Review

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Inverse Bremsstrahlung in Astrophysical Plasmas: The Absorption Coefficients and Gaunt Factors

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Abstract. The electron–ion inverse Bremsstrahlung is considered here as a factor of the influence on the opacity of the different stellar atmospheres and other astrophysical plasmas. It is shown that this process can be successfully described in the frames of cut-off Coulomb potential model within the regions of the electron densities and temperatures. The relevant quantum mechanical method of the calculation of the corresponding spectral coefficient processes is described and discussed. The results obtained for the plasmas with the electron densities from 10^{14} cm^{-3} to $2 \cdot 10^{19}$ cm^{-3} and temperatures from $5 \cdot 10^3$ K to $3 \cdot 10^4$ K in the wavelength region 100 nm $< \lambda <$ 3000 nm are presented. Also, these results can be of interest for different laboratory plasmas.

Key words. Atomic and molecular processes—plasmas—spectral lines.

1. Introduction

Since the Bremsstrahlung process is inevitable in the case of plasma spectroscopy, until now the entire literature was devoted to the subject (see e.g. Berger 1956; Karzas & Latter 1961; D'yachkov 1990; Hazak *et al.* 2002; van Hoof *et al.* 2014; Armstrong *et al.* 2014).

It could be seen in the literature that most of the papers are devoted to the determination of the Gaunt factor for the inverse Bremsstrahlung process. The reason behind this approach lies in the exact relation for the direct Bremsstrahlung process differential cross section (Sommerfeld 1953). This automatically led to the possibility of the exact term for the $\sigma_{i.b.}^{(ex)}(E, \varepsilon_{ph})$ cross section, where E is the free electron initial energy, ε_{ph} the absorbed photon energy, for the inverse Bremsstrahlung considered here. In relation with this, although mentioned exact term for $\sigma_{i.b.}^{(ex)}(E, \varepsilon_{ph})$ relates, strictly speaking, to the case of scattering of the free electron onto the Coulomb potential, it is important to mention that it could be applied onto any diluted enough plasma (e.g. plasma with considerably small density). The fact that the practical applicability of this term was rather complex led to difficulties in its application. However, for the same process, a simple and widely used quasi classical, Kramer's cross section $\sigma_{i.b.}^{\text{q.c.}}(E, \varepsilon_{\text{ph}})$ was known and used in practice. The meaningful idea of presenting $\sigma_{i.b.}^{(\text{ex})}(E, \varepsilon_{\text{ph}})$ in the form

$$\sigma_{i.b.}^{(\text{ex})}(E, \varepsilon_{\text{ph}}) = \sigma_{i.b.}^{\text{q.c.}}(E, \varepsilon_{\text{ph}}) \cdot g_{i.b.}(E, \varepsilon_{\text{ph}}), \tag{1}$$

was derived. Here $g_{i.b.}(E, \varepsilon_{\text{ph}})$ is the adequate Gaunt factor. A further step was to yield simple approximations for this quantity.

We mention that, in the general case, both cross sections as well as Gaunt factors are functions not only of E, ε_{ph}, but also of the positively charged center on which the free electrons scatter, e.g. ze, where e is the modulus of the electron charge and $z > 0$. However, only the singly charged plasma should be considered in this manuscript, taking $z = 1$ in the entire space.

Since, in the case of plasma, adequate absorption coefficient is governed by the inverse Bremsstrahlung process, the natural transition towards the averaged values of the plasma parameters occurs. Such an averaged value is an implicit function of the plasma electron and ion concentration, and explicitly depends on plasma temperature T as well as absorbed photon wavelength λ. Here, the exact coefficients are denoted by $k_{i.b.}^{(\text{ex})}(\lambda, T; Ne, Ni)$ and $k_{i.b.}^{\text{q.c.}}(\lambda, T; Ne, Ni)$, where Ne is free electron density, and Ni is the positive ion density.

