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Solar Magnetic Atmospheric Effects on Global Helioseismic Oscillations

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Abstract. Both the interior and the atmosphere of the Sun give a wide range of oscillations and waves. Interactions between waves and their highly structured and dynamic environment strongly influence the various properties of the waves. Understanding those possible interactions could provide priceless diagnostic tools in the search for hidden aspects of the solar interior and atmosphere. This article is an attempt to overview briefly our current understanding of how global helioseismic oscillations, f and p acoustic waves, interact with plasma flows and magnetic fields inthe solar atmosphere.

Key words. Sun—helioseismology—global oscillations—solar atmosphere—flow field—magnetic field—MHD.

1. Solar global oscillations

Solar photospheric dynamics at various scales were studied first by Leighton *et al.* (1962). Analysing Doppler difference images, they searched for uniform velocity fields with locally random radial components. Instead, they found that the temporal variation of the up and down motion of the photosphere showed about five-minute periodicity everywhere. The mechanisms that can result in such global oscillations of the photosphere were later explained by acoustic waves trapped in the solar interior due to the mirror-like effect of the sharp density drop of the plasma at the photosphere (Ulrich 1970; Leibacher & Stein 1971). The surprising discovery of *global photospheric oscillations* has opened a window to the unseen interior of the Sun.

The branch of solar physics that exploits the observed solar surface oscillations to understand the properties of the interior is called *helioseismology*, seismic investigation of the Sun. Spherical harmonics can be used to mathematically describe small-amplitude (linear) oscillations in a spherical, non-rotating medium with adiabatic stratification, that models the solar interior with no atmosphere (see details of such models in Christensen-Dalsgaard (1980), Campbell & Roberts (1989), and Pintér & Erdélyi (2011)). The frequencies of the possible oscillations are

$$\nu_{n,l} = \sqrt{\left(1 + \frac{2n}{\mu}\right) \frac{\sqrt{l\,(l+1)}g}{4\pi^2 R}}, \tag{1}$$

where $\mu \equiv 1/(\gamma - 1)$ is the polytropic index, γ is the ratio of specific heats, l is the harmonic degree, g is the gravity at the photosphere, and R is the solar radius. With radial order $n = 0$, eq. (1) gives the frequency of the fundamental (f) oscillation mode, also called surface gravity mode. With $n = 1, 2, 3, \ldots$, we obtain the pressure-mode (p_n) frequencies of different radial order n. The $\nu(l)$ graphs, based on eq. (1), in Figure 1 reproduce very well the parabola-like rigdes of observed frequency spectra of f and p oscillation modes. Compare Figure 1 with, for example, Figure 2 in Rabello-Soares (2012).

2. Observational evidence for solar atmospheric effects on helioseismic oscillations

The overall structure of the helioseismic frequency spectrum, see Figure 1, has not been changing since its first observation. However, by the eighties, enough data set had been collected to detect temporal variations of the order of μHz, and small changes in the frequency spectrum were reported.

Woodard & Noyes (1985) observed low-degree ($l = 0$ and 1) oscillations from near solar minimum (1980) and near solar maximum (1984), and noticed that the p-mode eigenfrequencies were systematically higher by about 0.4 μHz at maximum. The 0.01% change in p-mode frequency over a solar cycle was well above the observational errors. Bachmann and Brown (1993), analysing helioseismic data, observed between 1984 and 1990, found correlation of p-mode frequency variations

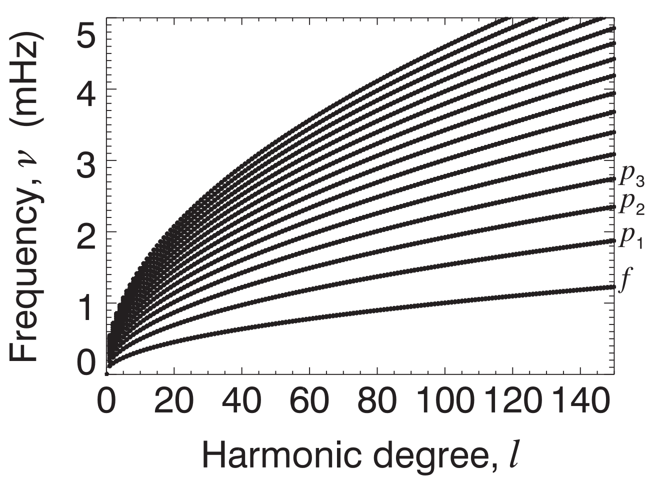

Figure 1. Frequency spectrum of the f and first fifteen p eigenmodes of a model of the solar interior for harmonic degrees below $l = 150$ and with polytropic index $\mu = 1.5$.

with large (solar cycle minimum to maximum) and small (over one month intervals) variations of magnetic activity indices, such as magnetic plage strength index, Kitt peak magnetic index, or solar EUV flux.

Jefferies *et al.* (1990) compared p-mode frequencies and widths from observations in 1981 and 1987, when the solar activity was low and high, respectively. In the frequency domain $2.4 \leq \nu \leq 4.8$ mHz and harmonic degree interval $3 \leq l \leq 98$, they measured 224 ± 19 nHz frequency decrease from solar maximum to solar minimum. The width of the spectral lines also decreased from high to low activity systematically, and it also varied with ν and l.

Libbrecht and Woodard (1990a) found an increase in p-mode frequencies during increasing solar activity. They proposed that the source of measured frequency changes was confined to the layers right beneath the photosphere.

Komm *et al.* (2000) derived line widths and oscillation amplitudes from 108-day GONG (Global Oscillation Network Group) observations over a period between 1995 and 1999. From minimum activity in 1995 to October 1998, they recorded a 3% increase in mode width, and 7% and 6% decrease in amplitude and in the product of amplitude and width, called *mode area*. The changes were the largest in the frequency interval between 2.7 and 3.3 mHz. No measurable l-dependence was found in the frequency variations.

