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Radio Recombination Lines of Hydrogen

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Abstract. The impact theory of spectral line broadening is used to obtain complete profiles for radio recombination lines perturbed by electron and proton impact. The collisions can be divided into two types: inelastic, where transitions take place between hydrogen levels with different principal quantum number n and elastic, where the transitions are only between degenerate levels for a particular value of n. The widths of the radio lines are essentially determined by inelastic electron collisions and elastic proton collisions with the emitting hydrogen atom occupying either the upper or lower levels of the line.

Here, earlier work is extended to examine the contribution from proton collisions to the line width in more detail, and it is shown that the trends in the behaviour of the widths again confirm previous results.

Key words. H II regions: ISM—profiles: radio lines—spectral line broadening: calculations.

1. Introduction

Radio lines are observed in regions with number densities $10^3 \leq N \leq 10^4\,\mathrm{cm}^{-3}$ and $T \simeq 10^4$ K. Under these conditions hydrogen is mostly ionized and so free electrons and protons can produce broadening of hydrogen lines emitted in transitions between highly excited states of neutral hydrogen. These are the so-called H II regions. In 1945, van de Hulst was the first astronomer to consider the possibility of observing radiation emitted in transitions between highly excited states of hydrogen. He made some estimates of the Stark broadening and concluded that such lines were unlikely to be detected. Other astronomers were similarly pessimistic.

Kardashev (1959) first predicted that these lines could be observed and gave some estimates for the Doppler and Stark broadening to be expected. His work inspired two Russian groups at the Pulkovo Observatory and the Lebedev Physical Institute to carry out observations and they were the first to obtain definitive results for hydrogen radio lines. Their results were reported at the XII General Assembly of the IAU in Hamburg in 1964 and subsequently other observatories detected lines.

The actual estimates of the line broadening made by Kardashev proved to be inconsistent with observation. However Griem (1967) realised that under these conditions both proton and electron collisions should be treated using the impact theory of line broadening and did obtain consistent results. He found proton impact to be unimportant and essentially the same conclusions were reached by Peach (1972, 2006). Many observations of radio lines emitted from various sources have been published since then.

The book by Gordon & Sorochenko (2009) gives a comprehensive review of observations and theory.

2. Reasons for the re-examination of the line broadening theory

Bell *et al.* (2000) published observations of radio lines emitted at frequencies around 6 GHz and 17.6 GHz by Orion A and W51 and more details have been given by Bell *et al.* (2011) and Bell (2011). The results for 17.6 GHz did not present any surprises but the ones at 6 GHz did. The transitions $n' = n + \Delta n \rightarrow n$ observed were: $(n, \Delta n) =$ (102,1), (129,2), (147,3), (174,5), (184,6), (194,7), (202,8), (210,9), (217,10), (224,11), (230,12), (236,13), (241,14), (247,15), (252,16), (257,17), (261,18), ((266,19), (270,20), (274,21), (278,22), (282,23), (286,24) and (289,25).

Lines above $(n, \Delta n) = (202,8)$ showed unexpected narrowing. Alexander & Gulyaev (2012) criticized the method of line width measurement adopted by Bell *et al.* and questioned their conclusions; this work was in turn refuted by Bell (2012). Alexander & Gulyaev (2015) have now made new observations of lines from the Orion Nebula and do not observe any line narrowing. Hence, recently, the line broadening theory was examined by several authors (Oks 2004; Griem 2005; Watson 2006; Peach 2014). In previous calculations complete profiles for the lines have not been calculated. In the present calculations, complete profiles are obtained and the widths extracted. A new analysis of the data of Bell *et al.* has been published by Hey (2013) who concluded that the electron impact contribution to the line widths does not require much correction and does not indicate any line narrowing. Thus the outstanding theoretical problem remains the relative importance of proton collisions in producing broadening of the radio lines and this is the subject of this paper.

3. The impact approximation

Broadly speaking, the impact assumption is that, on average, collisions are weak or well separated in time. For H II regions where the densities are very low and the temperatures are relatively high, this approximation is valid for collisions with both electrons and protons.

