Review

J. Astrophys. Astr., Vol. 36, No. 4, December 2015, pp. 539–553

Possible Alternatives to the Supermassive Black Hole at the Galactic Center

A. F. Zakharov[1,2,3,4,5,*]

[1] *National Astronomical Observatories of Chinese Academy of Sciences, Datun Road 20A, Beijing, 100012 China.*
[2] *Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya 25, 117218 Moscow, Russia.*
[3] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia.*
[4] *National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), 115409, Moscow, Russia.*
[5] *North Carolina Central University, Durham, NC 27707, USA.*
[*] *e-mail: alex_f_zakharov5@mail.ru*

Received 19 June 2015; accepted 12 August 2015
DOI: 10.1007/s12036-015-9345-x

Abstract. Now there are two basic observational techniques to investigate a gravitational potential at the Galactic Center, namely, (a) monitoring the orbits of bright stars near the Galactic Center to reconstruct a gravitational potential; (b) measuring the size and shape of shadows around black hole giving an alternative possibility to evaluate black hole parameters in mm-band with VLBI-technique. At the moment, one can use a small relativistic correction approach for stellar orbit analysis (however, in the future the approximation will not be precise enough due to enormous progress of observational facilities) while for smallest structure analysis in VLBI observations one really needs a strong gravitational field approximation. We discuss results of observations, their conventional interpretations, tensions between observations and models and possible hints for a new physics from the observational data and tensions between observations and interpretations. We discuss an opportunity to use a Schwarzschild metric for data interpretation or we have to use more exotic models such as Reissner–Nordstrom or Schwarzschild–de-Sitter metrics for better fits.

Key words. Black holes—supermassive black holes—gravitational lensing—the Galactic Center—large telescopes—VLBI interferometry.

1. Introduction

Soon after the discovery of general relativity (GR) (Einstein 1915; Hilbert 1916), a vacuum solution of GR equations has been found (Schwarzschild 1916), however, Albert Einstein was rather skeptical concerning physical applications of the solution, for instance, at the end of the paper he wrote: "...The essential result of this investigation is a clear understanding as to why the 'Schwarzschild singularities'[1] do not exist in physical reality..." (Einstein 1939), see also similar opinions in a textbook (Bergmann 1942) in spite of the fact that results on maximal masses for white dwarfs (Stoner 1930; Chandrasekhar 1931; Landau 1932; Chandrasekhar 1934) and for neutron stars (Oppenheimer & Volkoff 1939) have been known in those times. Moreover, Oppenheimer & Snyder (1939) showed an opportunity of black hole formation[2].

There were only three famous tests of GR at the beginning, namely, deflection of light, the mercury anomaly and gravitational redshifts (Bergmann 1942), but there were a number of phenomena where predictions of GR have been checked or they will be checked in the future (Will 2014).

A rapid development of black hole physics started after Wheeler's lecture in 1967 and his corresponding article (Wheeler 1968), where the term 'black hole' has been introduced. In spite of the fact that black hole solutions of Einstein equations are known for almost a century, there have not been too many astrophysical examples where one really need a strong gravitational field approximation but not small relativistic corrections to a Newtonian gravitational field. One of the most important option to test a gravity in the strong field approximation is analysis of relativistic line shape as it was shown (Fabian *et al.* 1989; Stella 1990; Laor 1991; Matt *et al.* 1993). Such signatures of the Fe K_α-line have been found in the active galaxy MCG-6-30-15 (Tanaka *et al.* 1995). Analyzing the spectral line shape, the authors concluded that the emission region is so close to the black hole horizon that one has to use Kerr metric approximation to fit observational data (Tanaka *et al.* 1995). However, see also alternative scenarios and discussions (Karas *et al.* 2000; Turner *et al.* 2002; Dovčiak *et al.* 2004; Murphy *et al.* 2009).

Results of our simulations of iron K_α line formation are given in Zakharov & Repin (1999, 2002, 2003a, b, c, 2004, 2005, 2006), Zakharov *et al*. (2003, 2004), Zakharov (2007a) and Zakharov (2004, 2005, 2007a, b), where we used our approach (Zakharov 1994, 1995), see also Fabian & Ross (2010) and Jovanović (2012) for more recent reviews on the subject.

The natural way to evaluate a potential gives an analysis of test particle trajectories similar to the experiment when E. Rutherford got constraints on an atomic potential and showed a presence of nuclei in atoms analyzing paths of α-particles. In the case of massive black holes, tracers may be stars, (hot) spots, gas clouds or light trajectories (gravitational lensing). Below we discuss the issues in more detail.

[1]According to the widespread terminology at those times the event horizon was called as Schwarzschild singularity. In fact, the Schwarzschild solution has no singularities but singularities appeared in the variables chosen by Hilbert (1917), see also Eisenstaedt (1982).

[2]In 1939, Einstein, Oppenheimer, Volkoff and Snyder worked at the Institute for Advanced Studies in Princeton, see also other curious issues of an earlier GR history in a Preface written by Ashtekar (2005).

Supermassive black holes have been found in the center of 85 galaxies (Kormendy & Ho 2013), however, usually astronomers do not use GR approaches for such claims about black hole existence. Around 40 quasars with redshifts $z > 6$ have been found and each quasar has a supermassive black hole with a mass around one billion solar masses $10^9 M_\odot$ and recently (Wu *et al.* 2015) found the ultraluminous quasar SDSS, J010013.02 + 1280225.8, at redshift $z \sim 6.30$, and the quasar has a black hole with a mass of around $1.2 \times 10^{10} M_\odot$. Remarkably, the initial optical spectroscopy of the quasar was carried out with the Chinese Lijiang 2.4-m telescope (it means that discoveries may be done with relatively modest facilities). Later, spectroscopic observations were conducted for the object with the 6.5-m Multiple Mirror Telescope (MMT) and the twin 8.4-m mirror Large Binocular Telescope (LBT) in the USA and initial estimates of black hole mass and redshift have been confirmed.