Accordingly, those coefficients are connected with the relation

$$k_{i.b.}^{(\text{ex})}(\lambda, T; Ne, Ni) = k_{i.b.}^{\text{q.c.}}(\lambda, T; Ne, Ni) \cdot G_{i.b.}(\lambda, T), \tag{2}$$

where $G_{i.b.}(\lambda, T)$ is the sought Gaunt factor. The determination of such averaged Gaunt factor as a function of λ and T was the object of investigation in majority of the previous papers devoted to the inverse Bremsstrahlung process. This is illustrated in Figure 1, where the behavior of the Gaunt factor $G_{i.b.}(\lambda, T)$ is shown on the base of the results obtained in several earlier papers (Berger 1956; Karzas & Latter 1961; D'yachkov 1990; van Hoof *et al.* 2014).

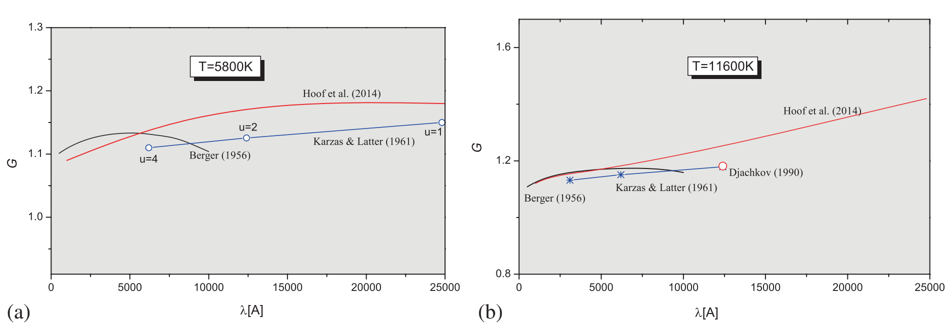

Figure 1. (**a**) Data for Gaunt factor from Berger (1956), Karzas & Latter (1961), D'yachkov (1990) and van Hoof *et al.* (2014) as a function of λ for $T = 5800$ K; (**b**) Same as in Figure 1(a) but for $T = 11600$ K.

It is clear that the application of the approximate relations, strictly applicable only on the case of diluted plasma, would lead to erroneous error in the case of plasma of higher densities, e.g. non-ideal plasma. The yielding of the applicable relations for the higher density plasma is related with the determination of the adequate coefficient $\sigma_{i.b.}^{(\mathrm{ex})}$. It represents a separate problem, and because of this, relatively small amount of published papers were devoted to the determination of the $\sigma_{i.b.}^{(\mathrm{ex})}$ applicable to the higher density plasma (see Hazak *et al.* 2002; Armstrong *et al.* 2014).

Since the objective of this work is exactly the determination of the absorption coefficient $k_{i.b.}^{(\mathrm{ex})}(\lambda, T; Ne, Ni)$ and Gaunt factor $G_{i.b.}(\lambda, T)$ applicable on the case of higher densities plasma, at this point it is necessary to make an observation on the method of their determination in the previous papers in the case of the mentioned dense plasma. Namely, the electron–ion scattering is treated there as a scattering of the electron onto the adequate Yukawa or Debye–Hückel potential. However, in Mihajlov *et al.* (1986), as well as in Mihajlov *et al.* (2011a, b) the fact that Debye-Hückel potential cannot be used for the description of the electron scattering in the case of dense plasma was taken into account. As a reminder, it should be mentioned that the Debye–Hückel potential is defined as a potential of the observed ion and its entire surrounding as a function of the distance from the ion, and as such could be used only for the determination of its average potential energy in the observed plasma. Because of this, in the papers of Mihajlov *et al.* (1986) and Mihajlov *et al.* (2011a, b), a model potential was applied, specially adopted for the description of the electron scattering onto the ion inside the plasma. Here the cut-off Coulomb potential, described by the relations

$$U_{\mathrm{cut}}(r) = \begin{cases} -\dfrac{e^2}{r} + \dfrac{e^2}{r_c}, & 0 < r \le r_c \\ 0, & r_c < r \end{cases}, \tag{3}$$

is pointed out. Here r_c is the cut-off radius, and $-e^2/r_c$ is the average potential energy of the electron in the considered plasma. Let us note that such a potential, which was introduced in the considerations in Suchy (1964) in connection with the transport plasma processes was investigated in detail in Mihajlov *et al.* (1986). Here we will show that in the case of non-ideal plasmas this potential could be successfully applied also for determination of spectral coefficients for the electron–ion inverse Bremsstrahlung processes.