Baranger (1958) developed impact theory for the case of overlapping lines, and for an isolated line. This reduces to the well-known expression for the full-half width (FWHM) of the Lorentzian profile for the transition $i \rightarrow f$ given by

$$W = \left[N v \left(\sum \sigma_{\rm i}({\rm in}) + \sum \sigma_{\rm f}({\rm in}) + \int {\rm d}\Omega |f_{\rm i}(\Omega) - f_{\rm f}(\Omega)|^2 \right) \right]_{\rm av} .$$

For lines in the visible spectrum the third term dominates, but as the wavelength of the line increases, the elastic scattering amplitudes $f_{\rm i}$ and $f_{\rm f}$ coherently cancel more and more efficiently and eventually only the sums over all the inelastic cross sections $\sigma_{\rm i}(\rm in)$ and $\sigma_{\rm f}(\rm in)$ contribute.

In the case of hydrogen we have to consider all the overlapping components in the $(n, \Delta n)$ transition. In the theory that follows the transition will be labelled by initial and final states $n_{\rm i}l_{\rm i}$ and $n_{\rm f}l_{\rm f}$ and we shall assume that the number densities and kinetic temperatures of the electrons and protons are the same and equal to N and T respectively.

The notation of Peach (1981) is adopted here and the formal expression for the line profile is given by

$$\begin{aligned} L(\omega) \;=\; & \frac{1}{\pi}\mathcal{R}\sum\langle\langle n_{\rm i}l_{\rm i}(n_{\rm f}l_{\rm f})^*||\delta||n_{\rm i}l'_{\rm i}(n_{\rm f}l'_{\rm f})^*\rangle\rangle \\ & \times\langle\langle n_{\rm i}l'_{\rm i}(n_{\rm f}l'_{\rm f})^*||[\mathbf{h}-\mathbf{i}(\omega-\omega_0)]^{-1}||n_{\rm i}l_{\rm i}(n_{\rm f}l_{\rm f})^*\rangle\rangle\,, \end{aligned} \tag{1}$$

where $(\omega - \omega_0)$ is the angular frequency separation from the centre of the line. The matrix elements are in reduced line space and $\mathcal{R}$ denotes 'real part of'. This has been used as the basis for the calculation of the complete line profile.

The dipole operator $\boldsymbol{\delta}$ is defined so that

$$\langle\langle n_{\rm i}l_{\rm i}(n_{\rm f}l_{\rm f})^*||\boldsymbol{\delta}||n_{\rm i}l'_{\rm i}(n_{\rm f}l'_{\rm f})^*\rangle\rangle \equiv \langle n_{\rm i}l_{\rm i}||\mathbf{d}||n_{\rm f}l_{\rm f}\rangle\cdot\langle n_{\rm i}l'_{\rm i}||\mathbf{d}||n_{\rm f}l'_{\rm f}\rangle^*, \tag{2}$$

where $\mathbf{d} = -e\mathbf{r}$ and on using the definition of a reduced matrix element given by Edmonds (1974), we have that

$$\begin{aligned} \langle n_{\rm i}l_{\rm i}||\mathbf{r}||n_{\rm f}l_{\rm f}\rangle \;=\; & (-1)^{l_{\rm i}}\,[(2l_{\rm i}+1)(2l_{\rm f}+1)]^{1/2}\begin{pmatrix} l_{\rm i} & 1 & l_{\rm f} \\ 0 & 0 & 0 \end{pmatrix} \\ & \times\int_0^\infty P_{n_{\rm i}l_{\rm i}}(r)\,r\,P_{n_{\rm f}l_{\rm f}}(r)\,{\rm d}r\,, \end{aligned} \tag{3}$$

where $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ is a 3-j coefficient. In (3) the radial wave function for the nl state of hydrogen is $(1/r)P_{nl}(r)$ and the operator $\mathbf{h}$ in (1) contains all the information about the broadening of the line. For either electron or proton impact we have that

$$\begin{aligned} \langle\langle n_{\rm i}l'_{\rm i}(n_{\rm f}l'_{\rm f})^*||\mathbf{h}||n_{\rm i}l_{\rm i}(n_{\rm f}l_{\rm f})^*\rangle\rangle \;=\; & N\int_0^\infty vf(v)\,{\rm d}v\sum(-1)^{l_{\rm i}-m_{\rm i}+l'_{\rm i}-m'_{\rm i}} \\ & \times\begin{pmatrix} l_{\rm i} & 1 & l_{\rm f} \\ -m_{\rm i} & \mu & m_{\rm f} \end{pmatrix}\begin{pmatrix} l'_{\rm i} & 1 & l'_{\rm f} \\ -m'_{\rm i} & \mu & m'_{\rm f} \end{pmatrix}[\langle i'f'|1-S_I S_F^*|if\rangle]_{\rm av}\,, \end{aligned} \tag{4}$$

