J. Astrophys. Astr. (2016) 37: 5

Gravitational Instability of Cylindrical Viscoelastic Medium Permeated with Non Uniform Magnetic Field and Rotation

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Received 27 July 2015; accepted 16 October 2015

DOI: 10.1007/s12036-016-9371-3

Abstract. The self-gravitating instability of an infinitely extending axisymmetric cylinder of viscoelastic medium permeated with non uniform magnetic field and rotation is studied for both the strongly coupled plasma (SCP) and weakly coupled plasma (WCP). The non uniform magnetic field and rotation are considered to act along the axial direction of the cylinder. The normal mode method of perturbations is applied to obtain the dispersion relation. The condition for the onset of gravitational instability has been derived from the dispersion relation under both strongly and weakly coupling limits. It is found that the *Jeans criterion* for gravitational collapse gets modified due to the presence of shear and bulk viscosities for the SCP, however, the magnetic field and rotation whether uniform or non uniform has no effect on the *Jeans criterion* of an infinitely extending axisymmetric cylinder of a self-gravitating viscoelastic medium.

Key words. Jeans criterion—viscoelastic medium—non uniform magnetic field—non uniform rotation—strongly coupled plasma (SCP)—weakly coupled plasma (WCP).

1. Introduction

One of the fundamental conditions for the onset of dynamical instability causing disperse, cold gas to collapse was given by Sir James Jeans (1902) and now known as *Jeans criterion* of gravitational instability. This criterion states that the medium becomes unstable and breaks up for perturbations of the wave number k less than Jeans wave number $k_j = \frac{\sqrt{G\rho}}{c}$. Here, ρ is the density, c is the velocity of sound and G is the gravitational constant. The problem of magneto gravitational instability of interstellar rotating medium is of considerable importance in connection with the protostar and star formation in magnetic dust clouds. Chandrasekhar (1961) studied the gravitational instability of the homogeneous medium and investigated the individual/simultaneous effects of uniform magnetic field and rotation on it. He found that *Jeans criterion* remains unaffected in the presence of uniform rotation and/or magnetic field. Further, the effects of rotation and magnetic field on Jeans instability in different medium have also been investigated by many authors and a general conclusion that can be drawn from these studies is that rotation and magnetic field have no effect on the *Jeans criterion*.

In natural and laboratory situations, many objects exist in strongly coupled state viz. neutron star, dwarf star, interior of planets, ultra cold neutral plasma and liquid plasma crystal (cf. Ikeji 1986). In these objects, the Coulomb potential energy is much greater than that of the average thermal energy of the plasma particles (Prajapati *et al.* 2012). The ratio of the Coulomb potential energy to the thermal energy is defined as the coupling parameter (Γ). Thus, for strongly coupled plasma (SCP), $\Gamma \gg 1$, whereas for decreasing value of coupling parameter $\Gamma \ll 1$, the plasma behaves as weakly coupled plasma (WCP).

Kaw *et al.* (2002) interpreted the experimental results of Sato *et al.* (1998, 2000, 2001) and Konopka *et al.* (2000) in which they have carried out some experiments in strongly coupled dusty plasma, with the help of a simple theoretical model. Further, Kaw *et al.* (2002) observed that the theoretical model developed by them was similar to that of Sato *et al.* (1998, 2000, 2001) and Konopka *et al.* (2000) to explain their experimental observations and reported that these studies are of importance in understanding the behavior of magnetized dusty plasma in various space, astrophysical and laboratory situations.

Kaw & Sen (1998) studied the dynamics of strongly coupled plasma (SCP) by using the generalized hydrodynamic model (GH). In GH model, the stress tensor is related to velocity gradients through bulk and shear viscosity coefficients for a fluid. Janaki & Chakrabarti (2010) investigated the shear wave vortex solution in strongly coupled dusty plasma and found that, in the kinetic limit ($\sigma\tau \gg 1$), SCP supports localized dipolar vortex like solutions with amplitude modulated periodically, where, σ is the wave frequency and τ is the viscoelastic relaxation time. Janaki *et al.* (2011) studied the Jeans instability in a viscoelastic (non Newtonian) fluid by using GH model and assumed that in the transition stage between the viscous liquid state and the elastic solid state, the characteristics of stellar matter are similar to that of viscoelastic fluid where both properties work together. They also observed that instability occurs at higher wavelengths in a viscoelastic medium in comparison to Newtonian fluid. Rosenberg & Shukla (2011) studied the instabilities in strongly coupled ultracold neutral plasmas and reported that in the field of cosmic physics the strongly coupled plasmas (SCP) is of considerable interest because of possible applications to various objects like white dwarf matter, interior of heavy planets, atmosphere of neutron star and ultra cold neutral plasma. Prajapati & Chhajlani (2013) studied the linear self-gravitational instability of finitely conducting magnetized viscoelastic fluid using the GH model and discussed Jeans instability in both SCP and WCP considering the longitudinal and transverse modes of wave propagation under the kinetic and hydrodynamic limits ($\sigma\tau \ll 1$) and drew some general conclusions pertaining to the onset of instability and growth rate of the unstable modes. Recently, Dhiman & Sharma (2014a) studied the problem of gravitational instability of an infinite homogeneous rotating self-gravitating viscoelastic medium permeated with the uniform magnetic field in the longitudinal and transverse modes of wave propagation under both the kinetic and hydrodynamic limits and found that the rotation has no effect on the gravitational instability criterion. They also reported that the rotation reduces the growth rate of Jeans instability. Dhiman & Sharma (2014b) also investigated the effect of non uniform magnetic field on the gravitational instability of strongly coupled plasma and observed that instability criterion gets modified due to the presence of non uniform magnetic field in transverse mode of wave propagation under both the kinetic and hydrodynamic limits, when the viscoelastic medium is infinitely electrically conducting