Eleven years of BiSON (Birmingham Solar Oscillations Network) data were investigated by Chaplin *et al.* (2004) to study the variations of low-l p-mode frequencies during the declining phase of cycle 22 and the ascending phase of cycle 23. Evidence was found in the frequency shifts for a response of the oscillation modes to the generation of magnetic flux at the beginning of the new cycle.

Tripathy *et al.* (2006) have studied p-mode frequencies, amplitudes, and line widths by analysing data from GONG observations over almost the entire period of solar cycle 23. The mode parameters were derived from time series of 9 to 108 days. It was found that the correlation between mode parameters and solar activity varies with the chosen length of the time series. Similarly, Komm *et al.* (2000) obtained significant correlation for the frequencies and anti-correlation for the amplitude and mode area.

Dziembowski and Goode (2005) detected differences between the activity minimum and maximum of solar cycle 23 in SOHO/MDI (Michelson Doppler Imager onboard Solar and Heliospheric Observatory) oscillation frequency data between 1996 and 2004. Frequencies of f and p modes have been found to correlate strongly with the solar magnetic activity.

An analysis of MDI data from 1996 to 2006 and GONG data from 2001 to 2006 by Howe *et al.* (2008) has verified that p modes typically show a positive correlation with the local surface magnetic field strength. The correlation was so persistent throughout the solar cycle, that the map of frequency shift against latitude and time derived by Howe *et al.* (2008) followed the magnetic butterfly diagram.

Salabert *et al.* (2010) has studied the shifts of three low-l oscillation mode frequencies over the 23rd solar cycle by using SOHO/GOLF (Global Oscillations at Low Frequencies) and GONG data. They obtained from the two data sets that, while the p mode with $l = 1$ showed a decrease in frequency during the extended minimum of the cycle (between 2007 and 2009), the frequencies of the $l = 0$ and $l = 2$ modes increased during the time period of decreasing surface magnetic activity. The authors have concluded that the difference in behaviour indicates that the $l = 0$

and $l = 2$ p modes, which are more sensitive to high-latitude environments than the $l = 1$ mode, were affected by an increasing magnetic field strength, related to the new solar cycle 24 in 2007. The magnetic field strength started to increase at high latitudes first.

Salabert *et al.* (2011) studied variations in a range of oscillation properties, such as low-l p-mode amplitudes, frequencies, line widths, peak asymmetry, rotational splittings, acoustic powers, and energy supply rates, during the period spanning the complete solar cycle 23 in GOLF and SOHO/VIRGO (Variability of solar Radiance and Gravity Oscillations) data taken between 1996 and 2010. Daily means of the 10.7 cm radio flux were used as a proxy of the solar surface activity. All the oscillation parameters, except for rotational splitting, showed *correlation* with solar cycle variations.

It can be concluded from the above observations that solar global oscillations are strongly affected by the *solar magnetic activity cycle*. The actual physical mechanisms responsible for the correlations, however, are still subject to controversy. It is most likely that several mechanisms play a role in the complex coupling between oscillations and their environment. In order to understand the underlying mechanisms and their contribution rate to the measured variations, intensive modelling of the solar environment is required. In the following sections, the progress of modelling the effects of atmospheric plasma flow fields and atmospheric magnetic fields on solar global oscillations is overviewed.

3. Modelling of coupling mechanisms

A standard way of studying global helioseismic oscillations is using magnetohydrodynamic (MHD) slab models with Cartesian geometry, that consist of a layer representing the solar interior and an overlying region of the atmosphere. The *interior*, for the general dominance of plasma pressure over magnetic pressure, can be approximated free of magnetic field, and it is usually a polytrope, in which the plasma pressure and density decrease with height so that

$$p(z) \propto \rho(z)^{(\mu+1)/\mu},$$

where μ is the same polytropic index that occurs in equation (1). The *atmospheric* layer, lying right above the interior, is significantly less dense than the interior, and it can be embedded in a magnetic field. Plasma flows can also be added to each layer in order to model their effects on global oscillations.

Such models allow us to investigate how individual or combined changes in the model parameters, such as variations in temperature, modify the oscillation properties, such as frequency, line width, penetration depth into the interior, etc. There are two major phenomena dominating large regions of the Sun that can have a strong influence on global oscillations. One is *global plasma flows*, that can be present at any scale and at any layer of the Sun. The other is the *magnetic field structure*, that governs most solar atmospheric dynamics.

The MHD equations consist of the *continuity equation*,

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \rho = 0, \tag{2}$$

where $\mathbf{v}$ and ρ are plasma flow velocity and plasma density; the *momentum equation*,

$$\rho\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{v}=-\nabla p+\frac{1}{\mu}(\nabla\times\mathbf{B})\times\mathbf{B}+\rho\mathbf{g}, \tag{3}$$

where p, $\mathbf{B}$, and $\mathbf{g}$ are plasma pressure, magnetic field strength, and gravity; the *induction equation*,

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{B}); \tag{4}$$

the adiabatic *energy equation*,

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\left(p\rho^{-\gamma}\right)=0; \tag{5}$$

and the constraint on the magnetic divergence,

$$\nabla\cdot\mathbf{B}=0. \tag{6}$$

Around a static equilibrium, small perturbations of the physical quantities from their equilibrium values are considered: $f(z,t)\approx f_0(z)+f_1(z,t)$ with $|f_1|\ll|f_0|$. This is a reasonable assumption as, for example, the observed photospheric radial velocity oscillations have amplitudes of only about 1 m s^{-1} (e.g. Elsworth *et al.* 1995), which are small compared to typical plasma flow and wave propagation speeds. (The rotational speed of the solar plasma at the equator, for example, is about 2 km s^{-1}. The sound speed at the solar surface is around 8–9 km s^{-1}. Alfvén waves can travel several megameters in a second in the thin photosphere even in a weak magnetic field.) This allows us to *linearize* the MHD equations.