2. Theory

Since the potential equation (3) is one of the finite radius for determination of the cross section $\sigma_{i.b.}^{(\mathrm{ex})}$ for the inverse Bremsstrahlung process, we can use the standard expressions from Sobelman (1979), namely

$$\sigma_{i.b.}^{(\mathrm{ex})}(E; E') = \frac{8\pi^4}{3}\frac{\hbar e^2 k}{q^2}\sum_{l'=l\pm 1} l_{\max}|\hat{D}_{E,l;E'l'}|^2,$$
$$\hat{D}_{E,l;E'l'} = \int_0^\infty P_{E';l'}(r)\cdot r \cdot P_{E;l}(r)\mathrm{d}r, \tag{4}$$

where r is the distance from the beginning of the coordinate system, $P_{E;l}(r)$ is the solution of the radial Schroedinger equation

$$\frac{\mathrm{d}^2 P_{E;l}(r)}{\mathrm{d}r^2} + \left[\frac{2m}{\hbar^2}\left(E - U_{\mathrm{cut}}(r)\right) - \frac{l(l+1)}{r^2}\right] P_{E;l}(r) = 0, \tag{5}$$

and $U_{\mathrm{cut}}(r)$ is the cut-off Coulomb potential given by equation (3).

Let us note that because of the well known properties of the free electron wave functions, the direct calculation of the dipole matrix element $\hat{D}_{E,l;E'l'}$ is practically impossible until now.

However, cut-off Coulomb potential model gives the possibility of direct determination of the cross section for the inverse Bremsstrahlung process without any additional approximations. For that purpose it is enough to use in equation (4) the matrix element of the gradient of the potential energy instead of the dipole matrix element Sobelman (1979) given by

$$|\hat{D}(r)_{ab}|^2 = \frac{\hbar^4}{m^2\left(E_a - E_b\right)^4}|\nabla U_{ab}|^2, \tag{6}$$

$$\nabla_r U_{ab} = \int_0^{r_{\mathrm{cut}}} P_b(r) \cdot \nabla_r U(r) \cdot P_a(r)\mathrm{d}r. \tag{7}$$

Namely, in the case of the cut-off potential (3) in the last expression, integration over r is carried out only in the interval from 0 to r_{cut}.

The given method enabled the fast and reliable calculation of the discussed cross section and, consequentially, the corresponding spectral absorption coefficient.

Here, it is common to use a dimensionless coupling parameter, the plasma non-ideality coefficient Γ, that characterizes the physical properties of the plasma. It is of special importance to describe dense, non-ideal plasmas, as the ones considered in this paper. The parameter $\Gamma = e^2/(akT)$ as such characterizes the potential energy of interaction at average distance between particles $a = (3/4\pi n_e)^{1/3}$ in comparison with the thermal energy. The well-known Brueckner parameter $r_s = a/a_B$ is the ratio of the Wigner–Seitz radii to the Bohr radius.

