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Magnetic Field in the Gravitationally Stratified Coronal Loops

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Abstract. We study the effect of gravitational stratification on the estimation of magnetic fields in the coronal loops. By using the method of MHD seismology of kink waves for the estimation of magnetic field of coronal loops, we derive a new formula for the magnetic field considering the effect of gravitational stratification. The fast-kink wave is a potential diagnostic tool for the estimation of magnetic field in fluxtubes. We consider the eleven kink oscillation cases observed by TRACE between July 1998 and June 2001. We calculate magnetic field in the stratified loops ($B_{\rm str}$) and compare them with the previously calculated absolute magnetic field ($B_{\rm abs}$). The gravitational stratification efficiently affects the magnetic field estimation in the coronal loops as it affects also the properties of kink waves. We find ≈22% increment in the magnetic field for the smallest ($L = 72$ Mm) while ≈ 42% increment in the absolute magnetic field for the longest ($L = 406$ Mm) coronal loops. The magnetic fields $B_{\rm str}$ and $B_{\rm abs}$ also increase with the number density, if the loop length does not vary much. The increment in the magnetic field due to gravitational stratification is small at the lower number densities, however, it is large at the higher number densities. We find that damping time of kink waves due to phase-mixing is less in the case of gravitationally stratified loops compared to nonstratified ones. This indicates the more rapid damping of kink waves in the stratified loops. In conclusion, we find that the gravitational stratification efficiently affects the estimation of magnetic field and damping time estimation especially in the longer coronal loops.

Key words. Sun: coronal loops—Sun: corona—Sun: magnetic field—Sun: MHD waves

1. Introduction

The inference of the complex coronal magnetic field is key to the understanding of the dynamics, structures and activity in different layers of the Sun's atmosphere. Radio diagnostics is the tool to determine the magnetic field of the order of a few tens of gauss, however, it is constrained by its low spatial resolution Lee *et al.* (1999). Measurements of the Faraday rotation of polarized radiation from the natural radio sources can measure magnetic field only in the upper corona (Mancuso & Spangler 2000). The extrapolation of the photospheric magnetic field is a well adopted technique to measure the coronal magnetic field (e.g., Seehafer 1978; Marsch *et al*. 2004; Wiegelmann *et al.* 2005; Valori *et al*. 2005; Aschwanden & Schrijver 2011; Aschwanden & Malanushenko 2013; Yang *et al*. 2015, and references cited therein). Such extrapolations provide a crude measurement of the coronal magnetic field, however, they are based on various model assumptions.

Using high resolution observations of EUV imaging telescope (EIT) onboard SoHO, TRACE, Hinode, STEREO, SDO/AIA, the signatures of transversal coronal loop oscillations have been obtained (e.g., Aschwanden *et al*. 1999, 2002, 2003; Nakariakov *et al.* 1999; Nakariakov & Ofman 2001; Schrijver & Brown 2000; Schrijver *et al.* 2002; Verwichte *et al.* 2009; Aschwanden & Schrijver 2011; White *et al*. 2012; Srivastava & Goossens 2013, and references cited therein). These studies provide a base for the development of a new method for the measurement of the coronal magnetic field by using 'MHD Coronal Siesmology' (Nakariakov & Verwichte 2005). Such oscillations have been interpreted as the fast-kink oscillations formed by the superimposition of two oppositely propagating kink waves. The oscillation frequency may be the same as the frequency of the component kink waves. Assuming various solar structures as a magnetic flux tube, the evolution and propagation properties of longitudinal (sausage) and transversal (kink and torsional) waves have been extensively studied (e.g., Spruit 1982; Roberts 1986, 1990; Roberts & Ulmschneider 1996; Erdélyi & Fedun 2006, 2007, 2010; Musielak *et al*. 2007; Van Doorsselaere *et al*. 2008b; Murawski *et al*. 2015; and references cited therein).