2. Observations of the Galactic Center

To evaluate a gravitational potential at the Galactic Center two teams of astronomers observed trajectories of bright stars in the IR band for several years. One group led by A. Ghez (UCLA, USA) used the twin 10-meter optical/infrared telescopes on Mauna Kea (Hawaii), and according to the Keck Strategic Mission, the first important goal is *high angular resolution astrophysics*[3] and practically it gives an opportunity to be a world leader in the field, see results of the observations of bright stars (Ghez *et al.* 2000, 2003, 2004, 2005; Weinberg *et al.* 2005; Meyer *et al.* 2012; Morris *et al.* 2012). Another group led by Genzel (ESO, MPE) used four 8.2-meter VLT telescopes at Paranal (Chile). The European group got very important results (Schödel *et al.* 2002; Genzel *et al.* 2003; Eckart *et al.* 2004, 2005a, b, 2006a, b, 2007; Gillessen *et al.* 2009, 2012) which are consistent results presented by the US team. The important case of G2 gas cloud is a very useful tracer of the gravitational potential at the Galactic Center (Gillessen *et al.* 2012) (later, the object has been called Dusty S-cluster Object (DSO/G2) since further detailed observations were not completely consistent with the gas cloud model). It occured that very likely this may be another example of a close peribothron[4] passage of a dust-enshrouded star (Phifer *et al.* 2013; Zajaček *et al.* 2014, 2015). The analysis showed that the DSO/G2 is rather a young star than a coreless gas and dust cloud (Valencia-M *et al*. 2015).

According to the list of the Top 10 ESO science discoveries, observations of stars orbiting the Milky Way black hole led to discovery # 1: ‘Several of ESO’s flagship telescopes were used in a 16-year long study to obtain the most detailed view ever of the surroundings of the monster lurking at the heart of our galaxy – a supermassive black hole’[5].

In 2012, the Royal Swedish Academy of Sciences decided to award the Crafoord Prize in Astronomy to Reinhard Genzel[6] and Andrea Ghez, ‘for their observations

[3] http://www.keckobservatory.org/

[4] The word ‘peribothron’ has been introduced by Frank & Rees (1976) following W. R. Stoeger’s suggestion. Greek bothros means pit or hole. So, the peribothron means the point of least distance of an object orbiting a black hole.

[5] http://www.eso.org/public/science/top10/

[6] In 2008, R. Genzel was awarded the Shaw prize for recognition of his outstanding contributions in demonstrating that the Milky Way contains a supermassive black hole at its Center.

of the stars orbiting the Galactic Centre, indicating the presence of a supermassive black hole'[7].

The ESO and MPE formed a team to construct the GRAVITY, the Very Large Telescope Interferometer for precise astrometry and interferometric imaging. The interferometer will provide precise astrometry of order 10 micro-arcseconds (Eisenhauer *et al.* 2011; Blind *et al.* 2015). The GRAVITY equipment will be shipped to the VLT observatory in Chile in 2015 and commissioning is planned to start in October 2015 (Blind *et al.* 2015).

3. Shadows for the black hole at the Galactic Center

Several years ago, Falcke *et al.* (2000a, b) and Melia & Falcke (2001) simulated formation of images for supermassive black holes. They used a toy model for their analysis and concluded that a strong gravitational field is bent trajectories of photons emitted by accreting particles and an observer can see a dark spot (shadow) around a black hole position. For the black hole at the Galactic Center, a size of shadow is around 50 μas. Based on the results of simulations, Falcke *et al.* (2000a, b) and Melia & Falcke (2001) concluded that the shadow may be detectable at mm and sub-mm wavelengths, however, scattering may be very significant at cm wavelength, so there are very small chances to observe the shadows at the cm band. Importantly, the results of Falcke *et al.* (2000a, b) and Melia & Falcke (2001) are rather general in spite of their specific model.

There is a tremendous progress to evaluate a minimal size of spot for the Sgr A^* (Shen *et al.* 2005; Doeleman *et al.* 2008), for instance, Doeleman *et al.* (2008) evaluated a shadow size as small as 37^{+16}_{-10} μas. Practically, a minimal size of bright spot was evaluated, but a boundary of dark spot (shadow) has to be bright, so, a size of bright boundary has been measured.

Holz & Wheeler (2002) considered retro-lensing, namely, they assumed the presence of a black hole near the Solar system, therefore, an observer could detect photons a direction toward to a black hole location due to a very strong deflection of light near the black hole.

Based on ideas introduced by Chandrasekhar (1983) and Holz & Wheeler (2002), Zakharov *et al.* (2005a) considered different types of shadow shapes for Kerr black holes and different position angles of a distant observer. Moreover, it was shown that for an equatorial plane position of a distant observer, maximal impact parameter $|\beta_{\max}|$ in the z-direction (which concides with a black hole rotation direction) is $\sqrt{27}$, while the corresponding impact parameter in the perpendicular direction for the $\beta_{\max}$ is $\alpha_{\max} = 2a$ (Zakharov *et al.* 2005a), if we consider the function $\beta(\alpha)$ for critical impact parameters separating a capture and scattering of photons.

Polarization measurements are very important to design an adequate model for the Galactic Center. The role of strong-gravity retro-lensing in polarized signal has been discussed by Horák & Karas (2006), and the specific observational signatures of polarisation in Galactic Center flares have been examined by Zamaninasab *et al.* (2010) and Shahzamanian *et al.* (2015). Promising future

[7] http://www.crafoordprize.se

prospects of X-ray polarimetry of molecular clouds surrounding Galactic Center have been recently discussed by Marin *et al.* (2014) in the context of past accretion activity of the supermassive black hole.

3.1 *Constraints on black hole parameters*

Theories with extra dimensions admit astrophysical objects (supermassive black holes, in particular) which are rather different from standard ones. There were proposed tests which may help to discover signatures of extra dimensions in supermassive black holes since the gravitational field may be different from the standard one in the GR approach. So, gravitational lensing features are different for alternative gravity theories with extra dimensions and general relativity. Sometime ago, Bin-Nun (2010a, b, c) discussed that the black hole at the Galactic Center is described by the tidal Reissner–Nordström metric (Dadhich *et al.* 2001) which may be admitted by the Randall–Sundrum II braneworld scenario. Bin-Nun suggested an opportunity of evaluating the black hole metric analyzing (retro-)lensing of bright stars around the black hole in the Galactic Center. Doeleman *et al.* (2008) evaluated a minimal size of a spot for the black hole at the Galactic Center. According to theoretical consideration and simulations, a minimal size of spot practically has to coincide with the shadow size (Falcke *et al.* 2000a, b; Melia & Falcke 2001).