In connection with this, we considered here plasma with the electron densities from 10^{14} cm^{-3} to $2 \cdot 10^{19}$ cm^{-3} and temperatures from $5 \cdot 10^3$ K to $3 \cdot 10^4$ K, where the corresponding coupling parameter $\Gamma \leq 1.3$. For such plasmas, in accordance with Mihajlov *et al.* (1993) and Adamyan *et al.* (1994), we have that the value of the chemical potential for electron component (treated as appropriate electron gas on the positive charged background) is practically equal to the value which is obtained in the classical case. This means that the distribution function for electrons may be taken as appropriate Maxwell's function. In accordance with this, we will look for $\kappa_{i.b.}^{(\mathrm{ex})}$ in the form

$$\kappa_{i.b.}^{(\mathrm{ex})}(\varepsilon_\lambda; N_e, T) = N_i N_e \cdot \int_0^\infty \sigma_{i.b.}^{(\mathrm{ex})}(E; E')v \cdot f_T(v) \cdot 4\pi v^2 \mathrm{d}v \cdot \left(1 - \exp\left[-\frac{\hbar\omega}{kT}\right]\right), \tag{8}$$

where $f_T(v)$ is the corresponding Maxwell-ova distribution function for a given temperature T, and the expression in parentheses was introduced in order to take into account the effect of stimulated emission. On the other hand, quasi classical Kramer's $k_{i.b.}^{\text{q.c.}}(\lambda, T; Ne, Ni)$ is given by the known expression (see e.g. Sobelman 1979), namely

$$k_{i.b.}^{\text{q.c.}}(\lambda, T; Ne, Ni) = N_i N_e \cdot \frac{16\pi^{5/2}\sqrt{2}e^6}{3\sqrt{3}cm^{3/2}\varepsilon_{\text{ph}}^3} \frac{\hbar^2}{(kT)^{1/2}} \left(1 - \exp\left[-\frac{\hbar\omega}{kT}\right]\right), \quad (9)$$

where $\varepsilon_{\text{ph}} = 2\pi\hbar c/\lambda$. According to this, averaged Gaunt factor $G_{i.b.}(\lambda, T)$ is determined here from equation (2), where $k_{i.b.}^{(\text{ex})}(\lambda, T; Ne, Ni)$ and $k_{i.b.}^{\text{q.c.}}(\lambda, T; Ne, Ni)$ are given by equations (8) and (9).

3. Results and discussion

In this work, the calculations of the Gaunt factor $G_{i.b.}(\lambda, T)$ was carried out for the electron densities in the range from 10^{14} cm^{-3} to $2 \cdot 10^{19}$ cm^{-3} and temperatures from $5 \cdot 10^3$ K to $3 \cdot 10^4$ K, where the coupling parameter is $0.01 \leq \Gamma \leq 1.3$. The observed wavelengths cover the region 100 nm $< \lambda <$ 3000 nm. The important quantity for the process of the light plasma interactions is critical plasma density. This quantity plays an important role because the critical plasma density is the free electron density at which the absorption tends to be the maximum $n_c = 1.113 \cdot 10^{21}(1/\lambda_{\mu m})^2$ cm^{-3}. Absorption occurs at densities less than the critical density (where the plasma frequency $\omega_p = (4\pi n_e e^2/m)^{1/2}$ equals the optical frequency). In connection with the investigation of inverse Bremsstrahlung processes in different stellar atmospheres (for e.g. solar and different dwarf atmospheres) in the range of the investigated wavelengths 100 nm $\leq \lambda \leq$ 3000 nm, the critical electron densities lie in the region of densities between $\sim 10^{20}$ cm^{-3} and 10^{23} cm^{-3}.

Here the obtained results are illustrated in Figures 2, 3 and 4. Figure 2 illustrates the behavior of the Gaunt factor $G_{i.b.}(\lambda, T)$ for the electron concentrations

