, (ii) critically damped case, and (iii) under-damped case, respectively. We give the solutions for these three special cases as follows. First, we consider the over-damped case. In this case, $\gamma > \omega$, so,

$$Z_O(t) = C_1 e^{a_1 t} + C_2 e^{a_2 t} - e^{a_1 t}\int \frac{e^{a_2 t} F(t)}{W} dt + e^{a_2 t}\int \frac{e^{a_1 t} F(t)}{W} dt \ , \tag{4}$$

where C_1 and C_2 are constants that depend on the initial values, $a_1 = -\gamma + \sqrt{\gamma^2 - \omega^2}$, $a_2 = -\gamma - \sqrt{\gamma^2 - \omega^2}$, $W = (a_2 - a_1)e^{(a_1+a_2)t}$. Then, we consider the critically damped case. In this case, $\gamma = \omega$, so,

$$Z_C(t) = C_1 e^{a_1 t} + C_2 t e^{a_2 t} - e^{a_1 t}\int \frac{t e^{a_2 t} F(t)}{W} dt + t e^{a_2 t}\int \frac{e^{a_1 t} F(t)}{W} dt \ , \tag{5}$$

where $a_1 = a_2 = -\gamma$, $W = e^{(a_1+a_2)t}$. Last, we consider the under-damped case. In this case, $\gamma < \omega$, so,

$$\begin{aligned} Z_U(t) \ = \ & e^{a_1 t}(C_1 \cos a_2 t + C_2 \sin a_2 t) - e^{a_1 t}\cos a_2 t \int \frac{e^{a_1 t}\sin a_2 t}{W} F(t) dt \\ & + e^{a_1 t}\sin a_2 t \int \frac{e^{a_1 t}\cos a_2 t}{W} F(t) dt \ . \end{aligned} \tag{6}$$

where a_1, a_2 is the real part and the imaginary part of $-\gamma \pm \sqrt{\gamma^2 - \omega^2}$, respectively, and $W = a_2 e^{2a_1 t}$. It can be seen from the above results, that the solution of the equation contains both the harmonic vibration term (no integral term) and random vibration term (integral term).

2.2 *The characteristic frequency and initial conditions*

We consider the oscillation of the accretion disk as a whole body under the influence of gravity of the central compact object. In this scenario, Shakura & Sunyaev (1973) approximate the surface density distribution in the geometrically thin disk by the formulae (see also, Titarchuk & Osherovich, 2000)

$$\begin{cases} \Sigma = \Sigma_0 = C \;\; \text{for} \;\; R_{\rm in} \le R \le R_{\rm adj} \;, \\ \Sigma = \Sigma_0 (\frac{R}{R_{\rm adj}})^{-\eta} \;\; \text{for} \;\; R_{\rm adj} \le R \le R_{\rm out} \;, \end{cases} \tag{7}$$

where C is a constant, $R_{\rm in}$ is the innermost radius of the disk, $R_{\rm adj}$ is an adjustment radius in the disk, $R_{\rm out}$ is the outer radius of the disk, and the surface density index η is either 3/5 or 3/4 (Shakura & Sunyaev 1973). We assume that the restoring force of the disk oscillation is caused by gravitational attraction of the central compact object. We can deduce the characteristic frequency of the disk using Hooke's law:

$$\omega_c = \frac{2.2 \times 10^3}{m} \left\{ \frac{2-\eta}{x_{\rm in}^3} \frac{[1 - \eta/((\eta+1) r_{\rm adj})]}{x_{\rm adj}^{\eta} x_{\rm out}^{2-\eta}} \right\}^{1/2} \;\; \text{Hz}, \tag{8}$$

where $x_{\rm in} = R_{\rm in}/3R_{\rm s}$, $r_{\rm out} = R_{\rm out}/R_{\rm in}$, $r_{\rm adj} = R_{\rm adj}/R_{\rm in}$, $m = M/M_{\odot}$ and $R_{\rm s} = 2GM/c^2$ is the Schwarzschild radius (for more details about equation (8), see Titarchuk & Osherovich (2000)).