where the sum is taken over the quantum numbers $m_{\rm i}, m_{\rm f}, m'_{\rm i}, m'_{\rm f}$ and μ and $j \equiv n_j l_j m_j$; $j = i, f$. In equation (4), S_J is the scattering operator that operates only on the levels j, where J is equal to either I or F and the subscript 'av' denotes an average over all orientations of the collision. The associated transition operator T_J is defined by

$$T_J \equiv 1 - S_J\,; \qquad J = I, F \tag{5}$$

and so

$$1 - S_I S_F^* = T_I + T_F - T_I T_F^* \,. \tag{6}$$

Unitarity of the scattering operators means that

$$\sum_k \langle i|T_I|k\rangle = 2\,\mathcal{R}\langle i|T_I|i\rangle\,; \qquad \sum_k \langle f|T_F|k\rangle = 2\,\mathcal{R}\langle f|T_F|f\rangle\,, \tag{7}$$

where the sums over final states k include transitions to the continuum. In equation (4), $f(v)$ is the Maxwell distribution for the relative velocity v defined by

$$f(v) = 4\pi v^2 \left(\frac{\mu}{2\pi kT}\right)^{\frac{3}{2}} \exp\left(-\frac{\mu v^2}{2kT}\right)\,; \quad \int_0^\infty f(v)\,\mathrm{d}v = 1\,, \tag{8}$$

where μ is the reduced mass of the emitter plus perturber.

On using equations (4)–(8) we obtain (see Peach 1981),

$$\langle\langle n_\mathrm{i} l'_\mathrm{i}(n_\mathrm{f} l'_\mathrm{f})^* ||\mathbf{h}|| n_\mathrm{i} l_\mathrm{i}(n_\mathrm{f} l_\mathrm{f})^*\rangle\rangle = N \int_0^\infty v f(v)\,\mathrm{d}v \times \left\{\frac{1}{2}\left[\sum_k \sigma_{\mathrm{i}k}(v) + \sum_k \sigma_{\mathrm{f}k}(v)\right]\delta_{\mathrm{ii}'}\delta_{ff'} - \sigma_{\mathrm{i'f'if}}(v)\right\}, \tag{9}$$

where $\sigma_{jk}(v)$; $j = i, f$ is the cross section for the transition $n_j l_j \to n_k l_k$ and $\sigma_{\mathrm{i'f'if}}(v)$ is a cross section arising from the mixed term $T_I T_F^*$ in (6).

4. Semi-classical impact parameter treatment

The dominant interaction between the emitting hydrogen atom and the electron/proton perturber is given by the dipole term

$$V(\mathbf{r},\mathbf{R}) = \pm e^2 \frac{\mathbf{r}\cdot\mathbf{R}}{R^3}\,; \qquad \mathbf{R} = \rho + \mathbf{v}t\,; \quad \rho\cdot\mathbf{v} = 0\,, \tag{10}$$

where $\mathbf{R}$ gives the position of the perturber relative to the emitter and it is assumed that the perturber follows a straight-line path where ρ is the impact parameter and t is the time. This expression for $V(\mathbf{r},\mathbf{R})$ gives the leading term in the interaction provided that $R > r$. Approximation (10) and second-order time-dependent perturbation theory is then used to calculate the cross sections in (9) for the emitter–perturber collisions. When only the dipole interaction is included the line profile is symmetric and unshifted from the line centre and the cross sections in (9) can be evaluated in terms of known functions.

In the analysis that follows atomic units are used; lengths are in units of a_0, $k = mva_0/\hbar$ and $M = \mu/m$. For an inelastic collision between states with energies E and E', energy conservation gives

$$E + \frac{M}{2}k^2 = E' + \frac{M}{2}k'^2 \tag{11}$$

and we define the dimensionless quantity β by

$$\beta = M\frac{|k'^2 - k^2|}{(k'^2 + k^2)}\kappa\rho\,. \tag{12}$$

In (12), if $E < E'$, $\kappa = k$ and if $E > E'$, $\kappa = k'$, the relative angular momentum $L = M\kappa\rho$ is conserved. This is approximately correct, since the total angular momentum is conserved and the initial and final relative angular momenta only differ by one unit.