Cylindrical structures of astronomical objects are of great importance in many ways. Nagasawa (1987) reported that there are some theoretical mechanisms on the formation of cylindrical objects, such as star formation process. Hunter *et al.* (1998) in their analysis studied the stability of the interface between two gaseous astrophysical media by including relative motion of the media, gravitational acceleration perpendicular to the interface, self-gravity of the media (self-gravitational instability) and extended their previous analysis (Hunter *et al.* 1997) by including ordered magnetic fields parallel to the interface. They also investigated the stability of long cylindrical interfaces in the absence of magnetic field and flow and derived some general results concerning the stability of the gaseous medium interface. Sharif & Abbas (2011) observed that in order to generalize the geometry of the star, many people worked on gravitational collapse using cylindrical symmetry. The existence of cylindrical gravitational waves provides a strong motivation in this regard. Hayward (2000) studied gravitational waves, black holes and cosmic strings in cylindrical symmetry. Sharif & Ahmad (2007) analyzed cylindrically symmetric gravitational collapse of two perfect fluids using high speed approximation scheme. Nakao *et al.* (2009) studied gravitational collapse of a hollow cylinder composed of dust.

Dibai (1958) studied the magnetogravitational instability of an infinite gaseous cylinder and showed that an axial magnetic field have no effect on the stability of the system, for longitudinal waves propagating through a cylindrical plasma medium. The effect of non uniform rotation on the onset of gravitational instability has been investigated by Bel & Schatzman (1958) and obtained a modified expression for the *Jeans criterion*, which is now known as the *Bel and Schatzman criterion*. Simon (1962) studied the gravitational instability in gaseous medium where rotation is considered as non uniform and they concluded that the gravitational field is exactly balanced by the centrifugal force. Devanathan (1962) studied the gravitational instability of a rotating mass of infinitely conducting inhomogeneous medium in the presence of non uniform magnetic field and generalized the results of Bel & Schatzman (1958). Radwan & Hasan (2009) considered the self-gravitating instability of a fluid cylinder pervaded by magnetic field and endowed with surface tension and showed that the presence of magnetic field does not affect the onset of instability.

Recently, Dhiman & Dadwal (2010, 2011) extended the analysis of Bel & Schatzman (1958) and Anand & Kushwaha (1962) to study the simultaneous effects of non uniform magnetic field, rotation and heat conduction of an infinitely extending axisymmetric cylinder in the self-gravitating gaseous medium and obtained some general results concerning the onset of gravitational instability. Hasan & Abdelkhalek (2013) studied the magnetogravitodynamic stability of streaming fluid cylinder under the effect of capillary force and concluded that the magnetic and capillary forces are stabilizing, but the streaming is destabilizing while the self-gravitating is stabilizing or destabilizing according to restrictions.

From the above studies, one can easily conclude that there are many astrophysical situations wherein the rotation and magnetic field may be regarded as non uniform or variable. Further, Argal *et al.* (2014) reported that the rotation plays a significant role in structure processes of compact systems such as neutron stars, white dwarf stars and supernovae. In recent years, many authors have studied the gravitational instability of a strongly coupled viscoelastic medium under various non-ideal effects such as rotation, magnetic field and electrical resistivity.

The above discussions and the analysis of Hunter *et al.* (1998), motivated us to consider the astrophysically important cylindrical geometry and to study the effect of non-uniform magnetic field and rotation on the onset of Jeans instability of an axisymmetric cylinder of viscoelastic medium. In the present paper, the self-gravitational instability of an infinitely extending axisymmetric cylinder permeated with nonuniform magnetic field and rotation is studied for both the strongly coupled plasma (SCP) and weakly coupled plasma (WCP) using the GH model. The nonuniform magnetic field and rotation are considered to act along the axial direction of the cylinder. The normal mode method of perturbations is applied to obtain the dispersion relation and is further discussed separately under both the strongly and weakly coupling limits.

2. Mathematical formulation of the problem

Consider an infinitely extending axisymmetric cylinder of a homogeneous, self-gravitating, infinitely electrically conducting viscoelastic medium permeated with non uniform magnetic field and rotation In order to study the gravitational instability of axisymmetric cylinder, the system is described by means of cylindrical coordinates. We shall use the GH model to treat the viscoelastic properties of the medium. Under these assumptions, the generalized basic equations that describe the above physical configuration are equation of continuity, motions, magnetic induction and Poisson equation and are given as (cf. Chapter V and Chapter XIII of Chandrasekhar 1961; Janaki *et al.* 2011; Dhiman & Dadwal 2010)

$$\frac{\partial \rho}{\partial t} + (\vec{u} \cdot \mathrm{grad})\, \rho + \rho\, (\nabla \cdot \vec{u}) = 0, \tag{1}$$

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \left[\left(\frac{\partial u_r}{\partial t} + (\vec{u} \cdot \mathrm{grad})\, u_r \right) - \frac{u_\theta^2}{r} - \frac{1}{4\pi\rho} \left((\vec{H} \cdot \mathrm{grad}) H_r - \frac{H_\theta^2}{r} \right) - \frac{\partial \phi}{\partial r} \right.$$

$$\left. + \frac{c_s^2}{\rho} \frac{\partial \rho}{\partial r} - 2\,(u_\theta \omega_z - u_z \omega_\theta) \right] = \nu \left(\frac{\partial^2 u_r}{\partial \mathrm{r}^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial \mathrm{z}^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r^2} \right)$$