Typical line widths of helioseismic oscillation modes are only around a microhertz (Chaplin *et al.* 1998), which indicates slow damping processes. Therefore, dissipation, such as due to viscosity, thermal conduction, or resistivity, can be ignored when modelling helioseismic oscillations. Dissipative effects, however, can play important roles at locations where oscillation amplitudes are significantly increased due to resonant coupling to MHD waves (see section 3.2). Using standard notation (ρ, p, $\mathbf{u}$, $\mathbf{B}$, $\mathbf{g}$, and t are plasma density, pressure, velocity, magnetic field, gravity, and time, respectively), the linearized governing equations in compressible ideal MHD are

$$\frac{\partial\rho_1}{\partial t}+\nabla\cdot(\rho_0\mathbf{u}_1+\rho_1\mathbf{u}_0)=0, \tag{7}$$

$$\rho_0\left[\frac{\partial\mathbf{u}_1}{\partial t}+\mathbf{u}_0\cdot\nabla\mathbf{u}_1+\mathbf{u}_1\cdot\nabla\mathbf{u}_0\right]+\rho_1\mathbf{u}_0\cdot\nabla\mathbf{u}_0$$
$$=-\nabla p_1+\frac{1}{\mu}(\nabla\times\mathbf{B}_1)\times\mathbf{B}_0+\frac{1}{\mu}(\nabla\times\mathbf{B}_0)\times\mathbf{B}_1+\rho_1\mathbf{g}, \tag{8}$$

$$\frac{\partial\mathbf{B}_1}{\partial t}=\nabla\times(\mathbf{u}_0\times\mathbf{B}_1)+\nabla\times(\mathbf{u}_1\times\mathbf{B}_0), \tag{9}$$

$$\frac{1}{\rho_0}\left[\rho_1\mathbf{u}_0\cdot\nabla p_0-\gamma p_1\mathbf{u}_0\cdot\nabla\rho_0\right]+\frac{\partial p_1}{\partial t}+\mathbf{u}_0\cdot\nabla p_1+\mathbf{u}_1\cdot\nabla p_0$$
$$-v_s^2\left[\frac{\partial\rho_1}{\partial t}+\mathbf{u}_1\cdot\nabla\rho_0+\mathbf{u}_0\cdot\nabla\rho_1\right]=0, \tag{10}$$

where all equilibrium quantities, denoted by the index 0, depend only on depth z and here $v_s(z) = \sqrt{\gamma p_0/\rho_0}$ is the local sound speed. The governing equations are supplemented by the solenoidal condition, $\nabla \cdot \mathbf{B} = 0$. The perturbations can be assumed to be harmonic oscillations, and, their local amplitude is only a function of height, z (or distance, for example, from the solar surface). By Fourier analysing the MHD equations in this way, the perturbations are basically approximated by *plane waves*. Using plane-wave approximation, all the linear perturbations take the form

$$f_1(x, y, z, t) = f(z; \omega, k_x, k_y)\mathrm{e}^{i(k_x x + k_y y - \omega t)}, \tag{11}$$

where k_x and k_y are the horizontal components of the wave vector, $\mathbf{k}$, perpendicular to the vertical (radial) direction, and its magnitude is

$$k = \sqrt{k_x^2 + k_y^2} = \frac{\sqrt{l(l+1)}}{r},$$

where r is the distance from the solar centre. The plane-wave approximation is valid for harmonic degrees $l \gg 2\pi$ (Pintér 1999) as for low-l modes, the curvatures of the paths are comparable to the wavelength of the modes, which cannot be ignored. Low-l oscillation modes have to be studied in full spherical (3D) geometry.

In equations (7)–(10), the explicit (exponential) dependence of the perturbed (both vector and scalar) quantities on the horizontal coordinate, x, y, and time, t (given in equation (11)) allows us to replace the partial derivatives, with respect to x and t, of the perturbations by algebraic functions of the perturbations themselves. Only derivatives with respect to z (vertical or radial direction) remain in the governing equations, which can be reduced to two coupled first-order ordinary differential equations, usually for the vertical component of the displacement vector, ξ_z, where $\mathbf{v} \equiv \mathrm{d}\underline{\xi}/\mathrm{d}t$, and the total (plasma plus magnetic) pressure perturbation, $P = p + \mathbf{B}_0\mathbf{B}/\mu$,

$$D\frac{\mathrm{d}\xi_z}{\mathrm{d}z} = C_1\xi_z - C_2 P, \qquad D\frac{\mathrm{d}P}{\mathrm{d}z} = C_3\xi_z - C_1 P, \tag{12}$$

where the z-dependent factors, D, C_1, C_2, and C_3, can be specified by the equilibrium profiles $\rho_0(z)$, $p_0(z)$, $\mathbf{v}_0(z)$, and $\mathbf{B}_0(z)$ of the particular model.

The resulting governing equations can be solved, to obtain the perturbations, either analytically or numerically in any layer of the model. A layer is defined by its equilibrium profile, for example, by $\rho_0(z)$, $p_0(z)$, $\mathbf{u}_0(z)$ and $\mathbf{B}_0(z)$, given in a range of z. In case of a multi-layer model, the solutions derived for each layer can be matched by *boundary conditions*: the vertical component of the Lagrangian displacement, ξ_z, and the Eulerian perturbation of the total pressure, $P + \rho_0 g \xi_z$, must be continuous at interfaces between plasma layers. Away from the solar surface, $z \to \pm\infty$, it is also required that the kinetic energy density of the perturbation, $E_{\mathrm{kin}}(z) = 2\pi^2\nu^2\rho_0(z)\xi^2(z)$, tend to zero as $z \to \pm\infty$. The governing equations (eqs. (7) to (10)) together with the above boundary conditions define an *eigenvalue problem*, where the eigensolutions are the perturbations (such as $\mathbf{v}(z)$, $P(z)$, $\mathbf{B}(z)$) around the equilibrium values ($\mathbf{v}_0$, P_0, $\mathbf{B}_0$, etc.) and the eigenvalue is the global (z-independent) frequency, ν. Note that the eigenvalue problem, defined above, describes *global helioseismic oscillations*, that are interacting with the magnetic atmosphere, as standing waves and not those countless atmospheric propagating waves that we can observe directly (e.g. Okamoto & De Pontieu 2011) or those upward-propagating waves that have been modelled and indirectly observed by, for example, Jess *et al.* (2012).