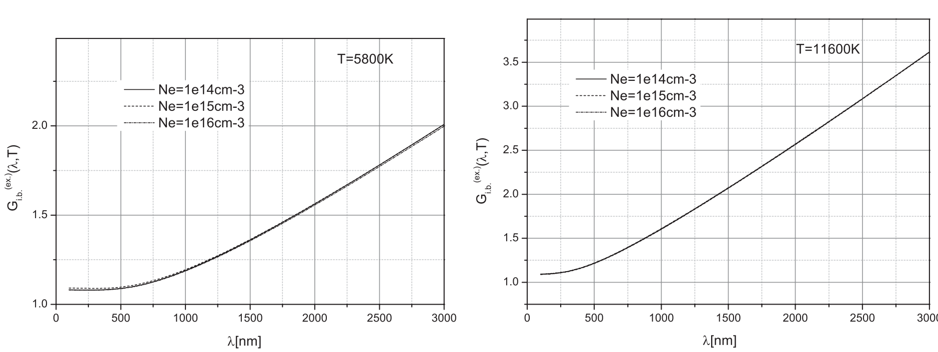

Figure 2. (**a**) Behavior of the Gaunt factor $G_{i.b.}^{(\text{ex.})}(\lambda, T)$ for temperature $T = 5800$ K and electron concentration $Ne = 10^{14}$ cm^{-3}, 10^{15} cm^{-3} and 10^{16} cm^{-3}, (**b**) Same as in Figure 2(a) but for $T = 11600$ K.

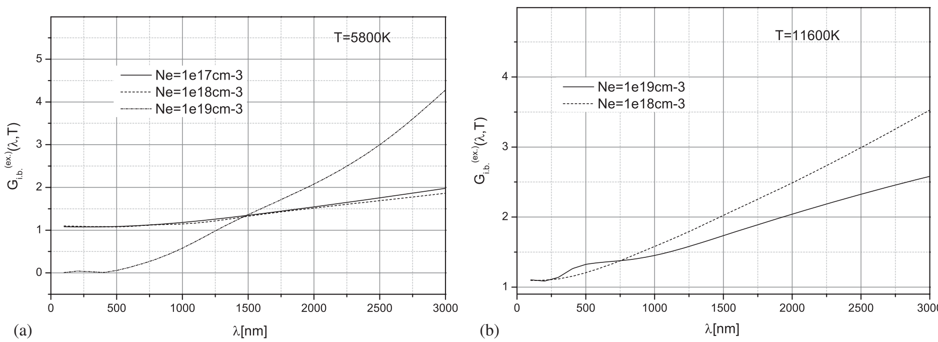

Figure 3. (**a**) Behavior of the Gaunt factor $G_{i.b.}^{(ex.)}(\lambda, T)$ for temperature $T = 5800$ K and $Ne = 10^{17}$ cm^{-3}, 10^{18} cm^{-3} and 10^{19} cm^{-3}. (**b**) Calculated values $G_{i.b.}^{(ex.)}(\lambda, T)$ at $T = 11600$ K for 10^{18} cm^{-3} and 10^{19} cm^{-3}.

10^{14} cm^{-3}, 10^{15}cm^{-3} and 10^{16} cm^{-3} and temperatures 5800 K and 11600 K, and Fig. 3 illustrates the electron concentrations 10^{17} cm^{-3}, 10^{18} cm^{-3} and 10^{19} cm^{-3} in the same temperature range as in Figure 2. Figure 4 illustrates the dynamics of the Gaunt factor change with the increase of the electron concentrations from 10^{18} cm^{-3} up to 10^{19} cm^{-3} in the case of $T = 11600$ K. Let it be noted that each of the figures shows a Gaunt factor behavior determined for the same temperature also, as in the case of an ideal plasma. It enables the estimate of the differences, bearing in mind the electron–ion influence on the inverse Bremsstrahlung in the case of non-ideal plasma. Our attention should be focused on the fact that the dependence of the Gaunt factor $G_{i.b.}^{(ex.)}(\lambda, T)$ on the electron density is governed by the dependence of the screening radius r_{cut} in equation (3) on the same electron density.