$$+ \frac{\left(\xi + \frac{\mu}{3}\right)}{\rho} \left[\left(\frac{\partial^2 u_r}{\partial \mathrm{r}^2} + \frac{1}{r} \frac{\partial^2 u_\theta}{\partial \mathrm{r} \partial \theta} + \frac{\partial^2 u_z}{\partial \mathrm{r} \partial \mathrm{z}} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{1}{r^2} \frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r^2} \right) \right], \tag{2}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\left(\frac{\partial u_\theta}{\partial t}+(\vec{u}\cdot\text{grad})\,u_\theta\right)-\frac{u_r u_\theta}{r}-\frac{1}{4\pi\rho}\left((\vec{H}\cdot\text{grad})H_\theta-\frac{H_r H_\theta}{r}\right)\right.$$
$$\left.-\frac{1}{r}\frac{\partial\phi}{\partial\theta}+\frac{1}{r}\frac{c_s^2}{\rho}\frac{\partial\rho}{\partial\theta}-2\left(u_z\omega_r-u_r\omega_z\right)\right]=\nu\left(\frac{\partial^2 u_\theta}{\partial \mathrm{r}^2}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2}+\frac{\partial^2 u_\theta}{\partial \mathrm{z}^2}+\frac{1}{r}\frac{\partial u_\theta}{\partial r}\right.$$
$$\left.+\frac{2}{r^2}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta}{r^2}\right)+\frac{\left(\xi+\frac{\mu}{3}\right)}{\rho}\left[\left(\frac{1}{r}\frac{\partial^2 u_r}{\partial\theta\partial \mathrm{r}}+\frac{1}{\mathrm{r}^2}\frac{\partial^2 u_\theta}{\partial\theta^2}+\frac{1}{r}\frac{\partial^2 u_z}{\partial\theta\partial \mathrm{z}}+\frac{1}{r^2}\frac{\partial u_r}{\partial\theta}\right)\right], \quad (3)$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\left(\frac{\partial u_z}{\partial t}+(\vec{u}\cdot\text{grad})\,u_z\right)-\frac{1}{4\pi\rho}(\vec{H}\cdot\text{grad})H_z-\frac{\partial\phi}{\partial z}+\frac{c_s^2}{\rho}\frac{\partial\rho}{\partial z}\right.$$
$$\left.-2(u_r\omega_\theta-u_\theta\omega_z)\right]=\nu\left(\frac{\partial^2 u_z}{\partial \mathrm{r}^2}+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial\theta^2}+\frac{\partial^2 u_z}{\partial \mathrm{z}^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right)$$
$$+\frac{\left(\xi+\frac{\mu}{3}\right)}{\rho}\left[\left(\frac{\partial^2 u_r}{\partial \mathrm{z}\partial \mathrm{r}}+\frac{\partial^2 u_z}{\partial \mathrm{z}^2}+\frac{1}{r}\frac{\partial^2 u_\theta}{\partial\theta\partial \mathrm{z}}+\frac{1}{r}\frac{\partial u_r}{\partial z}\right)\right], \quad (4)$$

$$\frac{\partial H_r}{\partial t}+(\vec{u}\cdot\text{grad})\,H_r-(\vec{H}\cdot\text{grad})u_r=0, \quad (5)$$

$$\frac{\partial H_\theta}{\partial t}+(\vec{u}\cdot\text{grad})\,H_\theta-(\vec{H}\cdot\text{grad})u_\theta+\frac{1}{r}\left(u_\theta H_r-u_r H_\theta\right)=0, \quad (6)$$

$$\frac{\partial H_z}{\partial t}+(\vec{u}\cdot\text{grad})\,H_z-(\vec{H}\cdot\text{grad})u_z=0, \quad (7)$$

$$\nabla^2\phi=-4\pi \mathrm{G}\rho. \quad (8)$$

Further, we have

$$\frac{\partial H_r}{\partial r}+\frac{H_r}{r}+\frac{1}{r}\frac{\partial H_\theta}{\partial\theta}+\frac{\partial H_z}{\partial z}=0 \quad (9)$$

which represents the solenoidal character of the magnetic field and the total stress tensor is given by

$$P=\frac{p}{\rho}-\frac{1}{2}\left|\omega\times r\right|^2+\frac{|H_0|^2}{8\pi\rho} \quad (10)$$

with $p=c_s^2\rho$.

Also, in the above equations, the following terms represent

$$\vec{u}\cdot\text{grad}=u_r\frac{\partial}{\partial r}+\frac{u_\theta}{r}\frac{\partial}{\partial\theta}+u_z\frac{\partial}{\partial z}, \quad (11)$$

$$\vec{H}\cdot\text{grad}=H_r\frac{\partial}{\partial r}+\frac{H_\theta}{r}\frac{\partial}{\partial\theta}+H_z\frac{\partial}{\partial z}, \quad (12)$$

$$\nabla^2\equiv\frac{\partial^2}{\partial \mathrm{r}^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial \mathrm{z}^2} \quad (13)$$

and

$$\nabla\left(\nabla\cdot\vec{u}\right)=\nabla\times\left(\nabla\times\vec{u}\right)+\nabla^2\vec{u}.$$

Further, in the above equations $\vec{u}$, $\vec{H}$ and $\vec{r}$ represent the velocity, magnetic field and position vector respectively and; τ, ρ, p, ϕ denote the viscoelastic relaxation time, density of fluid, the pressure, gravitational potential respectively; ξ, μ, G and c_s are the coefficient of bulk viscosity, coefficient of viscosity, the universal gravitational constant and the speed of sound respectively.

In the present analysis, we have assumed that both the non uniform rotation and magnetic field acts only in the axial direction of the cylinder, i.e. $\vec{H} = (0, 0, H_z)$ and $\vec{\omega} = (0, 0, \omega)$. Here H_z, ω and ϕ are the functions of the radial component r only.