The solutions to the dispersion relations in most of the solar models can be derived numerically only. In multi-layer models, due to their complexity, even the dispersion relations are not expressed by a single equation. In order to study the explicit mathematics of the different models reviewed in the present paper, the reader is kindly directed to the original articles, because, presenting the dispersion relations or the resulting frequency spectra would make the review lengthy and hardly readable.

3.1 *Atmospheric magnetic fields*

In order to understand how helioseismic f and p modes behave in a magnetic atmosphere, it is necessary to recall briefly what type of waves can arise in a magnetic plasma environment. Using the magnetohydrodynamic (MHD) approximation, wave propagation in magnetically structured atmospheres was first derived in its present, widely known form, in a series of papers (Roberts 1981a, b; Edwin & Roberts 1982; Miles & Roberts 1989). The first and last studies in this list consider two-layer models with a tangentially discontinuous interface, while the models in the two other papers have a slab between two semi-infinite layers. The two layers represent the field-free solar interior and the magnetic atmosphere, respectively. The plasma properties (such as density, temperature, or magnetic field strength) are constant in each layer; gravity is ignored. In such media, e.g., *slow* and *fast* magneto-acoustic and *Alfvén waves* can exist. The Alfvén frequency is $\nu_{\mathrm{A}}(z) = (2\pi)^{-1} k v_{\mathrm{A}}(z)$, where k is the wavenumber and $v_{\mathrm{A}}(z) \equiv \dfrac{B(z)}{\sqrt{\mu\rho(z)}}$ is the Alfvén speed, B is magnetic field strength, μ is magnetic permeability, and $\rho(z)$ is plasma density. The characteristic slow frequency is $\nu_{\mathrm{c}}(z) = (2\pi)^{-1} k v_{\mathrm{c}}(z)$, where $v_{\mathrm{c}}(z) \equiv \dfrac{v_{\mathrm{s}}(z) v_{\mathrm{A}}(z)}{v_{\mathrm{s}}(z) + v_{\mathrm{A}}(z)}$ is the slow speed, and $v_{\mathrm{s}}(z) \equiv \sqrt{\dfrac{\gamma p(z)}{\rho(z)}}$ is the sound speed. (For a detailed introduction to the characteristics of Alfvén, slow, and fast waves in an isothermal medium with uniform magnetic field and no gravitational stratification, see, for example, Lighthill 1960; Cowling 1976; Roberts 1981a, b.)

Gravity was added to the two-layer model, first by Miles and Roberts (1992), where the atmospheric magnetic layer had a constant Alfvén speed, and by Miles *et al.* (1992), where the atmospheric layer had a uniform magnetic field. Both (field-free and magnetic) layers were isothermal in each model. One of the consequences of gravity is that, besides slow and fast magneto-acoustic-gravity waves, a *gravity surface mode*, or fundamental, f, mode also appears. A characteristic property of this mode is that it can exist even in an incompressible medium and its velocity (or displacement) perturbation is free of divergence.

Jess *et al.* (2012) analysed data from the Dunn Solar telescope at Saxramento Peak, New Mexico and from Rapid Oscillations in the Solar Atmosphere (ROSA) and compared the results to simulations using the MuRAM radiative MHD code (Vögler *et al.* 2005; Shelyag *et al.* 2011). The model was a plane parallel slab simulating both photospheric convection and an atmospheric layer with a uniform vertical magnetic field. The observations and modelling revealed an abundance of upward propagating waves with 2 to 10 minute periods. The authors concluded from their results that the ubiquity of the oscillations could be traced back to magneto-convective processes occurring in the top of the solar convective zone.

Turova (2014) studied the dependence of oscillation properties on the location of the oscillation in a coronal hole and found that the low- and high-frequency components of the spectrum increased and decreased, respectively, with height. Within the coronal hole, the power of three-minute and five-minute oscillations increased towards the boundaries of bright magnetic network elements.

The upper, atmospheric, region of the model can be embedded in a homogeneous *uniform* or *non-uniform* magnetic field. In the model with non-uniform magnetic field in the atmosphere, the field strength decays to zero far away from the solar surface. The choice of making the magnetic field strength decrease exponentially together with the square root of plasma density makes the Alfvén speed constant in the atmosphere, reducing the mathematical complexity of the problem. It is important to note, however, that observations indicate that the Alfvén speed tends to increase with height in the solar atmosphere, particularly above coronal holes and quiet regions of the Sun, as observed by McIntosh *et al.* (2011). This certainly limits the applicability of atmospheric models that consider a constant Alfvén-speed profile.

The first attempt to model magnetic effects on helioseismic modes was made by Roberts & Campbell (1986) and Campbell & Roberts (1988) in a static equilibrium solar model. They considered the magnetic fields at the base of the convection zone. The two-layer model consisted of a field-free polytrope above an isothermal magnetic core. According to their predictions, at least $B = 5 \times 10^5$ G field strength is required to produce the frequency shift of p modes, observed by Woodard and Noyes (1985), and a decreasing field strength at the base of the convection zone results in decreasing p-mode frequency.

The pioneering model that successfully demonstrated the role of the *magnetic chromosphere* of the Sun in modifying p- and f-mode frequencies is introduced in the seminal paper by Campbell and Roberts (1989). The elegance of the mathematical structure of the model has since inspired many (Evans & Robert 1990, 1991; Murawski & Roberts 1993b; Jain & Roberts 1991; Tirry *et al.* 1998; Mędrek *et al.* 1999; Vanlommel & Čadež 2000; Varga & Erdélyi 2001; Vanlommel *et al.* 2002; Taroyan 2003; Čadež & Vanlommel 2005; Foullon 2002; Pinter *et al.* 2007; Petrovay *et al.* 2007, 2010; Mole *et al.* 2008; Foullon & Nakariakov 2010), to derive further details of the atmospheric effects on helioseismic modes, by enhancing the original model, which consists of a plane stratified polytrope and a magnetic chromosphere.