We apply the model to the BL Lac object S5 0716+714. In order to do so, we assume that $x_{\rm in} = 1$, $x_{\rm adj} = 3$, $r_{\rm out} = 10^4$ and $\eta = 3/5$. The mass range of the source from $4.79 \times 10^7 M_{\odot}$ to $2.40 \times 10^8 M_{\odot}$ was estimated by Fan *et al.* (2011), In our work, we adopted $M = 1.50 \times 10^8 M_{\odot}$. Substituting the above parameters into equation (8), we calculated the characteristic frequency of the disk as $\omega_c \approx 0.02$ Hz. Since we let $\omega = c\omega_c$, we obtained $\omega = 0.06$ Hz by adopting $c = 3$. Considering the initial displacement of the disk oscillation $Z(t = 0) = z_0$, and initial velocity of the disk $\frac{{\rm d}Z(t=0)}{{\rm d}t} = v_0$ as initial conditions (Wang *et al.* 2013, 2015), we could solve the equation of the stochastic oscillating accretion disk (equation (3)) in three special cases with the friction coefficient $\gamma > \omega$, $\gamma = \omega$ and $\gamma < \omega$, respectively. As a case, in our calculation, we adopt $\gamma = 0.35$, $\gamma = 0.06$ and $\gamma = 0.01$. The results are shown in Fig. 1.

3. The disk luminosity and PSD analysis

The disk luminosity, which mark the energy lost because of the viscous dissipation and the random force effect, is given by Harko & Mocanu (2012),

$$L(t) = c\mu \left(\frac{{\rm d}Z(t)}{{\rm d}t} \right)^2 - \frac{{\rm d}Z(t)}{{\rm d}t} F(t). \tag{9}$$

Using equation (9), we calculate the disk luminosity in the over-damped case, critically damped case and under-damped case, respectively. The results are shown in

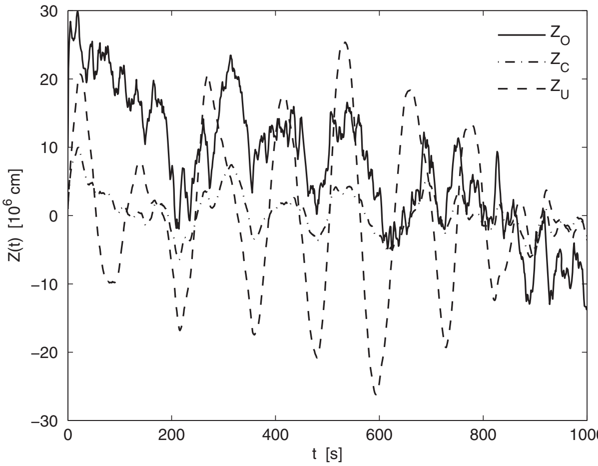

Figure 1. Variation of the vertical displacement Z of the stochastic oscillating disk as a function of timestep t for $\gamma > \omega$, $\gamma = \omega$ and $\gamma < \omega$ (labelled by $Z_{\rm O}$, $Z_{\rm C}$ and $Z_{\rm U}$), respectively.