Under these considerations the above basic equations and operators reduce to the following forms:

$$\frac{\partial \rho}{\partial t} + (\vec{u}\cdot\text{grad})\,\rho + \rho\,(\nabla\cdot\vec{u}) = 0, \tag{14}$$

$$\begin{aligned}\left(1+\tau\frac{\partial}{\partial t}\right)&\left[\left(\frac{\partial u_r}{\partial t} + (\vec{u}\cdot\text{grad})\,u_r\right) - \frac{u_\theta^2}{r} - \frac{\partial\phi}{\partial r} + \frac{c_s^2}{\rho}\frac{\partial\rho}{\partial r} - 2u_\theta\omega_z\right]\\ &= \nu\left(\frac{\partial^2 u_r}{\partial \text{r}^2} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial\theta^2} + \frac{\partial^2 u_r}{\partial \text{z}^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta} - \frac{u_r}{r^2}\right)\\ &+ \frac{\left(\xi+\frac{\mu}{3}\right)}{\rho}\left[\left(\frac{\partial^2 u_r}{\partial \text{r}^2} + \frac{1}{r}\frac{\partial^2 u_\theta}{\partial \text{r}\partial\theta} + \frac{\partial^2 u_z}{\partial \text{r}\partial \text{z}} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{1}{r^2}\frac{\partial u_\theta}{\partial\theta} - \frac{u_r}{r^2}\right)\right],\end{aligned} \tag{15}$$

$$\begin{aligned}\left(1+\tau\frac{\partial}{\partial t}\right)&\left[\left(\frac{\partial u_\theta}{\partial t} + (\vec{u}\cdot\text{grad})\,u_\theta\right) - \frac{u_r u_\theta}{r} + \frac{1}{r}\frac{c_s^2}{\rho}\frac{\partial\rho}{\partial\theta} + 2u_r\omega_z\right]\\ &= \nu\left(\frac{\partial^2 u_\theta}{\partial \text{r}^2} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2} + \frac{\partial^2 u_\theta}{\partial \text{z}^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} + \frac{2}{r^2}\frac{\partial u_r}{\partial\theta} - \frac{u_\theta}{r^2}\right)\\ &+ \frac{\left(\xi+\frac{\mu}{3}\right)}{\rho}\left[\left(\frac{1}{r}\frac{\partial^2 u_r}{\partial\theta\partial \text{r}} + \frac{1}{\text{r}^2}\frac{\partial^2 u_\theta}{\partial\theta^2} + \frac{1}{r}\frac{\partial^2 u_z}{\partial\theta\partial \text{z}} + \frac{1}{r^2}\frac{\partial u_r}{\partial\theta}\right)\right],\end{aligned} \tag{16}$$

$$\begin{aligned}\left(1+\tau\frac{\partial}{\partial t}\right)&\left[\left(\frac{\partial u_z}{\partial t} + (\vec{u}\cdot\text{grad})\,u_z\right) - \frac{1}{4\pi\rho}(\vec{H}\cdot\text{grad})H_z + \frac{c_s^2}{\rho}\frac{\partial\rho}{\partial z} - 2u_\theta\omega_z\right]\\ &= \nu\left(\frac{\partial^2 u_z}{\partial \text{r}^2} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial\theta^2} + \frac{\partial^2 u_z}{\partial \text{z}^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right)\\ &+ \frac{\left(\xi+\frac{\mu}{3}\right)}{\rho}\left[\left(\frac{\partial^2 u_r}{\partial \text{z}\partial \text{r}} + \frac{\partial^2 u_z}{\partial \text{z}^2} + \frac{1}{r}\frac{\partial^2 u_\theta}{\partial\theta\partial \text{z}} + \frac{1}{r}\frac{\partial u_r}{\partial z}\right)\right],\end{aligned} \tag{17}$$

$$(\vec{H}\cdot\text{grad})u_r = 0, \tag{18}$$

$$(\vec{H}\cdot\text{grad})u_\theta = 0, \tag{19}$$

$$\frac{\partial H_z}{\partial t} + (\vec{u}\cdot\text{grad})\,H_z - (\vec{H}\cdot\text{grad})u_z = 0, \tag{20}$$

$$\nabla^2\phi = -4\pi G\rho, \tag{21}$$

$$\frac{\partial H_z}{\partial z} = 0, \tag{22}$$

$$P = \frac{p}{\rho} - \frac{1}{2}|\omega \times r|^2 + \frac{|H_0|^2}{8\pi\rho}, \tag{23}$$

$$\vec{u}\cdot\text{grad} = u_r\frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial\theta} + u_z\frac{\partial}{\partial z}, \tag{24}$$

$$\vec{H}\cdot\text{grad} = H_z\frac{\partial}{\partial z}, \tag{25}$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial \mathrm{r}^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial \mathrm{z}^2}. \tag{26}$$

3. Initial stationary state and its solution

Initially, let ω denote the non uniform rotation, where $v = r\omega$ and the non uniform magnetic field is given as $H_z = H_0 H(r)$. Also $\frac{\partial}{\partial r}$ is the only non vanishing component as the propagation of wave is in the radial r direction of the cylinder and due to axial symmetry, we have

$$\frac{\partial}{\partial\theta}(..) = 0.$$

Since initially, there are no motions, we have

$$\left.\begin{array}{c} \vec{u}\left(u_r, u_\theta, u_z\right) = (0, v, 0)\,; \\ \phi = \phi_0;\ \rho = \rho_0; \\ \vec{H} = (0, 0, H_z) \text{ and } \vec{\omega} = (0, 0, \omega) \end{array}\right\}. \tag{27}$$

Using the above initial state in equations (14)–(22), we obtain the following equations representing the initial state solution;

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\left[-r\omega^2 + \frac{c_s^2}{\rho_0}\frac{\partial\rho_0}{\partial r} - \frac{\partial\phi_0}{\partial r} - 2\omega v\right] = 0, \tag{28}$$

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial t}\right) = \nu\left[\frac{\partial^2 v}{\partial \mathrm{r}^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right], \tag{29}$$

$$\nabla^2\phi_0 = -4\pi G\rho_0. \tag{30}$$

4. Perturbation equations

In order to investigate the instability of the above system, let the initial state be slightly perturbed by giving the infinitesimal small perturbations $\delta\rho$, $\delta\phi$, $\vec{h}(h_r, h_\theta, h_z)$, $\vec{u}(u_r, u_\theta, u_z)$ in the density ρ_0, gravitational potential ϕ_0, magnetic field $\vec{H}$ and velocity components $\vec{u}(0, v, 0)$ respectively.