The excitation of these kink modes is rooted in the photospheric granulation (Spruit 1981). These modes propagate in the upper solar atmosphere below the cutoff frequency. The propagating kink waves, either generated by photospheric drivers or *in situ*, when superimposed, trigger the standing oscillations in closed magnetic loops. These oscillations are strongly damped in the loops, and the most likely mechanisms may be either phase-mixing or resonant absorption (Aschwanden *et al.* 2002). The combined knowledge of oscillation properties (e.g., period, amplitude, temporal, spatial spectra etc.) and the properties of medium (e.g., density, temperature etc.), provide the diagnostic tool for the measurement of magnetic field (Nakariakov & Ofman 2001; Nakariakov & Verwichte 2005; Van Doorsselaere *et al.* 2008b; Yang *et al.* 2015). This also provides the seismic information about the coronal conditions (Roberts 2004).

Using the method of coronal seismology, we measure the magnetic field in the gravitationally stratified coronal loops and compare them with the nonstratified case by using the seismic properties of kink waves. In section 2, we describe the theoretical model. In section 3, we present results and discussions.

2. Theoretical model

When two oppositely propagating fast-kink waves are superimposed with each other, they form standing oscillation patterns in the coronal loops. The nodes are fixed, while the antinodes may oscillate with the period of the component kink waves which are engaged in the formation of standing wave patterns. We assume the loop length L, and observed oscillation period P_{obs}. The kink wave takes time t to reach the opposite footpoint of the loop, where it reflects and moves in the backward direction. Hence, the kink-wave trains take $2t$ second for each propagation and reflection between the two footpoints of the coronal loops during superposition. The interaction between the two oppositely oriented propagating kink-wave trains oscillate the bounded medium in different modes of standing waves. In the fundamental mode, the wavelength is twice the loop length. Hence, the wave propagation vector of the individual kink wave is $k_z \approx \frac{\pi}{L}$. The antinode of the fundamental mode will complete each oscillation in the same $2t$ seconds. Hence,

$$P_{\text{obs}} = 2t, \tag{1}$$

and the phase speed of kink-oscillation is

$$\frac{\omega}{k_z} \approx \frac{2L}{P_{\text{obs}}}. \tag{2}$$

It should be noted that propagating kink waves (in either direction) and their superimposed standing oscillations (fundamental mode) have the same role in perturbing the velocity and magnetic field of the coronal loop. The only difference is the phase between velocity and magnetic field perturbations. Component kink waves have either 0 and 180° between δV and δB, while, the standing wave has a phase difference of either -90 (antinodes) or $+90$ (nodes). Therefore, we aim to derive a dispersion relation for individual kink waves involved in the formation of standing wave pattern, which, further, can be converted in deriving the magnetic field expression.

The propagation of component kink-wave in the stratified atmosphere is governed by the general Klein–Gordon equation (Robert 2004),

$$\frac{\partial^2 \zeta}{\partial t^2} = {C_k}^2 \frac{\partial^2 \zeta}{\partial z^2} + g \frac{(1 - \rho_e/\rho_o)}{(1 + \rho_e/\rho_o)} \frac{\partial \zeta}{\partial z} - \omega_c^2 \, \zeta, \tag{3}$$

where $\zeta = \zeta(z, t)$ is the transversal displacement at height z and time t. The gravitational acceleration is g, while ρ_o and ρ_e are the densities inside and outside the coronal loop, respectively. The kink speed c_k under low beta plasma and field-free approximation is given as follows (Robert 2004; Nakariakov & Verwichte 2004):

$$C_k = \sqrt{2} \frac{C_{Ao}}{(1 + \rho_e/\rho_o)^{1/2}}, \tag{4}$$

where C_{Ao} is the Alfvénic velocity inside the loop. The cut-off frequency for the low beta plasma is given as follows (cf., Polner & Solanki 1997):

$$(\omega)_c^2 = \frac{g}{8\lambda_{\text{P}}} \frac{1}{2\beta + 1}, \tag{5}$$

where the pressure scale height, λ_{P}, is given by Aschwanden (2004) as follows:

$$\lambda_{\text{P}} = \frac{2k_{\text{B}}T}{\mu m_{\text{H}} g} \approx 4.7 \times 10^9 \, (T/1MK) \ \text{(cm)}. \tag{6}$$