Measurements of the shadow size around the black hole may help to evaluate parameters of black hole metric (Zakharov *et al.* 2005a, b). Another opportunity to evaluate parameters of the black hole is an analysis of trajectories of bright stars near the Galactic Center (Zakharov *et al.* 2007). We derive an analytic expression for the black hole shadow size as a function of charge for the tidal Reissner–Nordström metric. We conclude that observational data concerning shadow size measurements are not consistent with significant negative charges, in particular, the significant negative charge $Q/(4M^2) = -1.6$ (discussed by Bin-Nun 2010a, b, c) is practically ruled out with a probability (the charge is, roughly speaking, beyond 9σ confidence level, but a negative charge is beyond 3σ confidence level).

We could evaluate a shadow size for the black hole at the Galactic Center assuming that the black hole mass is about $4 \times 10^6 M_\odot$ and a distance toward the Galactic Center is about 8 kpc. In this case a diameter of shadow is about 52 μas for the Schwarzschild metric and about 40 μas for the extreme Reissner–Nordström metric.

Doeleman *et al.* (2008) evaluated a size of the smallest spot near the black hole at the Galactic Center such as 37^{+16}_{-10} microarcseconds at a wavelength of 1.3 mm with 3σ confidence level. Theoretical analysis and observations show that the size of the shadow can not be smaller than a minimal spot size at the wavelength (Falcke *et al.* 2000a, b; Melia & Falcke 2001; Zakharov *et al.* 2005a, b). Roughly speaking, it means that a small positive q is consistent with observations but a significant negative q is not. For $q = -6.4$ (as was suggested by Bin-Nun 2010a, b, c), we have a shadow size 84.38 μas. It means that the shadow size is beyond the shadow size with a probability corresponding to a deviation of about 9σ from an expected shadow size. Therefore, a probability to have significant tidal charge for the black hole at the Galactic Center is negligible. So, we could claim that the tidal charge is ruled out with observations and corresponding theoretical analysis (Zakharov *et al.* 2012; Zakharov 2013a, b, 2014a, c, d).

In Figure 1, shadow size is given as a function of charge (including possible tidal charge with a negative q and super-extreme charge $q > 1$). In Figure 2, radius of last unstable orbit for photons as a function of q is given.

3.2 *Shadows for a Kottler (Schwarzschild–de-Sitter) black hole*

The expression for the Kottler (Schwarzschild–de-Sitter) metric in natural units ($G = c = 1$) has the form (Kottler 1918; Stuchlik 1983)

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2\right) dt^2 + \left(1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 \\ &\quad + r^2(d\theta^2 + \sin^2\theta d\phi^2). \end{aligned} \tag{1}$$

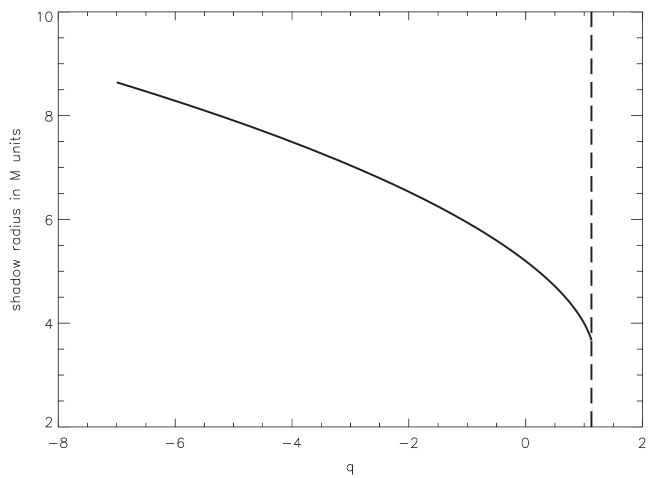

Figure 1. Shadow (mirage) sizes M units as a function of q.

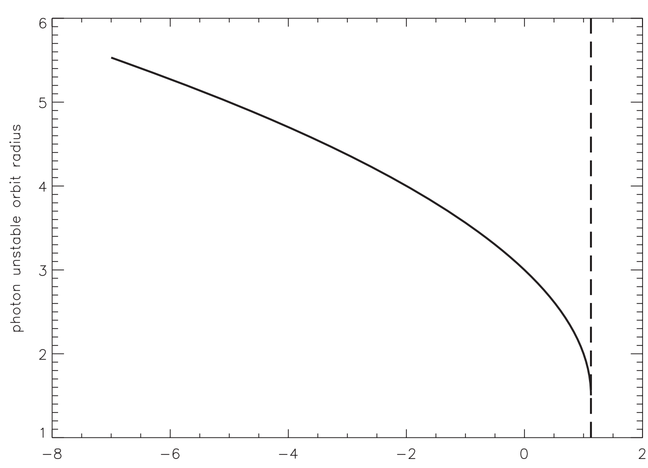

Figure 2. Radius of the last circular unstable photon orbits in M units as a function of q.

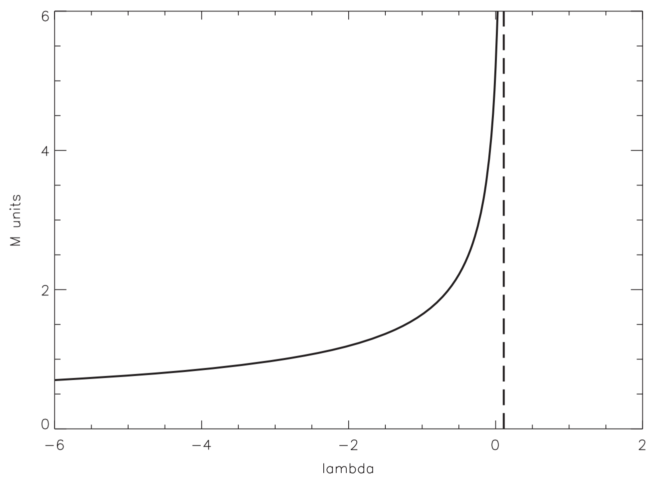

Figure 3. Shadow (mirage) radius (solid line) in M units as a function of dimensionless $\lambda = \Lambda M^2$. The critical value $\Lambda = 1/(9M^2)$ is shown with the dashed vertical line.

where we use the conventional nomination for the Λ-term.