The equilibrium state under perturbations is now represented as

$$\begin{aligned} \rho &= \rho_0 + \delta\rho, \phi = \phi_0 + \delta\phi, H'(h_r, h_\theta, H_z + h_z), \omega'(\omega_r, \omega_\theta, \omega + \omega_z), \\ &\quad u'\,(u_r, u_\theta + v, u_z). \end{aligned} \tag{31}$$

Using the above perturbed quantities from (31), in equations (14)–(22), we get the following equations, respectively:

$$\frac{\partial\left(\rho_0+\delta\rho\right)}{\partial t}+\left(\rho_0+\delta\rho\right)\frac{\partial u_r}{\partial r}+\left(\rho_0+\delta\rho\right)\frac{u_r}{r}=0, \tag{32}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}\right)-r\omega^2+\frac{c_s^2}{\rho_0}\frac{\partial}{\partial r}\left(\rho_0+\delta\rho\right)-\frac{\partial}{\partial r}\left(\phi_0+\delta\phi\right)\right.$$
$$\left.+2\left(-u_\theta\omega-u_\theta\omega_z-v\omega-v\omega_z+u_z\omega_\theta\right)\right]=\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)-\frac{u_r}{r^2}\right), \tag{33}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+u_r\frac{\partial v}{\partial r}\right)-\frac{u_r u_\theta}{r}-\frac{u_r v}{r}\right.$$
$$\left.+2\left(-u_z\omega_r+u_r\omega_z+u_r\omega\right)\right]=\nu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\left(u_\theta+v\right)}{\partial r}\right)-\frac{u_\theta}{r^2}-\frac{v}{r^2}\right), \tag{34}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial}{\partial r}u_z\right)-\frac{1}{4\pi\rho_0}h_r\frac{\partial}{\partial r}\left(H_z+h_z\right)\right]$$
$$=\nu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)-\frac{u_z}{r^2}\right), \tag{35}$$

$$\frac{\partial h_r}{\partial t}+u_r\frac{\partial h_r}{\partial r}-h_r\frac{\partial u_r}{\partial r}=0, \tag{36}$$

$$\frac{\partial h_\theta}{\partial t}+u_r\frac{\partial h_\theta}{\partial r}-h_r\frac{\partial\left(u_\theta+v\right)}{\partial r}+\frac{1}{r}\left(u_\theta h_r+v h_r-u_r h_\theta\right)=0, \tag{37}$$

$$\frac{\partial\left(H_z+h_z\right)}{\partial t}+u_r\frac{\partial\left(H_z+h_z\right)}{\partial r}=0, \tag{38}$$

$$\nabla^2\left[\phi_0+\delta\phi\right]=-4\pi G\left(\rho_0+\delta\rho\right), \tag{39}$$

$$\frac{\partial h_r}{\partial r}+\frac{h_r}{r}=0. \tag{40}$$

Using the initial state and its solution from (27)–(30) in equations (32)–(40) and neglecting the terms containing the second and higher order perturbed quantities from the resulting equations, we get the following linearized perturbed equations:

$$\frac{\partial\delta\rho}{\partial t}+\rho_0\frac{\partial u_r}{\partial r}+\rho_0\frac{u_r}{r}=0, \tag{41}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\frac{\partial u_r}{\partial t}+\frac{c_s^2}{\rho_0}\frac{\partial\delta\rho}{\partial r}-\frac{\partial\delta\phi}{\partial r}-2u_\theta\omega\right]=\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)-\frac{u_r}{r^2}\right), \tag{42}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial\,(r\omega)}{\partial r}\right)+u_r\omega\right]=\nu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)-\frac{u_\theta}{r^2}\right), \tag{43}$$

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[\left(\frac{\partial u_z}{\partial t}-\frac{1}{4\pi\rho_0}h_r\frac{\partial H_z}{\partial r}\right)\right]=\nu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)-\frac{u_z}{r^2}\right), \tag{44}$$

$$\frac{\partial h_r}{\partial t}=0, \tag{45}$$

$$\frac{\partial h_\theta}{\partial t}-h_r\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)=0, \tag{46}$$

$$\frac{\partial h_z}{\partial t}+u_r\frac{\partial H_z}{\partial r}=0, \tag{47}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\delta\phi}{\partial r}\right)=-4\pi G\delta\rho, \tag{48}$$

$$\frac{\partial h_r}{\partial r}+\frac{h_r}{r}=0. \tag{49}$$

In order to investigate the stability of the foregoing stationary state, we shall consider the dependence of the perturbations on r and t of the form

$$\psi^*(r)\exp(\sigma t).$$

Here, σ is the frequency of perturbation. Using the above dependence of the perturbations $(u_r, u_\theta, u_z, \omega_r, \omega_\theta, \omega_z, h_r, h_\theta, h_z, \delta\rho, \delta\phi)$, we have

$$\frac{\partial}{\partial t}\equiv\sigma \quad \text{and} \quad \frac{\partial}{\partial r}f\,(r)=\frac{\mathrm{d}}{\mathrm{d}r}f\,(r)\,.$$