We assume an isothermal atmosphere as well as the effect of gravitational stratification. Assuming all the variables with the phasor factor $\exp(i\omega t - ik_z z + z/4\lambda_{\text{P}})$, we solve equation (3) and find the dispersion relation (equations (7)–(11)). We solve the Klein–Gordon equation by inserting plane-wave solution and get some algebraic form of the dispersion relation. Thereafter, we solve it using equations (2) and (4) to get the final equation for $B_{\rm str}$ as expressed by equations (7)–(11). This relation yields the expression for the magnetic field in the gravitationally stratified coronal loops ($B_{\rm str}$),

$$B_{\rm str} = (4\pi)^{1/2}\,(M+N)^{1/2}\,(R/1-S)^{1/2}, \tag{7}$$

where

$$M = \frac{4L^2}{P_{\rm obs^2}} - \frac{\omega_c^2}{k_z^2}, \tag{8}$$

$$N = \frac{g}{4\lambda_{\rm P} k_z^2}\,\frac{(1-\rho_e/\rho_o)}{(1+\rho_e/\rho_o)}, \tag{9}$$

$$R = \rho_o\,(1+\rho_e/\rho_o), \tag{10}$$

$$S = 1 - \frac{1}{16\lambda_{\rm P}^2 k_z^2}. \tag{11}$$

3. Results and discussion

We take the parameters related to the eleven fast-kink oscillation events as observed by TRACE between the periods July 14, 1998 and June 15, 2001. These observed oscillation events have also been studied by Aschwanden *et al*. (2002, 2003), and summarized by Aschwanden (2004). We use equation (7) and its coefficients (equations (8)–(11)) as well as equations (5)–(6) to measure the magnetic field ($B_{\rm str}$) of the gravitationally stratified coronal loops and compare them with the absolute values of the magnetic field ($B_{\rm abs}$) as measured by Aschwanden (2004). We assume here that any transient (here flare) activity triggers the kink waves in the loops. These waves go into superposition quickly (here within the time $2t$) and form the observed fundamental oscillations. We diagnose the $B_{\rm str}$ based on kink waves excited impulsively, and compare them with $B_{\rm abs}$ deduced by Aschwanden *et al*. (2002, 2003), who used the seismic properties of observed fundamental kink waves.

Figure 1 compares $B_{\rm str}$ and $B_{\rm abs}$ with the conclusion that the magnetic field is higher for gravitationally stratified loop than that of the nonstratified one. The dotted rectangular strip shows the magnetic field values ($B_{\rm str}$ and $B_{\rm abs}$) for the longest coronal loop ($L = 406$ Mm), while the dotted eclipse shows the magnetic field values for the smallest coronal loop ($L = 72$ Mm). If we consider $B_{\rm abs}$ as a reference value of the magnetic fields, then we find $\approx 22\%$ increment in the magnetic field for the smallest ($L = 72$ Mm) while $\approx$42% increment for the longest ($L = 406$ Mm) gravitationally stratified coronal loops. Similarly, we find $\approx 41\%$ increment in the magnetic field for the loop length $L = 390$ Mm. We also examine the values of magnetic field ($B_{\rm str}$ and $B_{\rm abs}$) estimated for the loops of length between 162 Mm and 174 Mm. We find $\approx$25–26% increment in the magnetic field without any clear trend. We can see that only 1% increment occurs when the loop lengths increased by 16 Mm (in the case of 406 Mm and 390 Mm loops). Hence, we are not able to resolve the appreciable change in the effect of gravitational stratification for the loops having length