We now introduce the functions:

$$\begin{aligned}\zeta(\beta) &= \beta^2\{[K_0(\beta)]^2 + [K_1(\beta)]^2\}\,;\\ \phi(\beta) &= \int^{\beta}\frac{1}{\beta}\zeta(\beta)\mathrm{d}\beta = -\beta K_0(\beta)\,K_1(\beta)\,;\\ \phi'(\beta) &= \frac{1}{2}\zeta(\beta) + \phi(\beta)\,,\end{aligned} \tag{13}$$

where $K_0(\beta)$ and $K_1(\beta)$ are modified Bessel functions, see Abramowitz and Stegun (1972). Then the inelastic cross sections $\sigma(v)$ in (9) for transitions $nl \to n'l'$ can be evaluated to give

$$\sigma(v) = \frac{8\pi a_0^2}{3k^2}\frac{1}{(2l+1)}|\langle n'l'||\mathbf{r}||nl\rangle|^2\,[F(\beta)]_{\beta_a}^{\beta_m}\,, \tag{14}$$

where $F(\beta)$ is equal to either $\phi(\beta)$ for weak collisions or to $\phi'(\beta)$ for strong collisions. The functions $\zeta(\beta)$, $\phi(\beta)$ and $\phi'(\beta)$ tend exponentially to zero as $\beta \to \infty$ and as $\beta \to 0$,

$$\zeta(\beta) \to 1\,; \quad \phi(\beta) \to \ln\beta\,; \quad \phi'(\beta) \to \frac{1}{2} + \ln\beta\,. \tag{15}$$

For the cross section $\sigma_{i'f'if}$, $\beta = 0$ and we have that

$$\begin{aligned}\sigma_{\mathrm{i'f'if}}(v) \;=\; &\frac{8\pi a_0^2}{3k^2}\langle n_{\mathrm{i}}l'_{\mathrm{i}}||\mathbf{r}||n_{\mathrm{i}}l_{\mathrm{i}}\rangle\langle n_{\mathrm{f}}l'_{\mathrm{f}}||\mathbf{r}||n_{\mathrm{f}}l_{\mathrm{f}}\rangle\\ &\times(-1)^{l_{\mathrm{i}}+l'_{\mathrm{f}}}\,[\left\{\begin{matrix} l_{\mathrm{i}}\,l'_{\mathrm{i}}\,1\\ l'_{\mathrm{f}}\,l_{\mathrm{f}}\,1\end{matrix}\right\}\;[F(\beta)]_{\beta_a}^{\beta_m}\,,\end{aligned} \tag{16}$$

where $\left\{\begin{matrix} a & b & c\\ d & e & f\end{matrix}\right\}$ is a 6-j coefficient.

Given that (14) and (16) have been obtained using perturbation theory, in order to obtain realistic results, cutoff parameters must be introduced when evaluating the integrals over the impact parameter ρ. The cutoff parameters specified by Seaton (1962) have been used in the present calculations for both electron and proton impact. The inelastic cross sections given by (14) are calculated in two ways:

(a) *The weak coupling case*. $F(\beta_a) = \phi(\beta_0)$ with $\rho_0 = \min(\bar{r}, \bar{r'})$, where $\bar{r}$ and $\bar{r'}$ are the mean radii of the upper and lower states of the hydrogen atom and $\bar{r}$ is given by

$$\bar{r} = \frac{1}{2}[3n^2 - l(l+1)]\,. \tag{17}$$

(b) *The strong coupling case*. $F(\beta_a) = \phi'(\beta_1)$, where β_1 is calculated from the relation

$$\frac{8}{3}\frac{1}{(2l+1)}\frac{1}{(\kappa\rho_1)^2}|\langle nl||\mathbf{r}||n'l'\rangle|^2\zeta(\beta_1) = 1 \tag{18}$$

and ρ_1 and β_1 satisfy equation (12).

In both cases two possible choices are made for ρ_m; $\rho_m = \rho_{\rm D}$ and $\rho_m = \rho_n$, where $\rho_{\rm D}$ is the Debye radius and ρ_n is the nearest neighour distance. $\rho_{\rm D}$ and ρ_n are defined by

$$\rho_{\rm D} = \left(\frac{kT}{8\pi Ne^2}\right)^{\frac{1}{2}}; \qquad \rho_n = \left(\frac{3}{4\pi N}\right)^{\frac{1}{3}}. \tag{19}$$