Using this dependence in the linearized perturbed equations (41)–(49), we get the following equations in simplified forms:

$$\sigma\delta\rho+\rho_0\left(\frac{du_r}{dr}+\frac{u_r}{r}\right)=0, \tag{50}$$

$$(1+\tau\sigma)\left[\sigma u_r+\frac{c_s^2}{\rho_0}\frac{\mathrm{d}\delta\rho}{\mathrm{d}r}-\frac{\mathrm{d}\delta\phi}{\mathrm{d}r}-2u_\theta\omega\right]=\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u_r}{\mathrm{d}r}\right)-\frac{u_r}{r^2}\right), \tag{51}$$

$$(1+\tau\sigma)\left[\left(\sigma u_\theta+u_r\frac{\mathrm{d}\,(r\omega)}{\mathrm{d}r}\right)+u_r\omega\right]=\nu\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u_\theta}{\mathrm{d}r}\right)-\frac{u_\theta}{r^2}\right), \tag{52}$$

$$(1+\tau\sigma)\left[\left(\sigma u_z-\frac{1}{4\pi\rho_0}h_r\frac{\mathrm{d}H_z}{\mathrm{d}r}\right)\right]=\nu\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u_z}{\mathrm{d}r}\right)-\frac{u_z}{r^2}\right), \tag{53}$$

$$\sigma h_r=0, \tag{54}$$

$$\sigma h_\theta-h_r\left(\frac{\mathrm{d}\,(r\omega)}{\mathrm{d}r}-\omega\right)=0, \tag{55}$$

$$\sigma h_z + u_r \frac{\mathrm{d}H_z}{\mathrm{d}r} = 0, \tag{56}$$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\delta\phi}{\mathrm{d}r}\right) = -4\pi \mathrm{G}\delta\rho, \tag{57}$$

$$\frac{\mathrm{d}h_r}{\mathrm{d}r} + \frac{h_r}{r} = 0. \tag{58}$$

From equation (54), it is clear that $h_r = 0$. Using this value of h_r in equations (53), (55) and (58) we get

$$h_\theta = 0$$

and

$$(1+\tau\sigma)\,\sigma u_z = \nu\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u_z}{\mathrm{d}r}\right) - \frac{u_z}{r^2}\right). \tag{59}$$

Also, from equations (50) and (57), we get

$$u_r = -\left(\frac{\sigma}{\rho_0}\right)\frac{T}{r}, \tag{60}$$

$$\frac{\mathrm{d}\delta\phi}{\mathrm{d}r} = -\frac{4\pi G}{r}\mathrm{e}^{\sigma t}T, \tag{61}$$

where

$$T = \int_{r0}^{r} x\delta\rho\,(x)\,\mathrm{d}x, \tag{62}$$

$$F = -\left(\frac{\mathrm{d}\,(r\omega)}{\mathrm{d}r} + \omega\right) \tag{63}$$

and

$$\delta\rho = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\int_{r0}^{r} x\delta\rho\,(x)\,\mathrm{d}x = \frac{1}{r}\frac{\mathrm{d}T}{\mathrm{d}r}. \tag{64}$$

Using the values given in equations (61)–(64) in equations (51), (52) and (59), we have

$$(1+\tau\sigma)\,[(\sigma^2+4\pi G\rho_0-\mathrm{c}_\mathrm{s}^2k^2)T + 2u_\theta\omega] = -\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u_r}{\mathrm{d}r}\right) - \frac{u_r}{r^2}\right), \tag{65}$$

$$(1+\tau\sigma)\,[(\sigma u_\theta - u_r F)] = \nu\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u_\theta}{\mathrm{d}r}\right) - \frac{u_\theta}{r^2}\right), \tag{66}$$

$$(1+\tau\sigma)\,\sigma u_z = \nu\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u_z}{\mathrm{d}r}\right) - \frac{u_z}{r^2}\right). \tag{67}$$

Using the value of u_r from (60) in equation (66), we get

$$(1+\tau\sigma)\left[\left(\sigma u_\theta + \left(\frac{\sigma}{\rho_0}\right)\frac{T}{r}F\right)\right] = \nu\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u_\theta}{\mathrm{d}r}\right) - \frac{u_\theta}{r^2}\right). \qquad (68)$$

It is to be noted that, in the above equations, u_z, u_θ and T are the functions of r. Therefore the conditions for global instability cannot be obtained for the system of equations (65), (67) and (68). Hence, following Dhiman & Dadwal (2011), we shall investigate the local stability of the above system in the neighborhood of $r = r_0$. In such a situation, the coefficients of u_z, u_θ and T in equations (65), (67) and (68) are to be evaluated at $r = r_0$.

For this, let us assume that the perturbations have a periodic form in the neighborhood of $r = r_0$. In view of this, we have

$$\frac{\mathrm{d}}{\mathrm{d}r} \equiv -ik.$$

Now, using the above dependence in equations (65), (67) and (68) and then eliminating u_θ from the resulting equations, we obtain the following equations after some simplifications:

$$\Bigg[(1+\tau\sigma)^2\sigma\{\sigma^2 - 4\pi G\rho_0 + c_s^2k^2 - 2F\omega\} + (1+\tau\sigma) \times\left\{\nu\left(k^2+\frac{1}{r^2}\right)(\sigma^2 - 4\pi G\rho_0 + c_s^2k^2) + \sigma^2\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\left(k^2+\frac{1}{r^2}\right)\right\} + \sigma\nu\left(k^2+\frac{1}{r^2}\right)^2\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\Bigg]T = 0, \qquad (69)$$

$$(1+\tau\sigma)\sigma u_z = -\nu\left(k^2+\frac{1}{r^2}\right)u_z. \qquad (70)$$