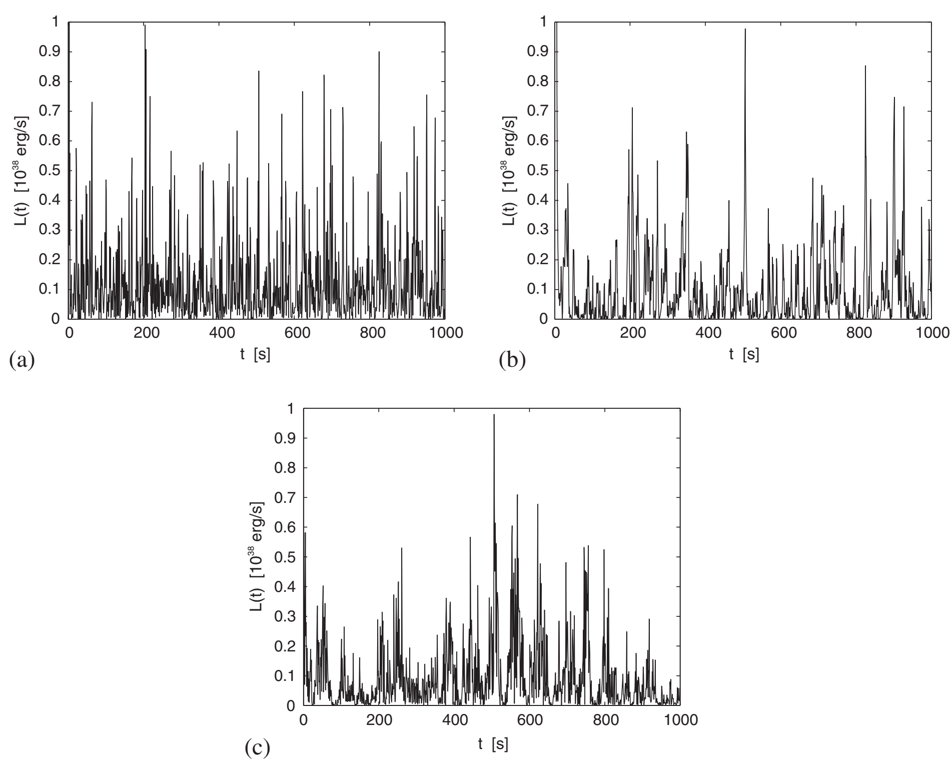

Figure 2. The luminosity of the stochastic oscillating disk for $\gamma > \omega$ (**a**), $\gamma = \omega$ (**b**) and $\gamma = \omega$ (**c**), respectively. The vertical axis represents the relative values of luminosity.

Fig. 2. Now the index of the PSD can be used to penetrate the variability mechanism of the accretion disk. In order to describe the stochastic features of the oscillating disk in three special cases, we first calculate the PSD of the luminosity, and then we fit the PSD by using a function as follows:

$$P(f) = \beta f^{-\alpha} + C_n, \tag{10}$$

where f is the frequency, α is the spectral index, and C_n describes the Poisson noise contribution to the PSD. In Fig. 3, we show the PSD of the disk luminosity (thin solid line) and the fitting results by equation (10) (dashed line). The fitting parameters are listed in Table 1, all of the errors that is quoted in this table are on the 68% confidence level. For comparison, the best fitting results of the PSD, which is fitted by Gompertz curve, are shown in this figure (bold solid line). As can be seen from our results, (i) the spectra index of PSD is in a wide range from 0.94 to 2.05; (ii) when the characteristic frequency ω keeps on an invariant value, the α becomes bigger with increasing the friction coefficient γ.

In order to examine the dependence of the PSD on the friction coefficient, in Fig. 4, we simulate the spectral index α of PSD with different friction coefficient γ. It can be seen that the α enlarges when γ increases. The relationship could be exhibited by a power law function with $\alpha = 2.54\gamma^{0.16}$. Now that the disk luminosity depends

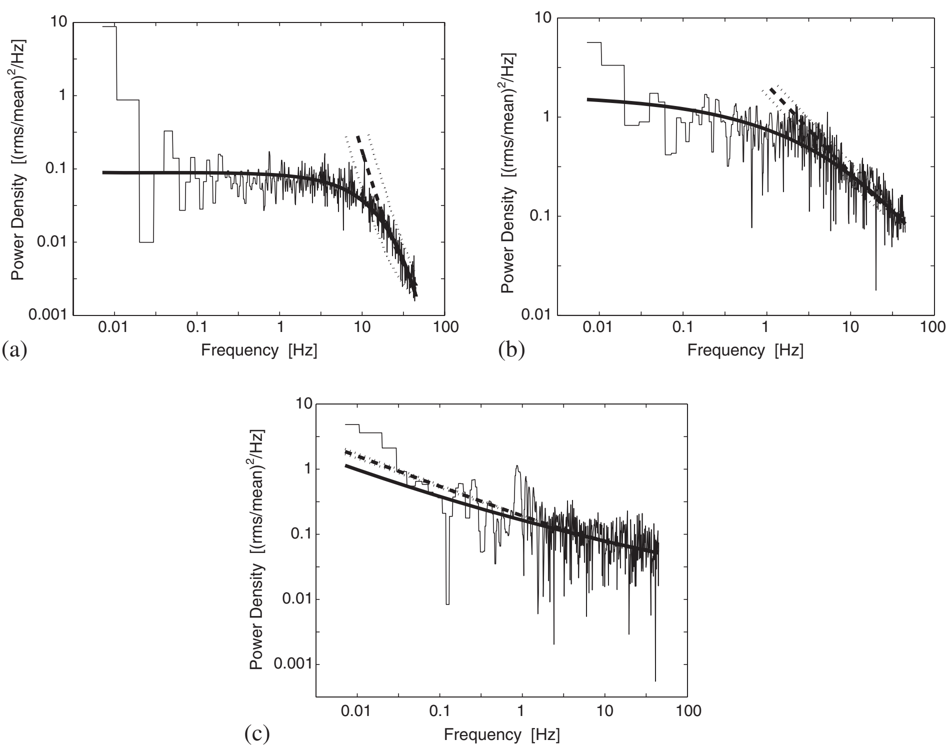

Figure 3. PSDs of the luminosity for $\gamma > \omega$ (**a**), $\gamma = \omega$ (**b**) and $\gamma < \omega$ (**c**), respectively. The dotted line shows the 68% confidence limits.