The conclusion from the model used by Campbell and Roberts (1989) is that the frequency shifts are proportional to the square of the chromospheric magnetic field strength and they increase with the harmonic degree. The f-mode frequency and for low and moderate l, the p_1-mode frequency increases, while the frequency of other p modes decrease for increasing field strength. In Campbell and Roberts (1989), the upper layer is isothermal (therefore, the sound speed, v_s, is constant), and it has a *constant Alfvén speed*. Due to the constant plasma beta, $\beta \equiv v_s^2(z)/v_A^2(z)$, the magnetic field strength rapidly decreases, together with the plasma density in this atmospheric layer.

In another atmospheric profile, which represents the presence of a strong magnetic field, $B_0(z)$ is kept constant. The mathematical description of analytical and numerical solutions of the governing equations for the perturbed quantities, equations (12), including short- and long-wavelength approximations, in atmospheres with constant Alfvén speed and also with constant magnetic field strength, are given in detail in Erdélyi (2006a, b) and Pintér & Erdélyi (2011).

Compared to the spectrum for the closed model (Figure 1), which consists of only an interior of the Sun, the existence of an atmospheric layer above the interior introduces an upper and a lower *cut-off frequency*, ω_I and ω_{II}, respectively. This means that the presence of an atmospheric layer of low plasma density above the solar interior does not allow the existence of global acoustic waves with oscillation frequency below ω_I or above ω_{II}. The frequencies of the f and p modes can occur only between these two cut-off frequencies. Another atmospheric effect is the appearance of the Lamb or a mode. The a mode is a pressure mode with frequency slightly lower than the characteristic sound frequency for any given harmonic degree (see also, Lamb 1932). The frequency vs. harmonic degree graph for the a mode avoids crossing with the f- and p-mode frequencies as the a-mode frequency increases with increasing l, more sharply than the f- and p-mode frequencies. With harmonic degrees around avoided crossings, the oscillation is in a transitional stage, showing mixed properties of the a and p modes. A set of *atmospheric gravity oscillations* is also a solution to the model. They are low-frequency modes and they can exist only for high harmonic degrees. Contrary to the p modes, for which the frequencies increase for any given l, g-mode frequencies decrease with increasing order number, n (Pintér & Goossens 1999). That is, g modes with lower frequencies have more nodes along the solar radius.

3.2 *Resonant coupling to MHD waves*

Models with a constant Alfvén speed throughout the atmosphere are realistic only in the sense that their atmospheric magnetic field strength decreases exponentially with the distance from the photosphere. However, as pointed out in the previous section, observations indicate increasing Alfvén speeds in the solar atmosphere (McIntosh *et al.* 2011). One of the reasons for such a magnetic structure to be chosen so often is that it avoids singularities in the governing MHD equations being singular.

In a magnetic environment, MHD slow and Alfvén waves can exist, as it was pointed out in section 3.1. Slow and Alfvén waves propagate basically along magnetic field lines, and their oscillation frequency is proportional to the local slow and Alfvén speed, respectively. At locations where the frequency of a helioseismic f or p mode equals the frequency of a slow wave or an Alfvén wave, they will be *coupled resonantly*, which results in a significant increase of the oscillation amplitudes in the vicinity of the resonant position.

Figure 2 is an example of a three-layer model, in which a global oscillation, with a frequency $\nu = 1.2$ mHz, is coupled resonantly to an Alfvén wave and to a slow wave at two different heights in the intermediate, transitional layer. The interior is free of magnetic field, hence, $\nu_A\,(z \leq 0) = \nu_c\,(z \leq 0) = 0$. The atmosphere is isothermal, and the magnetic field strength and the plasma density decrease with z at the same rate, which makes both the Alfvén and slow frequencies constant. In the transitional

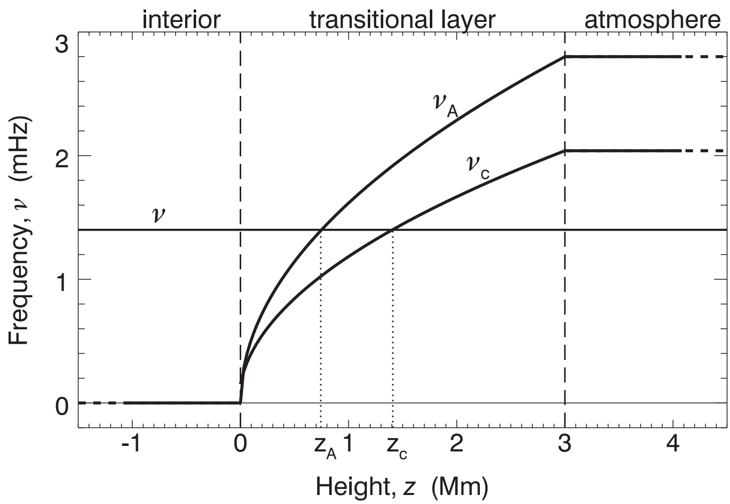

Figure 2. Resonance at heights z_A and z_c due to $\nu_A(z_A) = \nu$ and $\nu_c(z_c) = \nu$ in a three-layer model. The thickness of the intermediate, transitional layer is $L = 3$ Mm.

layer, the Alfvén and slow frequencies increase with height from zero to their atmospheric values. For each of the Alfvén and slow waves, there is a height where its frequency is equal to ν, the frequency of a global f or p oscillation mode. Those two resonant heights are denoted by z_A and z_c, respectively.