The above equations are linear and homogeneous and thus for the non-trivial solution of these equations the determinant of coefficients matrix must vanish. Hence, we have

$$\Bigg[(1+\tau\sigma)^2\sigma\{\sigma^2 - 4\pi G\rho_0 + c_s^2k^2 - 2F\omega\} + (1+\tau\sigma) \times\left\{\nu\left(k^2+\frac{1}{r^2}\right)(\sigma^2 - 4\pi G\rho_0 + c_s^2k^2) + \sigma^2\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\left(k^2+\frac{1}{r^2}\right)\right\} + \sigma\nu\left(k^2+\frac{1}{r^2}\right)^2\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\Bigg]\left[(1+\tau\sigma)\sigma + \nu\left(k^2+\frac{1}{r^2}\right)\right] = 0. \qquad (71)$$

Equation (71) is the required dispersion relation which includes the effects of rotation, viscoelastic relaxation time, shear and bulk viscosities. Equation (71) clearly yields the following couple of equations:

$$\left[(1+\tau\sigma)^2 \sigma\{\sigma^2 - 4\pi G\rho_0 + c_s^2 k^2 - 2F\omega\} + (1+\tau\sigma) \right.$$

$$\times \left\{ \nu\left(k^2+\frac{1}{r^2}\right)(\sigma^2 - 4\pi G\rho_0 + c_s^2k^2) + \sigma^2 \frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\left(k^2+\frac{1}{r^2}\right)\right\}$$

$$\left. +\sigma\nu\left(k^2+\frac{1}{r^2}\right)^2 \frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\right] = 0, \tag{72}$$

$$(1+\tau\sigma)\,\sigma + \nu\left(k^2+\frac{1}{r^2}\right) = 0. \tag{73}$$

Now, we shall discuss the above dispersion relations, separately under both strongly and weakly coupling limits.

4.1 *Strongly coupling limit* ($\sigma\tau \gg 1$)

Under the strongly coupling limit, equation (73) reduces to

$$\sigma^2 + \frac{\nu}{\tau}\left(k^2+\frac{1}{r^2}\right) = 0 \tag{74}$$

which represents the damped shear viscous mode modified by the viscoelastic relaxation time. Also, under this limit, equation (72) reduces to the following dispersion relation:

$$\sigma^4 + \sigma^2\left\{c_s^2k^2 - 4\pi G\rho_0 - 2F\omega + \left(\nu_c^2 + \frac{\nu}{\tau}\right)\left(k^2+\frac{1}{r^2}\right)\right\}$$
$$+\frac{\nu}{\tau}\left(k^2+\frac{1}{r^2}\right)\left(c_s^2k^2 - 4\pi G\rho_0 + \nu_c^2\left(k^2+\frac{1}{r^2}\right)\right) = 0, \tag{75}$$

Here, $\nu_c^2 = \frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0\tau}$ is the velocity of compressional viscoelastic mode. The constant term of the above equation yields

$$c_s^2k^2 + \nu_c^2\left(k^2+\frac{1}{r^2}\right) < 4\pi G\rho_0. \tag{76}$$

This represents the modified *Jeans criterion* for the onset of gravitational instability of an axially symmetric cylinder of viscoelastic fluid for SCP, under the strongly coupling limit. The criterion gets modified due to the presence of shear and bulk viscosities i.e. due to the viscoelastic behavior of the medium.

4.2 *Weakly coupling limit* ($\sigma\tau \ll 1$)

Under weakly coupling limit, equation (72) reduces to the following dispersion relation:

$$\sigma^3+\sigma^2\left(k^2+\frac{1}{r^2}\right)\left(\nu+\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\right)$$
$$+\sigma\left\{c_s^2k^2-2F\omega-4\pi G\rho_0+\nu\left(k^2+\frac{1}{r^2}\right)^2\frac{\left(\xi+\frac{4\mu}{3}\right)}{\rho_0}\right\}$$
$$+\nu\left(k^2+\frac{1}{r^2}\right)(c_s^2k^2-4\pi G\rho_0)=0. \qquad (77)$$

From the constant term of the above equation, we have

$$c_s^2k^2<4\pi G\rho_0. \qquad (78)$$

This represents the *Jeans criterion* of gravitational instability for a viscoelastic medium. Further, the inequality (78) yields an identical criterion of gravitational instability as obtained by Chandrasekhar (1961) for gaseous medium.

5. Results and conclusions

In the present paper, we have investigated the Jeans instability of an infinitely extending axisymmetric cylinder of viscoelastic medium permeated with non uniform magnetic field and rotation for both the strongly coupled plasma (SCP) and weakly coupled plasma (WCP) under the strongly and weakly coupling limits. A similar type of analysis has also been carried out by Hunter *et al.* (1998) by investigating the problem of stability of interfaces with self-gravity, relative flow and magnetic field. As a particular case in the absence of magnetic field and flow, they analyzed the interface stability of gaseous medium for a three-dimensional configuration, namely; a long, axisymmetric cylinder of radius r_0 with density ρ_1 on the inside and ρ_2 on the outside and observed that instability occurs for long axial wavelengths, $\lambda_z=3.8d$, where d is the diameter of the cylinder. The main difference between the two studies is that, in the present paper the medium considered is a viscoelastic fluid and analyzes the effect of non-uniform magnetic field on the self-gravitating medium in the presence of rotation, whereas Hunter *et al.* (1998) have studied the gravitational stability of the long axisymmetric cylindrical interface in gaseous medium with constant gravitational acceleration by neglecting the magnetic field.

From inequality (76), we observed that for the case of viscoelastic medium in the presence of non uniform rotation the criterion gets modified due to the presence of shear and bulk viscosities. This means that, the shear and bulk viscosities decrease the critical Jeans wave number and thus have the stabilizing effect on the onset of gravitational instability. Further, it is to be noted that if the effects of shear and bulk viscosity are ignored and medium is considered to be gaseous, then one can deduce a criterion from inequality (76) which is identical to the *Bel and Schatzman criterion* i.e. the effect of non-uniform rotation can be observed in the criterion.