This is to test the sensitivity of the final results for the width to the choice made for ρ_m. Finally the inelastic cross sections in (14) are given by

$$F(\beta) = \min[[\phi'(\beta)]_{\beta_1}^{\beta_m}, [\phi(\beta)]_{\beta_0}^{\beta_m}] \tag{20}$$

and for the 'elastic' cross sections (20) simplifies to

$$F(\beta) = \frac{1}{2} + \ln\left(\frac{\rho_m}{\rho_1}\right). \tag{21}$$

For $\sigma(v)$ in (14), ρ_1 is given by

$$\rho_1 = \rho_j = \min(\bar{r_j}, \bar{r_{j'}}); \quad j = i, f \tag{22}$$

and for $\sigma_{\rm i'f'if}$ in (16),

$$\rho_1 = \sqrt{\rho_i\,\rho_f}\,. \tag{23}$$

5. Results and discussion

The present calculations take into account both electron and proton broadening and use both Debye and nearest neighbour cutoffs for ρ_m. The definition of the function $F(\beta)$ in (20) has been slightly modified as compared with earlier work, see Peach (2014), and here $W(e) \leq W(ep)$ everywhere as expected.

Hey (2013) carried out a new analysis of the data of Bell *et al.* (2000, 2011), taking into account the relative intensities of the lines with frequencies near to 6 GHz, out of which five transitions were omitted as being clearly inconsistent with the plasma conditions deduced from the rest. Here results are presented for the remaining eight transitions analysed by Hey which were observed by Bell *et al.* in the spectra of Orion A.

The calculated line profiles are approximately Lorentzian and in Tables 1–4 results are presented for the FWHM impact widths divided by density N as a function of

Table 1. FWHM impact widths, $W_n(e)/N$, in units of Hz cm^3 as a function of $_{10}N$ and for $T = 10^4$ K. The data are listed in the form 1.04231(1) = 1.04231 $\times 10^1$.

	$\log_{10}(N)$			
Line	3.0	3.5	4.0	4.5
(102,1)	1.04231(1)	1.04219(1)	1.04207(1)	1.04195(1)
(129,2)	2.60448(1)	2.60354(1)	2.60259(1)	2.60151(1)
(147,3)	4.32726(1)	4.32446(1)	4.32143(1)	4.31640(1)
(174,5)	8.42632(1)	8.41452(1)	8.39611(1)	8.35118(1)
(184,6)	1.05356(2)	1.05150(2)	1.04783(2)	1.03895(2)
(202,8)	1.53999(2)	1.53445(2)	1.52320(2)	1.49829(2)
(252,16)	3.90676(2)	3.84211(2)	3.72778(2)	3.54506(2)
(274,21)	5.63803(2)	5.48724(2)	5.24822(2)	4.90565(2)

Table 2. FWHM impact widths, $W_{\mathrm{D}}(e)/N$ in units of Hz cm^3 as a function of $\log_{10}N$ and for $T = 10^4$ K. The data are listed in the form 1.04403(1) = 1.04403 $\times 10^1$.

	$\log_{10}(N)$			
Line	3.0	3.5	4.0	4.5
(102,1)	1.04403(1)	1.04386(1)	1.04368(1)	1.04350(1)
(129,2)	2.61797(1)	2.61657(1)	2.61516(1)	2.61376(1)
(147,3)	4.36721(1)	4.36305(1)	4.35889(1)	4.35472(1)
(174,5)	8.58088(1)	8.56479(1)	8.54869(1)	8.53259(1)
(184,6)	1.07837(2)	1.07579(2)	1.07321(2)	1.07063(2)
(202,8)	1.59311(2)	1.58761(2)	1.58211(2)	1.57660(2)
(252,16)	4.25207(2)	4.21801(2)	4.18394(2)	4.14985(2)
(274,21)	6.36819(2)	6.29859(2)	6.22895(2)	6.15927(2)

Table 3. FWHM impact widths, $W_n(ep)/N$ in units of Hz cm^3 as a function of $\log_{10}N$ and for $T = 10^4$ K. The data are listed in the form 1.11844(1) = 1.11844 $\times 10^1$.

	$\log_{10}(N)$			
Line	3.0	3.5	4.0	4.5
(102,1)	1.11844(1)	1.11531(1)	1.11216(1)	1.10898(1)
(129,2)	3.06798(1)	3.04091(1)	3.01372(1)	2.98628(1)
(147,3)	5.53905(1)	5.45758(1)	5.37560(1)	5.29128(1)
(174,5)	1.24866(2)	1.21670(2)	1.18396(2)	1.14843(2)
(184,6)	1.67346(2)	1.62185(2)	1.56847(2)	1.50961(2)
(202,8)	2.76506(2)	2.65332(2)	2.53550(2)	2.40350(2)
(252,16)	1.02751(3)	9.54172(2)	8.75684(2)	7.90150(2)
(274,21)	1.77954(3)	1.62689(3)	1.46514(3)	1.29274(3)