Then we have for shadow size (Zakharov 2014b),

$$\xi_{\rm cr}^2 = \frac{27}{1 - 9\Lambda M^2}. \qquad (2)$$

As one can see from equation (2), shadows disappear for $\Lambda > 1/(9M^2)$ and there exist for $\Lambda < 1/(9M^2)$ and for positive Λ its presence decreases shadow dimension while for negative Λ, we have an opposite tendency (see Figure 3).

4. Constraints on black hole parameters and gravity theories from trajectories of bright stars at the Galactic Center

4.1 *Constraints on black hole parameters and extended mass distribution*

As it was mentioned earlier an enormous progress in monitoring bright stars near the Galactic Center has been reached (Ghez *et al.* 2000, 2003, 2004, 2005; Weinberg *et al.* 2005; Meyer *et al.* 2012; Morris *et al.* 2012). The astrometric limit for bright stellar sources near the Galactic Center with 10-m telescopes is today $\delta\theta_{10} \sim 1$ mas and the Next Generation Large Telescope (NGLT) will be able to improve this number at least down to $\delta\theta_{30} \sim 0.5$ mas or even to $\delta\theta_{30} \sim 0.1$ mas (Weinberg *et al.* 2005) in the K-band (see also perspectives for observations with GRAVITY facilities (Eisenhauer *et al.* 2011) and TMT[8] or E-ELT[9]. Therefore, it will be possible to measure the proper motion for about ~100 stars with astrometric errors several times smaller than errors in current observations.

[8] http://www.tmt.org/sites/default/files/documents/application/pdf/srd%20ccr20_final.pdf
[9] http://www.eso.org/sci/facilities/eelt/science/doc/eelt_sciencecase.pdf

GR predicts that orbits about a massive central body suffer peribothron shifts yielding rosette shapes. However, the classical perturbing effects of other objects on inner orbits give an opposite shift (Nucita *et al.* 2007; Zakharov *et al.* 2007) (the effect is rather general and it weakly depends on a choice of extended mass distribution between peribothron and apobothron). Since the peribothron advance depends strongly on the compactness of the central body, the detection of such an effect may give information about the nature of the central body itself. This would apply for stars orbiting close to the GC, where there is a 'dark object', the black hole hypothesis being the most natural explanation of the observational data. A cluster of stars in the vicinity of the GC (at a distance <1 arcsec) has been monitored by ESO and Keck teams for several years.

For a test particle orbiting a Schwarzschild black hole of mass $M_{\rm BH}$, the peribothron shift is given by (see e.g. Weinberg 1972)

$$\Delta\phi_S \simeq \frac{6\pi G M_{\rm BH}}{d(1-e^2)c^2} + \frac{3(18+e^2)\pi G^2 M_{\rm BH}^2}{2d^2(1-e^2)^2c^4}\ , \tag{3}$$

d and e being the semi-major axis and eccentricity of the test particle orbit, respectively. For a rotating black hole with spin parameter $a = |\mathbf{a}| = J/GM_{\rm BH}$, the space-time is described by the Kerr metric and, in the most favorable case of equatorial plane motion $((\mathbf{a}, \mathbf{v}) = 0)$, the shift is given by (Boyer & Price 1965)

$$\Delta\phi_K \simeq \Delta\phi_S + \frac{8a\pi M_{\rm BH}^{1/2} G^{3/2}}{d^{3/2}(1-e^2)^{3/2}c^3} + \frac{3a^2\pi G^2}{d^2(1-e^2)^2c^4}\ , \tag{4}$$

which reduces to equation (3) for $a \to 0$. In the more general case, $(\mathbf{a}, \mathbf{v}) \neq 0$, the expected peribothron shift has to be evaluated numerically.

The expected peribothron shifts (mas/revolution), $\Delta\phi$ (as seen from the center) and $\Delta\phi_E$ (as seen from the Earth at a distance $R_0 \simeq 8$ kpc from the GC), for the Schwarzschild and the extreme Kerr black holes, for the S2 and S16 stars turn out to be $\Delta\phi^{S2} = 6.3329 \times 10^5$ and 6.4410×10^5 and $\Delta\phi_{\rm E}^{S2} = 0.661$ and 0.672 respectively, and $\Delta\phi^{S16} = 1.6428 \times 10^6$ and 1.6881×10^6 and $\Delta\phi_{\rm E}^{S16} = 3.307$ and 3.399 respectively. Recall that

$$\Delta\phi_{\rm E} = \frac{d(1+e)}{R_0}\Delta\phi_{\rm S,K}\ . \tag{5}$$

Notice that the differences between the peribothron shifts for the Schwarzschild and the maximally rotating Kerr black hole is at most 0.01 mas for the S2 star and 0.009 mas for the S16 star. In order to make these measurements with the required accuracy, one needs to know the S2 orbit with a precision of at least 10 μas.

The cluster mass and density distribution, that is to say, its mass and core radius, is still unknown. The presence of this cluster affects the peribothron shift of stars orbiting the central black hole (Rubilar & Eckart 2001; Nucita *et al.* 2007; Zakharov *et al.* 2007, 2010).