For the WCP under the hydrodynamic limit, inequality (78) yields a criterion governing the onset of gravitational instability of the viscoelastic medium, which is identical to the criterion as obtained by Chandrasekhar (1961) for the gaseous medium. Thus, we can conclude that the shear and bulk viscosities do not affect the onset of gravitational instability in the WCP. This may be due to the reason explained by Prajapati & Chhajlani (2013) that under the hydrodynamic limit, the wave frequency is much lower than the inverse of the viscoelastic relaxation time τ and the medium behaves like a fluid. Further, we can deduce from inequality (78) that when the rotation is uniform, the gravitational instability of the viscoelastic medium is governed by the *Jeans criterion*, as obtained by Chandrasekhar (1961). It is also observed that under the hydrodynamic limit and in the absence of magnetic field, the gravitational instability criterion derived in the present analysis appears to be similar as derived by Hunter *et al.* (1998) in the static limit in which there is no flow.

We also observed that magnetic field, whether uniform or non uniform and when acts along the axial direction of the cylinder, does not affect the Jeans instability criterion under both strongly and weakly coupling limits of a self gravitating viscoelastic medium. This behavior of magnetic field was explained/supported by Chandrasekhar (1961) for a mode in which the density waves are perpendicular to the lines of force and the motion of particles is parallel to the lines of force. Hence mode of wave propagation remains unaffected by the magnetic field.

The above derived results are of qualitative nature and generalize the earlier results obtained by Bel & Schatzman (1958), Anand & Kushwaha (1962), Simon (1962), Hunter *et al.* (1998) and Dhiman & Dadwal (2010, 2011) for the case of self-gravitating viscoelastic medium.

References

Anand, S. P. S, Kushwaha, R. S. 1962, *Proc. Phys. Soc.*, **79**, 1089.

Argal, S., Tiwari, A., Sharma, P. K. 2014, *A Letters Journal Exploring The Frontiers of Physics, EPL*, **108**, 35003.

Bel, N., Schatzman, E. 1958, *Rev. Mod. Phys.*, **30**, 1015.

Chandrasekhar, S. 1961, *Hydrodynamic and hydromagnetic stability*, Oxford University Press, Oxford.

Devanathan, C. 1962, *Annales d'Astrophysique*, **25**, 400.

Dhiman, J. S., Dadwal, R. 2010, *Astrophys. Space Sci.*, **325(2)**, 195.

Dhiman, J. S., Dadwal, R. 2011, *Astrophys. Space Sci.*, **332**, 373.

Dhiman, J. S., Sharma, R. 2014a, *Int. J. Tech.*, **4(1)**, 7.

Dhiman, J. S., Sharma, R. 2014b, *Phys. Scr.*, **89**, 125001.

Dibai, E.A. 1958, *SvA*, **2**, 226.

Hasan, A. A., Abdelkhalek, R.A. 2013, *Boundary Value Problems*, **2013**, 48.

Hayward, S. A. 2000, *Class. Quantum Grav.*, **17**, 1749.

Hunter, J. H. Jr., Whitaker, R.W., Lovelace, R. V. E. 1997, *The Astrophysical Journal*, **482**, 852.

Hunter, J. H. Jr., Whitaker, R.W., Lovelace, R. V. E. 1998, *The Astrophysical Journal*, **508**, 680.

Ikeji, H. 1986, *Phys. Fluids*, **29(6)**, 1764.

Janaki, M. S., Chakrabarti, N. 2010, *Phys. Plasmas*, **17**, 053704.

Janaki, M. S., Chakrabarti, N., Benerjee, D. 2011, *J. Phys. Plasmas*, **18**, 012901.

Jeans, J. H. 1902, *Philos. Trans. R. Soc. Lond.*, **199**, 1.

Kaw, P. K., Sen, A. 1998, *Phys. Plasmas*, **5**, 3552.

Kaw, K. K., Nishikawa, K., Sato, N. 2002, *Phys. Plasmas*, **9(2)**, 387.

Konopka, U., Samsonov, D., Iviev, A. V., Goree, V., Steinberg, V., Morfill, G. E. 2000, *Phys. Rev. E.*, **61**, 1890.

Nagasawa, M. 1987, *Prog. Theor. Phys.*, **77(3)**, 635.
Nakao, K., Harada, T., Kurita, Y., Morisawa, Y. 2009, *Prog.Theor. Phys.*, **122**, 52.
Prajapati, R. P., Chhajlani, R. K. 2013, *Astrophys. Space Sci.*, **344**, 371.
Prajapati, R. P., Sharma, P. K., Sanghvi, R. K., Chhajlani, R. K. 2012, *Physics. J. Conference Series*, **365**.
Radwan, A. E., Hasan, A. A. 2009, *Appl. Math. Model.*, **33(4)**, 2121.
Rosenberg, M., Shukla, P. K. 2011, *Phys. Scr.*, **83**, 015503.
Sato, N., Uchida, G., Kamimura, T., Lizuka, S. 1998, *Physics of Dusty plasmas*, edited by M. Horanyi *et al*. (American Institute of Physics, New York), 239.
Sato, N., Uchida, G., Kamimura, T., Uchida, G., Lizuka, S. 2000, *Frontiers in Dusty Plasmas*, edited by Y. Nakamura *et al.* (Elsevier Science, New York), 329.
Sato, N., Uchida, G., Kaneko, T., Shimizu, S., Lizuka, S. 2001, *Phys. Plasmas*, **8**, 1786.
Sharif, M., Abbas, G. 2011, *J. Phys. Soc. Jpn.*, **80**, 104002.
Sharif, M., Ahmad, Z. 2007, *Gen. Relativ. Gravit.*, **39**, 1331.
Shore, S. N. 1992, *An Introduction to Astrophysical Hydrodynamics*, Academic Press, New York.
Simon, R. 1962, *Annales d'Astrophysique*, **25**, 405.