The linear nature of the oscillations can be maintained by taking into account some kind of *dissipation* in the MHD equations around the resonant position. Everywhere else, *ideal* MHD equations are used. The frequencies that solve the new set of MHD equations are complex. Oscillations with *negative imaginary part* in their frequency describe modes that, due to the local dissipation, decay exponentially in time (compare with equation (11)). The greater the damping of a mode, the wider the mode's peak in the frequency-spectrum, the line-width being related to the imaginary part of the frequency of a damped mode by $\Gamma = -2\ \mathrm{Im}(\nu)$. An elegant way of handling the resulting fourth order partial differential equations was derived in Sakurai *et al.* (1991b). The authors present a method to derive simple algebraic formulae which connect the solutions below and above the resonant heights. The *connection formulae* are derived for a magnetic cylinder in Sakurai (1991a, b), to investigate the absorption of sound waves by sunspots. The mathematical model is further extended by Goossens & Ruderman (1995) and Erdélyi (1997) on slow resonant absorption to stationary equilibria. The analogue derivations for slab geometry can be found in, for example, Tirry *et al.* (1998) and Pintér (2008a, b).

Studying the coupling of solar global oscillations to the non-uniform and magnetic solar atmosphere, where the coupling may involve resonant absorption, is an important step forward in modelling, as it allows investigating the contributions, for example, to the *damping* of the global acoustic oscillations and link the modelling predictions to measurable physical quantities such as line-width.

The actual physical process of resonant absorption, and the modelling of resonant coupling between local MHD waves and global oscillations under solar atmospheric circumstances are reviewed in detail in Goossens *et al.* (2010). Here, we focus on its applications only with regard to the coupling of solar global oscillations to the magnetized atmosphere of the Sun.

A three-layer model with an intermediate zone, where the magnetic field, together with the Alfvén speed, increases continuously from zero, was first introduced by Tirry *et al.* (1998). In the intermediate zone, global helioseismic modes may interact resonantly to local boundary layer Alfvén and/or slow oscillations at the height where their frequency matches the frequency of the global mode, hence, the model can be used to investigate the effects of resonant coupling between global modes and local atmospheric MHD oscillations. Pintér and Goossens (1999) discussed the case of parallel propagation in such three-layer models. For propagation parallel to the magnetic field, global oscillation modes couple to slow modes only. In addition, to the damping of global oscillation modes due to resonant absorption, it was also found that the interaction of global eigenmodes with slow continuum modes leads to an unanticipated behaviour in the global eigenmodes. A rather strange behaviour in the slow continuum involves the disappearance, appearance, and splitting and merging of global modes.

The examples shown so far were global modes that propagate horizontally along magnetic field lines. Helioseismic modes propagating horizontally with a wave vector it tilted to the magnetic field are modelled in Jain & Roberts (1994) and Pintér *et al.* (2007). Jain and Roberts (1994) studied a two-layer atmospheric model, in which neither slow nor Alfvén continuum arises. A follow-up work by Pintér *et al.* (2007) is based on a three-layer model with a continuous transitional layer between the interior and the atmosphere, in which, resonant coupling to slow and Alfvén waves occur. Pintér *et al.* (2007) investigated the magnetic effects on global mode frequencies for oblique propagating resonant waves, especially the frequency shifts and damping rates caused by the resonant interaction with local MHD waves. They demonstrated that global modes propagating obliquely to the magnetic field lines can couple also to local MHD Alfvén and slow continuum modes. Due to the presence of a number of characteristic frequencies in the system, they found not only the f- and p-modes, but also Lamb modes, with frequencies near the characteristic cut-off frequencies. Resonantly coupled atmospheric gravity modes also appeared as solutions to the linear MHD equations. Pintér *et al.* (2007) found that an increase in the propagation angle relative to the magnetic field lines generally decreases the magnetic shift of frequency. It was also shown by Pintér *et al.* (2007) that the line-widths of damped modes are smaller for propagations closer to and/or perpendicular to the field lines. This is because the slow and Alfvén waves are propagating along the field lines, hence, resonant interaction is the strongest with parallel propagating global modes.

3.3 *Atmospheric flow fields*

Besides atmospheric magnetism, another major global solar phenomenon that potentially can modify properties of global oscillations of the Sun and can cause observable shifts in the oscillation frequencies is *plasma flows*, more particularly, photospheric and sub-surface flows, such as photospheric granulation, meridional flows and differential rotation. Large-scale plasma motions can be *random* or *coherent*, long-term flows.

The influences of surface granulation on f modes have been modelled by random horizontal flows in Murawski and Roberts (1993a), Edélyi *et al.* (2004), and Mole *et al.* (2008). They obtained that the mode frequency is lower than the fundamental value, $\nu_{0,l}$ (see equation (1)). The frequency decrease was particularly significant in the high-l domain. Similar effects were found when random vertical velocity fields in the convection zone were added to horizontal flows (Murawski & Roberts 1993bb, and Kerebes *et al.* 2008a, b). Rosenthal *et al.* (1999) investigated effects of turbulent convection on f and p oscillation modes and also concluded that turbulence generally lowers the mode frequencies. These model results correspond well with those observed by Libbrecht *et al.* (1990).

Erdélyi *et al.* (1999) has extended the solar model by an atmospheric layer to study effects of sub-surface flows in a more realistic environment. Erdélyi and Taroyan (1999, 2001a, b) improved the temperature profile in the above multi-layer model, and derived a dispersion formula, $f(\nu, l) = 0$, for it. They concluded that the effects of steady plasma flows and those of an atmospheric magnetic field on frequencies of global solar oscillations compete with each other and the dominance of one above the other depends on the harmonic degree, or wavelength, of the oscillation modes.