We model the stellar cluster by a Plummer model density profile (Binney & Tremaine 2008)

$$\rho_{\rm CL}(r) = \rho_0 f(r)\ , \qquad \text{with } f(r) = \left[1 + \left(\frac{r}{r_{\rm c}}\right)^2\right]^{-\alpha/2}\ , \tag{6}$$

where the cluster central density ρ_0 is given by

$$\rho_0 = \frac{M_{\rm CL}}{\int_0^{R_{\rm CL}} 4\pi r^2 f(r)\, {\rm d}r}\,, \tag{7}$$

$R_{\rm CL}$ and $M_{\rm CL}$ being the cluster radius and mass, respectively. According to dynamical observations towards the GC, we require that the total mass $M(r) = M_{\rm BH} + M_{\rm CL}(r)$ contained within $r \simeq 5 \times 10^{-3}$ pc is $M \simeq 3.67 \times 10^6 M_\odot$. Useful information is provided by the cluster mass fraction, $\lambda_{\rm CL} = M_{\rm CL}/M$ and its complement, $\lambda_{\rm BH} = 1 - \lambda_{\rm CL}$. As one can see, the requirement given in equation (7) implies that $M(r) \to M_{\rm BH}$ for $r \to 0$. The total mass density profile $\rho(r)$ is given by

$$\rho(r) = \lambda_{\rm BH} M \delta^{(3)}(\vec{r}) + \rho_0 f(r) \tag{8}$$

and the mass contained within r is

$$M(r) = \lambda_{\rm BH} M + \int_0^r 4\pi r'^2 \rho_0 f(r')\, {\rm d}r'\,. \tag{9}$$

Based on results of simulations, one could conclude that now even around 5% of the BH mass (or around $2 \times 10^5 M_\odot$) distributed in bulk between apobothron and peribothron could lead to measurable shift for S2 star orbit (Zakharov *et al.* 2007), so one could constrain total extended mass distribution for the stellar cluster and dark matter concentration near the Galactic Center. These conclusions weakly depend on assumptions about specific extended mass distributions of stellar cluster and dark matter concentrations. Therefore, in the future precise astrometrical measurements of bright star trajectories could be treated as a tool for indirect searches of dark matter or these measurements could rule out the hypothesis about the significant concentration of dark matter near the Galactic Center (astroparticle physicists assumed that γ-flux from the Galactic Center region is caused by neutralino annihilation).

There are more advanced models for the stellar cluster at the Galactic Center. One of the currently most sophisticated modelling of the central cluster (taking into account careful modelling of the effects of significant light extinction and possible non-sphericity of the central cluster) has been developed recently (Schödel *et al.* 2014; Kunneriath *et al.* 2014).

In the framework of the modified Hořava gravity, Stuchlik & Schee (2014) obtained different exact solutions for the central dark mass for the so-called Kehagias–Sfetsos (KS) metric.

4.2 *Constraints on R^n theory*

The presence of dark matter and dark energy in the conventional cosmological approach may be treated as an anomaly. As it was noted by Zakharov *et al.* (2009), Le Verrier not only discovered the Mercury pericenter anomaly and explained 93% of the observed value, but a supplementary advance of 38 arcseconds/century was without explanation (later on, the value was corrected to 43 arcseconds/century), but also Le Verrier (1859a, b), analyzed the following options in order to explain the anomaly, namely, Le Verrier had analyzed the following options to explain an anomaly.

- The gravitational field of an invisible matter (planet, asteroids near the Sun). This means that new objects are being introduced.

- A deviation from the Newtonian law. It means that there is a change of the law of Nature.
- There is a lack in precision, implying that the model has to be clarified.

It was a rather general approach to resolve any anomaly. Le Verrier was very successful, when in 1846, he suggested that the Uranus anomaly could be explained by unknown planet, and following his suggestion, Neptune has been discovered. Le Verrier explained the Mercury anomaly with a presence of additional planet, and he called it Vulcan, however, now we know that it was necessary to introduce a new fundamental gravity law as it was done by Einstein in 1915.

Similarly for the Mercury anomaly, dark matter and dark energy problems may be explained by a change of the fundamental gravity law, for instance, one could change Einstein–Hilbert Lagrangian R with a function $f(R)$ and we have $f(R) = R$ for the standard GR (Capozziello 2002; Capozziello *et al.* 2004, 2006a, b, 2007a; Capozziello & Faraoni 2010; Capozziello & de Laurentis 2011). For instance, in the framework of $f(R) = R^n$ (we have GR limit for $n = 1$), one could explain acceleration of the Universe without dark energy and rotation curves for spiral galaxies without dark matter, however, we have $n \approx 1$ from solar system data (Zakharov *et al.* 2006; Maruccia *et al.* 2007). One could arrive at the same conclusion from S2 orbit data (Borka *et al.* 2012) (see also, Zakharov *et al.* 2014), where an extended distribution near the Galactic Center has been taken into account.

4.3 *Constraints on Yukawa gravity theory*

Borka *et al.* (2013) constrained parameters of Yukawa gravity from the S2 star trajectory (see Capozziello *et al.* 2007b, 2009; Cardone & Capozziello 2011; Napolitano *et al.* 2012), where it was shown that the Yukawa potential may be a weak gravitational field limit for a wide class of alternative gravitational theories

$$\Phi(r) = -\frac{GM}{(1+\delta)r}\left[1 + \delta e^{-\left(\frac{r}{\Lambda}\right)}\right], \tag{10}$$

where Λ is a parameter with a length dimension and δ is a dimensionless constant.

Borka *et al.* (2013) obtained the most probable Λ in the case of the S2 star, which is around 5000–7000 AU and that from the current observations it is very hard to obtain the reliable constraints on the universal constant δ. Borka *et al.* (2013) also found the same universal constant $\delta = 1/3$ which was successfully applied to clusters of galaxies (Capozziello *et al.* 2007b, 2009) and rotation curves of spiral galaxies (Cardone & Capozziello 2011) also give a good agreement in the case of observations of the S2 star orbit.

4.4 *Constraints on massive graviton theory*

A version of a Lorentz invariant massive gravity has been introduced by Fierz & Pauli (1939) (see also a popular review on the subject by Visser 1998), however, later people found a number of problems with such theories such as existence of ghosts, vDVZ discontinuity (Zakharov 1970; van Dam & Veltman 1970) and some other technical problems, however, some of them may be overcome (Rubakov & Tinyakov 2008).

Assuming a natural modification of the Newtonian potential corresponding to massive graviton theory (Visser 1998; Will 2014),

$$V(r) = \frac{GM}{r} \exp(-r/\lambda_g). \tag{11}$$

An opportunity to evaluate a graviton mass analyzing a time delay in electromagnetic waves such as supernova or gamma-ray burst has been analyzed (Will 2014), moreover, earlier it was demonstrated that a graviton mass can be constrained from gravitational wave signal alone (Will 1998) ($\lambda_g > 2.8 \times 10^{12}$ km).