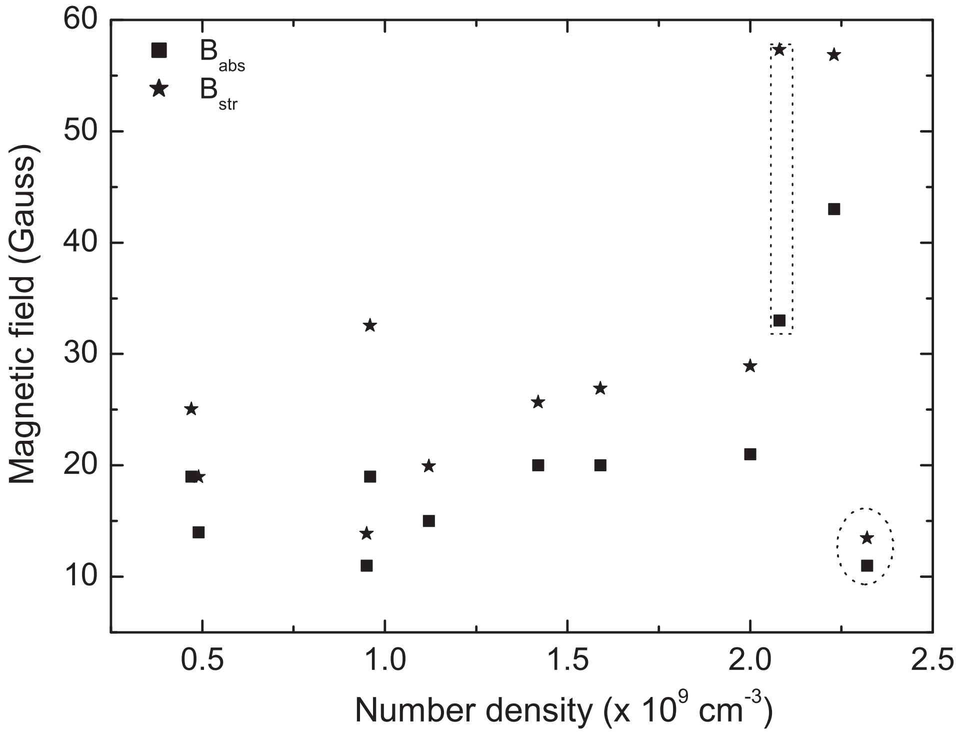

Figure 1. Comparison of B_{str} and B_{abs} for the eleven oscillatory coronal loops observed by TRACE between July 14, 1998 and June 15, 2001. The dotted rectangular strip shows the values of magnetic field for the longest coronal loop ($L = 406$ Mm). The dotted eclipse shows the values of magnetic field for the smallest coronal loop ($L = 72$ Mm).

difference less than ≈16 Mm. In conclusion, the longer loops with any typical electron density show significant increment of estimated coronal magnetic field, while this is small for the smaller loops. It should be noted that the wavelength of fundamental kink wave in larger loops may be greater than typical scale height at coronal temperature. Therefore, the wave is affected more by the gravitational stratification, and increased resultant kink speed may reflect on the increment in the enhanced magnetic field. The explicit study of oscillation events indicates the importance of stratification in the measurement of the coronal magnetic field based on kink waves. Gravitational stratification highly affects the properties of the kink waves especially in larger loops (McEwan *et al.* 2008), and therefore, play a crucial role in the estimation of coronal magnetic field. The consideration of gravitational stratification is crucial especially in the magnetic field measurement of longer and long-lived coronal structures. The negligence of this factor causes a large amount of uncertainty in the estimation of magnetic field. This result is very important in keeping with the view of the evolution of kink waves in gravitationally stratified coronal loops.

Figure 2 shows the variation of B_{str} and B_{abs} with number density when loop length does not vary much. B_{str} and B_{abs} both increase with increasing number density, which is consistent with the findings of Nakariakov & Ofman (2001). The difference between B_{str} and B_{abs} is small for lower number densities,