3.3.1 *Steady state and resonant damping.* In this section, we recall an application of resonant absorption in steady state, where *changes* in a steady state may be accounted for frequency shifts of resonantly coupled helioseismic eigenoscillations. A generalization of the work by Tirry *et al.* (1998) and Pintér & Goossens (1999) on resonant coupling and damping of helioseismic modes in a *steady state*, are the studies by Pintér *et al.* (2001a, b, c). Their three-layer solar model represents a solar interior including a homogeneous constant equilibrium flow, the chromosphere, and the corona, with a unidirectional horizontal atmospheric magnetic field). The authors computed the eigenfrequencies, ν, and the line-widths, Γ, of global mode oscillations as a function of the harmonic degree, l, for fixed values of a sub-photospheric flow, v. The frequencies and the line-widths of eigenmodes are affected by sub-surface flows *and* atmospheric magnetic fields. A key contribution to the effects comes from the universal mechanism of resonant absorption in steady state. Pintér *et al.* (2001a, b) propose a model which shows in detail the influence of steady state (e.g. representing differential rotation or meridional flows) on the frequencies and line widths of global p-modes of resonant coupling with local MHD waves in the solar chromosphere. Pintér *et al.* (2001a) demonstrated the potential for this effect to cause frequency shifts and damping for a range of magnetic field geometries, by incorporating the effects of a large-scale flow on this system. The results unveil the principle that resonant coupling with MHD waves in a steady state might be, in part, responsible for the solar cycle variation of p-mode parameters. When both atmospheric magnetic fields and sub-surface flows are present, a complex picture of competition between these two effects has been found.

Pintér *et al.* (2001b) derived frequency shifts due to changing the direction of a global sub-surface flow and they obtained shifts by up to tens of μHz. Such a large effect would be easily detectable. Table 1 in Pintér *et al.* (2001b) also shows the sensitivity of the line-width of the f- and p_1-modes to an equilibrium flow varying between $-0.1v_s$ and $0.1v_s$. The ratio $(\Gamma(v;\ l) - \Gamma(v = 0;\ l))/v$ is given for different values of l. The line-width of the f-mode increases or decreases linearly with v in the interval $v \in [-0.1v_s,\ 0.1v_s]$, while that of the p-mode always increases, also linearly, in the given interval of v and l. For larger l, the effects are more complicated, and the line-width of p-modes can also decrease with v. In addition, for larger values of v, the dependence becomes non-linear, where the model may break.

A straightforward application of the resonant coupling of acoustic oscillations in a *steady state* is the rotational splitting of helioseismic modes influenced by a magnetic atmosphere. Pintér *et al.* (2001b) studied the splitting of sectoral ($m = \pm l$) helioseismic eigenmodes (f- and p-modes) in the presence of a magnetic atmosphere. The solar interior was in a steady state, with sub-photospheric plasma flow along the equator representing solar rotation for co- and counter-propagating modes in surface regions. The Cartesian geometry employed restricted the study to sectoral modes with $l \geq 50$, which guarantees that the modes do not penetrate deeply into the solar interior, and, therefore, experience an approximately uniform rotation. They evaluated the rotational splitting of sectoral modes, $\Delta\nu_{nl\pm l} \equiv \nu_{nl\text{-}l}(v) - \nu_{nll}(v)$ by calculating the frequency difference for reversed flow velocities, $\nu_{nll}(-v) - \nu_{nll}(v)$.

They found, for example, that the frequency increase for the f and p_1 modes with $l = 100$ is $\Delta\nu_{n=0,l=100} \approx 270$ nHz and $\Delta\nu_{n=1,l=100} \approx 810$ nHz, respectively, as the atmospheric magnetic field strength increases from 0 to about 150 G, while the increase in frequency splitting for the two modes for the same change in the magnetic field is $\Delta\nu_{n=0,l=100,m=\pm 100} \approx 30$ nHz and $\Delta\nu_{n=1,l=100,m=\pm 100} \approx 50$ nHz (see Figures 2–4 in Pintér *et al.* 2001c). For a comparison with observed data, these contributions to frequency splitting are about 10–20% of those observed by Jefferies *et al.* (1990). The changes in frequency splitting obtained in the model are on the verge of detectability, and ought to be detectable by combining a number of modes. On the other hand, there are other competing effects — such as those due to zonal flows — which are of the same order. A possible way of helping to differentiate between the several competing shifts would be to evaluate the modelled rotational splitting for all m. This requires a move to spherical geometry.

The above brief theoretical outline and the three (somewhat selective) branches of resonant MHD wave studies implementing steady states (steady state effect on resonant heating and flow effect on helioseismic eigenmodes) clearly indicate the richness and complexity of the interaction of MHD waves when equilibrium bulk motions are present. The sensitivity of results also show that there is a strong need for more accurate observations and diagnostics of solar *steady* magnetic structures. Variations with solar activity of the characteristic parameters of solar acoustic vibrations (e.g. p-modes) have been tracked over two decades with a much increased range of independent observations in the current activity cycle. Again, the changes in frequency of up to about 0.4 μHz over the solar cycle were established in the early 1990s (e.g. Elsworth *et al.* 1990; Libbrecht & Woodard 1990a, b), and detailed studies have been made possible by the high quality data sets now available from the ground-based GONG and BiSON networks and instruments such as MDI on the SOHO spacecraft (e.g. Chaplin *et al.* 1998; Howe *et al.* 1999). The correlations between solar activity and mode amplitudes and widths are intrinsically more difficult to study, but a consensus between observers now appears to have been reached. Modes with angular degree, l, up to 150 in the frequency range 1700 to 4100 mHz show lower amplitudes and greater widths at solar maximum, compared with solar minimum. Both effects are of the order of 10%, with the detailed values depending somewhat upon the modes studied.

Helioseismic techniques can also be used to study large-scale flows in the Sun, such as meridional circulation. Meridional flows with velocities in the range 10–20 m/s have been detected, applying time-distance helioseismic techniques to MDI data obtained in 1996 (Giles *et al.* 1997), and there are some indications in the literature that such flows may be *slower* at sunspot maximum (Komm *et al.* 1993). Torsional oscillations were first reported in the early 1980s (Howard & Labonte 1980), and manifest themselves as bands in the rotation profile, the relative speed of adjacent bands being of the order of 5 m/s. GONG (Howe *et al.* 2000) and MDI (Toomre *et al.* 2000) observations over recent years show a solar cycle variation in these oscillations, with the bands drifting towards the equator during the rising phase of the Sun's current activity cycle.