Pulsar timing may be used to evaluate a graviton mass (Lee *et al.* 2010), where the authors used the idea to find gravitational waves analyzing pulsar timing (Sazhin 1978). In the paper, it was concluded that massless gravitons can be distinguished from gravitons heavier than 3×10^{-22} eV (Compton wavelength $\lambda_g = 4.1 \times 10^{12}$ km).

A preliminary analysis of S2 orbit data from VLT and Keck telescopes showed that one can constrain the Compton length and graviton mass, namely $\lambda_g > 2350$ AU= 3.5×10^{11} km or $m_g < 3.5 \times 10^{-21}$ eV (Zakharov *et al.* 2015).

5. Conclusions

One can conclude that there are tensions between a size of the smallest spot at the Galactic Center and an expected shadow size, therefore one should use Reissner–Norström or/and the Kottler (Schwarzschild–de-Sitter) metrics or there are systematic effects.

Concerning the best fits for trajectories S2 like stars with alternative theories of gravity, one concludes that R^n has to be practically ruled out with the observational data, and that there are hints for the Yukawa potential from an analysis of these data, because the Yukawa potential provides a slightly better fit in comparison to the Newtonian fit.

One needs more precise observations (such as VLBI in mm band, GRAVITY interferometer or/and forthcoming large telescopes (E-ELT and TMT)) for more definite claims on the discussed issues.

Acknowledgements

The author would like to thank D. Borka, V. Borka Jovanović, F. De Paolis, S. Gillessen, G. Ingrosso, P. Jovanović, S. M. Kopeikin, A. A. Nucita, S. Simić, B. Vlahovic for useful discussions, and the anonymous referee for constructive criticism, and Prof. S. Mao for his important remarks. He also thanks the Chinese Academy of Sciences for partial support and for a senior scientist fellowship.

References

Ashtekar, A. 2005, in: *100 Years of Relativity. Space-Time Structure: Einstein and Beyond*, edited by A. Ashtekar, World Scientific, Singapore, p. V.

Bergmann, P. G. 1942, *Introduction to the theory of relativity*, New York, Prentice-Hall.

Binney, J., Tremaine, S. 2008, *Galactic Dynamics*, Princeton, NJ, Princeton University Press.
Bin-Nun, A. Y. 2010a, *Phys. Rev. D*, **81**, 123011.
Bin-Nun, A. Y. 2010b, *Phys. Rev. D*, **82**, 064009.
Bin-Nun, A. Y. 2010c, arXiv:1011.5848.
Blind, N. *et al.* 2015, arXiv:1503.07303 [astro-ph.IM].
Borka, D., Jovanović, P., Borka Jovanović, V., Zakharov, A. F. 2012, *Phys. Rev. D*, **85**, 124004.
Borka, D., Jovanović, P., Borka Jovanović, V., Zakharov, A. F. 2013, *JCAP*, **11**, 050.
Boyer, R. H., Price, T. G. 1965, *Proc. Cambridge Phil. Soc.*, **61**, 531.
Capozziello, S. 2002, *Int. J. Mod. Phys. D*, **11**, 483.
Capozziello, S., Cardone, V. F., Carloni, S., Troisi, A. 2004, *Phys. Lett. A*, **326**, 292.
Capozziello, S., Cardone, V. F., Troisi, A. 2006a, *JCAP*, **08**, 1.
Capozziello, S., Cardone, V. F., Troisi, A. 2006b, *Phys. Rev. D*, **73**, 104019.
Capozziello, S., Cardone, V. F., Troisi, A. 2007a, *MNRAS*, **375**, 1423.
Capozziello, S., Cardone, V. F., Troisi, A. 2007b, *Phys. Rev. D*, **76**, 104019.
Capozziello, S., de Filippis, E., Salzano, V. 2009, *MNRAS*, **394**, 947.
Capozziello, S., Faraoni, V. 2010, *Beyond Einstein Gravity (Fundamental Theories of Physics) A Survey of Gravitational Theories for Cosmology and Astrophysics*, Springer.
Capozziello, S., De Laurentis, M. 2011, *Phys. Rep.*, **509**, 167.
Cardone, V. F., Capozziello, S. 2011, *MNRAS*, **414**, 1301.
Chandrasekhar, S. 1931, *ApJ*, **34**, 81.
Chandrasekhar, S. 1934, *Observatory*, **57**, 373.
Chandrasekhar, S. 1983, *Mathematical Theory of Black Holes*, Clarendon Press, Oxford.
Dadhich, N., Maartens, R., Papadopoulos, Ph., Rezania, V. 2001, *Phys. Lett. B*, **487**, 1.
Doeleman, S. S. *et al.* 2008, *Nature*, **455**, 78.
Dovčiak, M., Karas, V., Yaqoob, T. 2004, *ApSS*, **153**, 205.
Eckart, A., Moultaka, J., Viehmann, T., Straubmeier, C., Mouawad, N. 2004, *ApJ*, **605**, 760.
Eckart, A., Schödel, R., Straubmeier, C. 2005a, *The black hole at the center of the Milky Way*, World Scientific, Singapore.
Eckart, A., Schödel, R., Straubmeier, C., Viehmann, T., Pott, J.-U., Mouawad, N. 2005b, in: *Dark matter in astro- and particle physics*, Proc. of the International Conference DARK 2004, College Station, Texas, USA, October 3–9, 2004, edited by Klapdor-Kleingrothaus, H.-V. & Arnowitt R., Springer, Berlin, p. 3.
Eckart, A., Schödel, R., Meyer, L., Trippe, S., Ott, T., Genzel, R. 2006a, *A&A*, **455**, 1.
Eckart, A., Baganoff, F. K., Schödel, R., Morris, M. *et al.* 2006b, *A&A*, **455**, 1.
Eckart, A., Schödel, R., Meyer, L. *et al.* 2007, in: *Black Holes from Stars to Galaxies Across the Range of Masses*, Proc. of the IAU Symposium No. 238, Cambridge University Press, p. 181.
Eisenhauer, F. *et al.* 2011, *GRAVITY: Observing the Universe in Motion, The Messenger*, **143**, 16.
Eisenstaedt, J. 1982, *Arch. Hist. Exact Sci.*, **27**, 157.
Einstein, A. 1915, *Sitzungsberichte der Koniglich Preussischen Akademie der Wissenschaften (Berlin)*, 844; see English translation in *The collected papers of Albert Einstein, Volume 6: The Berlin Years: Writings, 1914-1917* (English translation supplement) p. 117.
Einstein, A. 1939, *Ann. Math.*, **40**, 922.
Fabian, A. C., Rees, M. J., Stella, L., White, N. E. 1989, *MNRAS*, **238**, 729.
Fabian, A. C., Ross, R. R. 2010, *Space Sci. Rev.*, **157**, 167.
Falcke, H., Melia, F., Agol, E. 2000a, *ApJ*, **528**, L13.
Falcke, H., Melia, F., Agol, E. 2000b, in: *Cosmic Exposions: Tenth Astrophysics Conference*, edited by W. W. Zhang & S. S. Holt, AIP Conference Proceedings, **522**, 317.
Fierz, M., Pauli, W. 1939, *Proc. Royal Soc. London A*, **173**, 211.
Frank, J., Rees, M. J. 1976, *MNRAS*, **176**, 633.
Genzel, R., Schödel, R., Ott, T. *et al.* 2003, *Nature*, **425**, 934.