$\log_{10}N$. Both the nearest neighbour and Debye cutoff parameters ρ_n and ρ_{D} are used and for the cases considered here, ρ_{D} is two orders of magnitude greater than ρ_n and $\rho_n >> \bar{r}$. The choice of $\rho_m = \rho_n$ would seem to be a more appropriate choice of cutoff, but of course it is still to some extent arbitrary. The temperature chosen is $T = 10^4$ K and Tables 1 and 2 are for electron impact only, whereas in Tables 3 and 4 both electron and proton impacts are included. For each line, W/N decreases gradually with increasing density. Table 5 gives results for $T = 2 \times 10^4$ K and comparison with the results in Table 3 shows that the widths also gradually decrease with increasing temperature.

Table 4. FWHM impact widths, $W_{\rm D}(ep)/N$ in units of Hz cm^3 as a function of $\log_{10}N$ and for $T = 10^4$ K. The data are listed in the form 1.16119(1) = 1.16119 $\times 10^1$.

	$\log_{10}(N)$			
Line	3.0	3.5	4.0	4.5
(102,1)	1.16119(1)	1.15689(1)	1.15256(1)	1.14819(1)
(129,2)	3.44745(1)	3.40855(1)	3.36950(1)	3.33031(1)
(147,3)	6.68634(1)	6.56823(1)	6.44981(1)	6.33107(1)
(174,5)	1.69957(2)	1.65296(2)	1.60627(2)	1.55950(2)
(184,6)	2.40038(2)	2.32513(2)	2.24978(2)	2.17432(2)
(202,8)	4.32680(2)	4.16482(2)	4.00269(2)	3.84039(2)
(252,16)	2.01720(3)	1.91434(3)	1.81145(3)	1.70853(3)
(274,21)	3.82068(3)	3.60862(3)	3.39652(3)	3.18438(3)

Table 5. FWHM impact widths, $W_n(ep)/N$ in units of Hz cm^3 as a function of $\log_{10}N$ and for $T = 2 \times 10^4$ K. The data are listed in the form 1.02554(1) = 1.02554 $\times 10^1$.

	$\log_{10}(N)$			
Line	3.0	3.5	4.0	4.5
(102,1)	1.02554(1)	1.02312(1)	1.02068(1)	1.01822(1)
(129,2)	2.72886(1)	2.70920(1)	2.68939(1)	2.66879(1)
(147,3)	4.84426(1)	4.78568(1)	4.72572(1)	4.65950(1)
(174,5)	1.06767(2)	1.04451(2)	1.01960(2)	9.90028(1)
(184,6)	1.41723(2)	1.37957(2)	1.33841(2)	1.28941(2)
(202,8)	2.30599(2)	2.22319(2)	2.13104(2)	2.02249(2)
(252,16)	8.21905(2)	7.66824(2)	7.06869(2)	6.41429(2)
(274,21)	1.39956(3)	1.28589(3)	1.16474(3)	1.03621(3)

The following results are clear:

(a) For electron impact only, the results are totally insensitive to the choice of ρ_m for smaller values of Δn, but $W_{\rm D}(e)$ and $W_n(e)$ begin to differ for higher values of Δn.
(b) For electron and proton impact the trend is the same as in (a), but significant differences are already observed for $\Delta n \geq 2$.
(c) For the transition $(n, \Delta n) = (102,1)$, the choice of cutoff makes no difference and the contribution from proton impact is unimportant. This agrees with the conclusion of Griem (1967) who only considered $\Delta n = 1$ transitions.

Additional tests have been carried out to check whether any appreciable effects arise when correlation between the collisional and Doppler processes are included, but none were found. The possibility of a contribution to the impact widths from quenching collisions (see Peach 1981), was also found to be negligible.

6. Conclusions

(a) For a fixed value of Δn, only the contributions from inelastic collisions contribute significantly as n increases. This is because there is increasing cancellation between the upper and lower levels, $n_{\rm i}$ and $n_{\rm f}$, of the contributions from elastic collisions.

(b) Proton impact is not effective for inelastic collisions until very large values of n are reached, but dominates for elastic collisions.
(c) For sufficiently large values of n condition (a) holds for both electron and proton impact.
(d) For lines within a narrow frequency band, the cancellation between contributions to the width from elastic collisions becomes less severe as Δn increases and then proton impact starts to make a significant contribution.

The trends in the behaviour of the lines studied here are very similar to those obtained previously; the widths increase monotonically with increasing values of n.

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