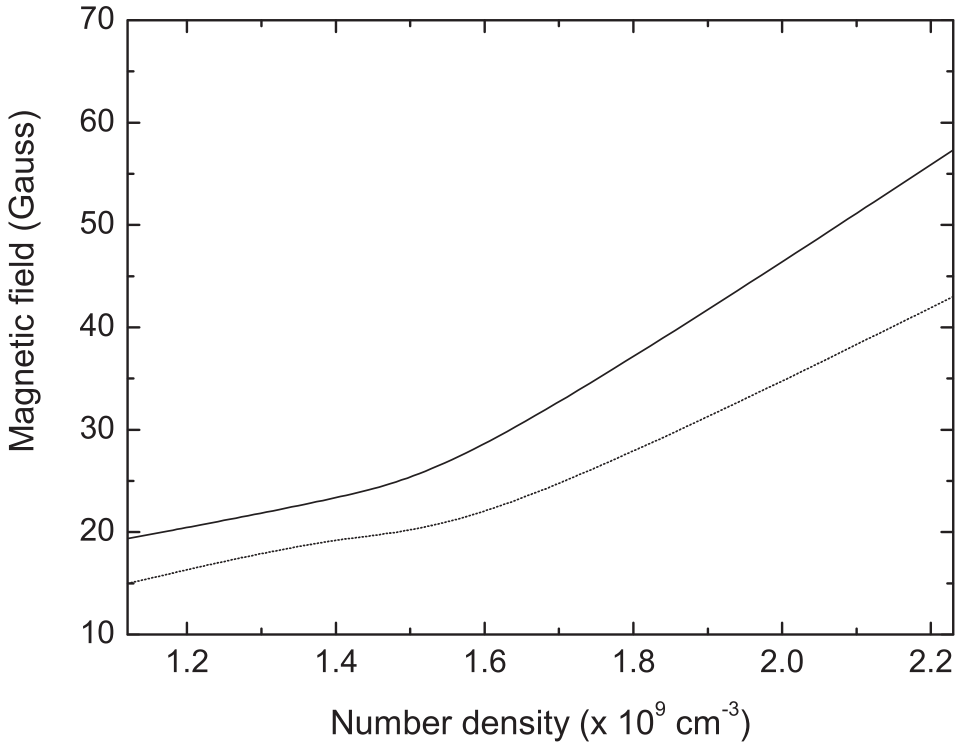

Figure 2. Variation of magnetic fields with number density in the coronal loops. Dotted line corresponds to B_{abs}, while solid line corresponds to B_{str}.

however, this is large for higher number densities. For example, B increases by $\approx$25% at $\approx 1.2 \times 10^9$ cm^{-3} and increases by $\approx$34% at $\approx 2.0 \times 10^9$ cm^{-3}, when we include the effect of gravitational stratification. Hence, the accurate density measurement is also crucial to the estimation of the magnetic field of coronal loops using kink waves and related seismology. The height-dependent electron density diagnostics with EUV spectral line ratio only takes collisional excitation into account which is a viable mechanism for the formation of EUV lines in the inner corona. O'Shea *et al.* (2005) have reported that radiative excitation becomes stronger than collisional excitation in the corona at $\approx 1.21 R_o$ (solar radius). Hence, the line ratio calculations of electron densities using collision excitation calculations, may be less accurate at larger heights, where radiative excitation becomes stronger than collisional excitation. Hence, the accurate density estimation, particularly at larger heights, may provide correct estimation of the magnetic field in the corona (Van Doorsselaere *et al.* 2008a).

The presence of density inhomogeniety on the small length-scale in loops, may be one of the likely mechanisms for the damping of kink-oscillations in the coronal loops. Using the phenomenon of phase mixing, Roberts (2000) has estimated the damping time τ_{d}. The ratio of the damping time for stratified and non-stratified coronal loops can be written as follows:

$$(\tau_{\text{d}})_{\text{str}} = [B_{\text{abs}}/B_{\text{str}}]^{2/3}\, (\tau_{\text{d}})_{\text{nonstr}}. \tag{12}$$

Equation (12) clearly indicates the shorter damping time for the gravitationally stratified coronal loops. This gives rise to enhanced dissipation of the kink waves. For the longer loops, the ratio of absolute to stratified magnetic fields will be the lowest. The damping time in such loops will also be the lowest when certain non-classical damping mechanism (here phase-mixing) is at work. Hence, the dissipation of kink-oscillation will be quick in these loops.

In conclusion, we find that gravitational stratification efficiently affects the properties of the kink waves excited in the loops, and therefore, the estimation of the loop's magnetic field using the principle of MHD seismology.

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