Gillessen, S., Eisenhauer, F., Trippe, S. *et al.* 2009, *ApJ*, **692**, 1075.

Gillessen, S., Genzel, R., Fritz, T. K. *et al.* 2012, *Nature*, **481**, 51.

Ghez, A. M., Morris, M., Becklin, E. E., Tanner, A., Kremenek, T. 2000, *Nature*, **407**, 349.

Ghez, A. M. *et al.* 2003, *ApJL*, **586**, L127.

Ghez, A. M. *et al.* 2004, *ApJL*, **601**, L159.

Ghez, A. M. *et al.* 2005, *ApJ*, **620**, 744.

Hilbert, D. 1916, *Nachrichten von der Kuniglichen Gesellschaft der Wissenschaften zu Gottingen. Mathematisch-physikalische Klasse*, **3**, 395; English translation in: *100 Years of Gravity and Accelerated Frames The Deepest insights of Einstein and Yang-Mills* (Advanced Series on Theoretical Physical Science, v. 9) edited by Hsu J.-P. & Fine D., World Scientific (2005, Singapore) p. 120.

Hilbert, D. 1917, *Nachr. Ges. Wiss. Gottingen, Math. Phys. Kl.*, 53.

Holz, D., Wheeler, J. A. 2002, *ApJ*, **578**, 330.

Horák, J., Karas, V. 2006, *MNRAS*, **365**, 813.

Jovanović, P. 2012, *New Astron. Rev.*, **56**, 37.

Karas, V., Czerny, B., Abrassart, A., Abramowicz, M. A. 2000, *MNRAS*, **318**, 547.

Kormendy, J., Ho, L. C. 2013, *ARAA*, **51**, 511.

Kottler, F. 1918, *Annalen der Physik*, **361**, 401.

Kunneriath, D., Schödel, R., Stolovy, S., Feldmeier, A. 2014, in *The Galactic Center: Feeding and Feedback in a Normal Galactic Nucleus*, Proc. of the International Astronomical Union Symposium S303, **9**, p. 228.

Landau, L. D. 1932, *Phys. Z. Sowjetunion*, **1**, 285.

Laor, A. 1991, *ApJ*, **376**, 90.

Lee, K. J. *et al.* 2010, *ApJ*, **722**, 1589.

Le Verrier, U. J. J. 1859a, *R. Acad. Sci. Paris*, **59**, 379.

Le Verrier, U. J. J. 1859b, *Annales de l'Observatoire imperial de Paris, Annales de l'Observatoire de Paris. Memoires*, vol. 5.

Marin, F., Karas, V., Kunneriath, D., Muleri, F. 2014, *MNRAS*, **441**, 3170.

Maruccia, Y., Zakharov, A. F., De Paolis, F., Nucita, A. A., Ingrosso, G. 2007, *Nuovo Cimento*, **122 B**, 607.

Matt, G., Perola, G. C., Stella, L. 1993, *A & A*, **267**, 643.

Melia, F., Falcke, H. 2001, *ARAA*, **39**, 309.

Meyer, L., Ghez, A. M., Schödel, R. *et al.* 2012, *Science*, **338**, 84.

Morris, M. R., Meyer, L., Ghez, A. M. 2012, *RAA*, **12**, 995.

Murphy, K. D., Yaqoob, T., Karas, V., Dovčiak, M. 2009, *ApJ*, **701**, 635.

Napolitano, N. R. 2012, *ApJ*, **748**, 87.

Nucita, A. A., De Paolis, F., Ingrosso, G., Qadir, A., Zakharov, A. F. 2007, *PASP*, **119**, 349.

Oppenheimer, J. R., Snyder, H. 1939, *Phys. Rev.*, **56**, 455.

Oppenheimer, J. R., Volkoff, G. M. 1939, *Phys. Rev.*, **55**, 374.

Phifer, K., Do, T., Meyer, L., Ghez, A.M. *et al.* 2013, *ApJ*, **773**, L13.

Rubakov, V. A., Tinyakov, P. G. 2008, *Phys. – Usp.*, **51**, 759.

Rubilar, G. F., Eckart, A. 2001, *A & A*, **374**, 95.

Sazhin, M. V. 1978, *Sov. Astron.*, **22**, 36.

Schödel, R., Ott, T., Genzel, R. *et al.* 2002, *Nature*, **419**, 694.

Schödel, R., Feldmeier, A., Kunneriath, D., Stolovy, S. *et al.* 2014, *A & A*, **566**, A47.

Schwarzschild, K. 1916, *Sitzungsberichte der Koniglich Preussischen Akademie der Wissenschaften*, 189; see English translation in *Gen. Rel. Grav.* **35**, 951 (2003).

Shahzamanian, B., Eckart, A., Valencia-M., S. *et al.* 2015, *A &A* , **576**, A20.

Shen, Z. Q., Lo, K. Y., Liang, M.-C., Ho, P. T. P., Zhao, J.-H. 2005, *Nature*, **438**, 62.

Stella, L. 1990, *Nature*, **344**, 747.