The data shown in Figure 5 enables the comparison of the results obtained here with the results of electron–ion inverse Bremsstrahlung processes in the case of

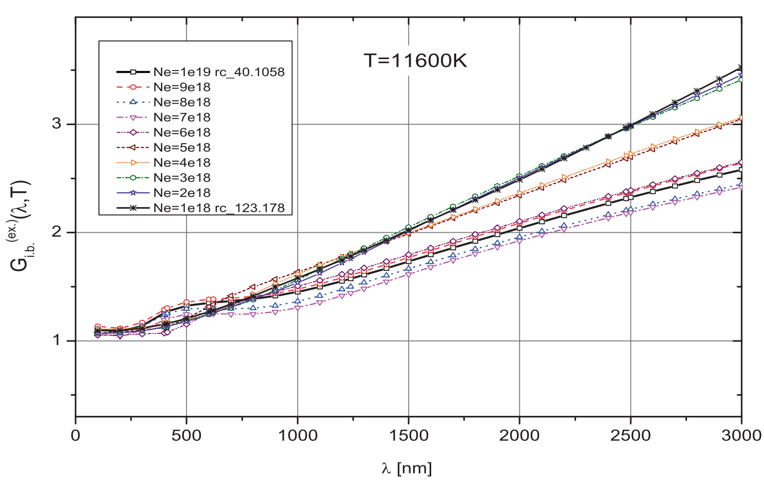

Figure 4. Dynamics of the Gaunt factor change with increase of the electron density from $Ne = 10^{18}$ cm^{-3} to 10^{19} cm^{-3} in the case of temperature $T = 11600$ K.

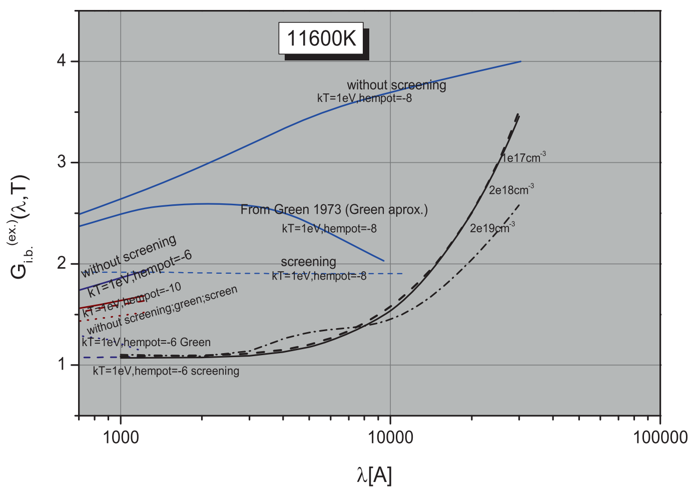

Figure 5. The behavior of the mean Gaunt factor for few cases that differ in the values of the chemical potential of electronic components (μ_{el}), i.e. concentration of free electrons at $T = 11600$ K (Armstrong *et al.* 2014), together with our corresponding data (solid, dashed and dotted black lines).

non-ideal plasma by other authors. This figure shows the behavior of the averaged Gaunt factor $G_{i.b.}(\lambda, T)$ determined in Armstrong *et al.* (2014) for $T = 11600$ K in several cases of different chemical potential of the electron component (μ_{el}), that implies different electron concentrations also. The following cases are considered: $\mu_{\text{el}} = -10, -8, -6$. The same figure also shows the behavior of the Gaunt factor $G_{i.b.}^{(\text{ex.})}(\lambda, T)$ determined for the corresponding region of electron concentration and temperature. The differences of the obtained results and those from Armstrong *et al.* (2014) are evident in the figure and expected. It is clear that these differences are affected by the principal differences in the way of describing the electron–ion scattering in the rest of the plasma.

4. Conclusion

The exact quantum mechanical method presented here could be used to obtain the spectral coefficients for inverse Bremsstrahlung process for the broad class of weakly non-ideal plasmas as well as for plasma of higher non-ideality. It is expected that the cut-off Coulomb potential model results are more accurate in comparison with other methods, which is so far used for cases of non-ideal plasma. Certainly, this method can be of interest in connection with the investigation of inverse Bremsstrahlung processes in different stellar atmospheres. Also, these results can be of interest for different high energy laboratory plasma research.

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