Table 1. Results of spectral analysis.

Cases	$\lg \beta$	α	$\lg C_n$	χ^2/dof
$\gamma > \omega$	$8.04^{+1.35}_{-0.63}$	2.05 ± 0.04	$9.92^{+0.45}_{-0.86}$	45.4/1297
$\gamma = \omega$	$4.08^{+1.19}_{-0.26}$	1.81 ± 0.07	$4.64^{+1.08}_{-0.04}$	1302.2/1882
$\gamma < \omega$	$3.32^{+0.49}_{-1.01}$	0.93 ± 0.16	$4.42^{+0.37}_{-0.60}$	3455.0/1943

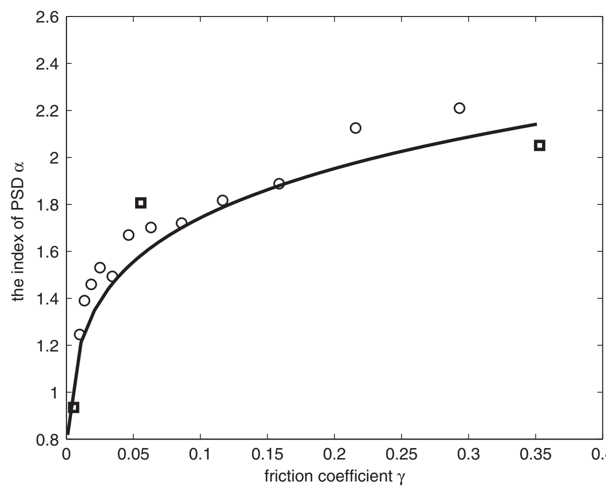

Figure 4. The dependence of the spectral index of α PSD on the friction coefficient γ. The squares denote the values in Table 1 and the solid line represents the best fit of all γ vs. α points.

on the random noise $\xi(t)$, we argue that the relationship between the PSD and the friction coefficient will be affected by the the random noise.

4. Discussion and conclusion

On the mathematical concepts, correlation degree is a statistical correlation between random variables at two different points, and could be usually perceived by the spectral index α of the PSD. As shown in Harko & Mocanu (2012), a PSD, $P(f) \sim f^{-\alpha}$ with $\alpha = 2$. The $\alpha = 2$ is referred to as red noise or Brownian noise, and it could be produced by a simple Brownian motion model for the accretion disk oscillations. While a PSD, $P(f) = f^0 =$ const, the $\alpha = 0$ is referred to as the white noise, and the white noise was produced by the completely uncorrelated interaction of the disk with the external medium (Harko & Mocanu 2012).

However, the observational data for a large percentage of AGN show that their optical IDV has PSD for which α is neither 0 nor 2. For example, the spectral index of BL Lac object S5 0716+714 was found to vary from 1.083 to 2.65 (Poon *et al.* 2009; Mocanu & Marcu 2012), from 0.6 to 2.22 (Carini *et al.* 2011), from 0.8 to

1.4 (Azarnia *et al.* 2005) and from 0.6 to 1.7 in the data set discussed in Leung *et al.* (2011b). As shown in the literature, the spectral index variability with wide ranges is observed in S5 0716+714. Our results show that the simulated PSD curves of luminosity depend on the intrinsic property of accretion disk, and it could be produced in a wide interval ranging from 0.935 to 2.051 (see Table 1) by changing the friction coefficient in a stochastic oscillating accretion disk model.

In summary, we briefly introduce an existing model that describes the vertical stochastic oscillations of the accretion disks by general Langevin type differential equation, and solve this equation for three cases: (i) over-damped, (ii) critically damped, and (iii) under-damped. From this, the light curve and hence the power spectrum are calculated for the three model cases. Finally, we obtain the spectral index from these cases by fitting a power law to the power spectrum and conclude that the variability in the spectral index from an observed object could be caused by a change between one model case to another. The stochastic oscillating accretion disk model could be an alternative interpretation of the observed PSD variability.

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