Stoner, E. C. 1930, *Philos. Mag.*, **9**, 944.

Stuchlik, Z. 1983, *Bull. Astron. Inst. Czechoslov.*, **34**, 129.
Stuchlik, Z., Schee, J. 2014, *Class. Quan. Grav.*, **31**, 195013.
Tanaka, Y., Nandra, K., Fabian, A. C. *et al.* 1995, *Nature*, **375**, 659.
Turner, T. J., Mushotzky, R. F., Yaqoob, T. *et al.* 2002, *ApJL*, **574**, L123.
van Dam, H., Veltman, M. 1970, *Nucl. Phys. B*, **22**, 397.
Valencia-M., S., Eckart, A., Zajaček, M., Peissker, F. *et al.* 2015, *ApJ*, **800**, 125.
Visser, M. 1998, *Gen. Rel. Grav.*, **30**, 1717.
Wheeler, J. A. 1968, *American Scientist*, **56**, 1; reprinted in *American Scholar*, **37** 248 (1968).
Weinberg, S. 1972, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, Wiley and Sons, New York.
Weinberg, N. N., Milosavljević, M., Ghez, A. M. 2005, *ApJ*, **622**, 878.
Will, C. 1998, *Phys. Rev. D*, **57**, 2061, [gr-qc/9709011].
Will, C. 2014, *Living Reviews in Relativity*, **17**, 4, arXiv:1403.7377v1[gr-qc].
Wu, X.-B., Wang, F., Fan, X.-H. *et al.* 2015, *Nature*, **518**, 512.
Zajaček, M., Karas, V., Eckart, A. 2014, *A&A*, **565**, A17.
Zajaček, M., Eckart, A. M., Peissker, F., Karssen, G. D., Karas, V. 2015, arXiv: 1507.00237v1[astro-ph.GA].
Zakharov, A. F. 1994, *MNRAS*, **269**, 283.
Zakharov, A. F. 1995, in: *Proceedings of 17th Texas Symposium on Relativistic Astrophysics*, Ann. NY Academy of Sciences, **759**, 550.
Zakharov, A. F. 2004, in: *The Physics of Ionized Gases*, edited by L. Hadzievski, T. Gvozdanov & N. Bibic, AIP Conference Proceedings, **740**, p. 398.
Zakharov, A. F. 2005, *Intern. J. Mod. Phys. A*, **20**, 2321.
Zakharov, A. F. 2007a, *Phys. Atom. Nucl.*, **70**, 159.
Zakharov, A. F. 2007b, *Nuovo Cimento*, **122 B**, 505.
Zakharov, A. F. 2013a, in: *Proceedings of the School and Conference on Modern Mathematical Physics, SFIN year XXVI Series A: Conferences* No. A1, 375.
Zakharov, A. F. 2013b, in: *Proceedings of the XII Congress of Serbian Physicists*, Belgrade, p. 40.
Zakharov, A. F. 2014a, *Phys. Rev. D*, **90**, 062007.
Zakharov, A. F. 2014b, arXiv:1407.2591[astro-ph.GA].
Zakharov, A. F. 2014c, *Facta Universitatis, Series: Physics, Chemistry and Technology, Special Issue – Advances in Theoretical and Mathematical Physics, Ten years of Southeastern European Network in Mathematical and Theoretical Physics*, **12**, 201.
Zakharov, A. F. 2014d, in: *Proc. of the XXIXth International Workshop on High Energy Physics, New Results and Actual Problems in Particle & Astroparticle Physics and Cosmology*, edited by V. A. Petrov & R. Ryutin, World Scientific, p. 141.
Zakharov, A. F., Kardashev, N. S., Lukash, V. N., Repin, S. V. 2003, *MNRAS*, **342**, 1325.
Zakharov, A. F., Ma, Z., Bao, Y. 2004, *New Astron.*, **9**, 663.
Zakharov, A. F., Mao, S., Borka, D., Jovanović, P., Borka Jovanović, V. 2015, in preparation.
Zakharov, A. F., Nucita, A. A., De Paolis, F., Ingrosso, G. 2005a, *New Astron.*, **10**, 479.
Zakharov, A. F., Nucita, A. A., De Paolis, F., Ingrosso, G. 2005b, *A&A*, **442**, 795.
Zakharov, A. F., Nucita, A. A., De Paolis, F., Ingrosso, G. 2006, *Phys. Rev. D*, **74**, 107101.
Zakharov, A. F., Nucita, A. A., De Paolis, F., Ingrosso, G. 2007, *Phys. Rev. D*, **76**, 062001.
Zakharov, A. F., Capozziello, S., De Paolis, F., Ingrosso, G., Nucita, A. A. 2009, *Space Sci. Rev.*, **48**, 301.
Zakharov, A. F., Nucita, A. A., De Paolis, F., Ingrosso, G. 2010, *Phys. At. Nucl.*, **73**, 1870.
Zakharov, A. F., Nucita, A. A., De Paolis, F., Ingrosso, G. 2012, *New Astron. Rev.*, **56**, 64.
Zakharov, A. F., Borka, D., Borka Jovanović, V., Jovanović, P. 2014, *Adv. Space Res.*, **54**, 1108.
Zakharov, A. F., Repin, S. V. 1999, *Astron. Rep.*, **43**, 705.
Zakharov, A. F., Repin, S. V. 2002, *Astron. Rep.*, **46**, 360.
Zakharov, A. F., Repin, S. V. 2003a, *A&A*, **406**, 7.

Zakharov, A. F., Repin, S. V. 2003b, *Astron. Rep.*, **47**, 733.
Zakharov, A. F., Repin, S. V. 2003c, *Nuovo Cimento*, **118B**, 1193.
Zakharov, A. F., Repin, S. V. 2004, *Advances in Space Res.*, **34**, 2544.
Zakharov, A. F., Repin, S. V. 2005, *Mem. S. A. It. della Suppl.*, **7**, 60.
Zakharov, A. F., Repin, S. V. 2006, *New Astron.*, **11**, 405.
Zakharov, V. I. 1970, *JETP Letters*, **12**, 447.
Zamaninasab, M., Eckart, A., Witzel, G. *et al.* 2010, *A&A*, **510**, A3.