The theoretical interpretation of the shifts of f and p modes has attracted a considerable effort in *static* models. Campbell & Roberts (1989), Evans & Roberts (1992), and Jain & Roberts (1993, 1994) considered coherent magnetic field in the atmosphere, while Erdélyi *et al.* (2005) incorporated the random magnetic carpet. One common key feature of these modelling efforts is that the profile of the magnetic field was selected such that the Alfvén speed was *constant* in the solar atmosphere, excluding the possibility of resonant interaction (including, e.g., efficient energy transfer) of the solar global oscillations with the solar atmosphere. In the following, we relax this condition on the Alfvén speed and review efforts made to include the physics of resonant absorption of solar acoustic oscillations in a background equilibrium, where steady flows are also allowed.

To implement this new physics in the modelling efforts, one of the first tasks is to understand the role of the presence of the *boundary layer* between the solar interior and the corona, where the actual resonant interaction will take place. When compared to the extent of the interior (the solar radius is about 700 Mm) or of the corona, the transition between the solar interior and the corona occurs in a rather narrow layer. This boundary layer, that includes the photosphere, chromosphere and TR, is around 2–3 Mm thick and contains *coherent and random magnetic and velocity fields*, giving a very difficult task to describe even in approximate terms the wave perturbations. Random flows, for example, turbulent granular motion (see, e.g. Petrovay *et al.* 2007, 2010), coherent flows (meridional flows or the near-surface component of the differential rotation), random magnetic fields (e.g., the continuously emerging tiny magnetic fluxes or magnetic carpet) and coherent fields (large loops and their magnetic canopy region), each have their own effect on wave perturbations. Some of these effects may be more important than the others. The magnitude of these corrections has to be estimated one by one and, it is suspected that, unfortunately, they all may contribute to line-widths or frequency shifts of the global acoustic oscillations on a rather equal basis. Various attempts are made to estimate these effects. Murawski and Roberts (1993a, b) studied random velocity field corrections of the f mode, while Kerekes *et al.* (2008a, b) and Mole *et al.* (2007, 2008) re-visited this problem and has explained some inconsistencies found in Murawski and Roberts (1993a, b). Kerekes *et al.* (2008a) introduce a new approach to study the interaction of solar global eigenoscillations, with particular emphasis on the f-mode, with random inhomogeneities caused by flows *and* magnetic field near the solar surface. They suggest the approach of an initial value method to derive a general dispersion relation for a class of models where the magnetic atmosphere is overlying an arbitrary static solar interior. The strength of this approach is that, in these models, the interior part is treated parametrically and does not need to be specified before we obtain the dispersion relation. In order to demonstrate the applicability of their method, an analytical solution of the dispersion relation is given for an incompressible interior with constant density. The powerful conclusion of this study is that it can, perhaps, more easily incorporate resonant absorption as a damping mechanism. In helioseismology, corrections from this transitional boundary layer are called the surface term (Basu 2002) and in many helioseismologic modelling, the surface term is taken in some *ad-hoc* functional form.

4. Conclusion

The effects of a magnetic atmosphere on global helioseismic modes have been studied intensively in the solar physics community, but still, there are a number of questions that are not answered conclusively. The currently existing models contain an interface or have simple slab geometry, and they are often one dimensional. Only very simple magnetic fields can be studied in such models, which hardly describe the true complexity of the magnetic solar atmosphere. However, recent discoveries of ubiquitous small-scale horizontal magnetic fields above the photosphere (Tsuneta *et al.* 2008; Lites *et al.* 2008; Ishikawa & Tsuneta 2010; Ishikawa *et al.* 2010) somewhat justify the studies of the effects of horizontal atmospheric magnetic fields. Although horizontal magnetic fields have been observed everywhere in the solar atmosphere, they are generally small-scale structures and obviously not only horizontal fields are present in the Sun, so the applicability of models with horizontal magnetic fields have their limitations.

Another unsettled problem related to these models is how realistic the existence of atmospheric gravity modes, which arise as eigensolutions in gravitationally stratified three-layer models, can be. They might be discovered by future observations.

A more extensive non-linear study is also essential to understand helioseismic global oscillations. Although the mode amplitudes are small, the assumption of an equilibrium, in which perturbations arise, is a great reduction of the real situation, as the state of the Sun cannot be described by an equilibrium, instead, it is a rather dynamic system continuously changing in complex ways. Moving away from the 1D efforts to 2D and 3D modelling in spherical geometry is more challenging mathematically and computationally, but will be very important both for global and local helioseismology. Studying the effects of random flows can be considered the first attempt to move in this direction.

Moving away from the 1D efforts to 2D and 3D modelling in spherical geometry is also more challenging mathematically and computationally, but will be very important both for global and local helioseismology. The inclusion of the rather complex, often time-dependent magnetic effects in the models on which the inversion techniques are based will make it possible to derive not only high-resolution maps of sub-photospheric plasma flows, but even magnetic maps from helioseismic data, which will further contribute to the success of helioseismology in understanding the physics of the Sun.

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References

Bachmann, K. T., Brown, T. M. 1993, *Astrophys. Lett.*, **411**, 45–48.
Basu, S. 2002, What does helioseismology tell us about solar cycle related structural changes in the Sun?, in: *From Solar Min to Max: Half a Solar Cycle with SOHO*, edited by Wilson, A., ESA Special Publication, vol. 508, pp. 7–14.
Čadež, V. M., Vanlommel, P. 2005, *Serbian Astronomical J.*, **171**, 19–27.
Campbell, W. R., Roberts, B. 1988, Magnetic Effects on Solar *P*-Modes, in: *Advances in Helio- and Asteroseismology*, edited by J. Christensen-Dalsgaard & S. Frandsen, IAU Symposium, vol. 123, pp. 161–165.
Campbell, W. R., Roberts, B. 1989, *J. Astrophys.*, **338**, 538–556.
Chaplin, W. J., Elsworth, Y., Isaak, G. R., Lines, R., McLeod, C. P., Miller, B. A., New, R. 1998, *Mon. Not. Roy. Astron. Soc.*, **300**, 1077–1090.
Chaplin, W. J., Elsworth, Y., Isaak, G. R., Miller, B. A., New, R. 2004, *Mon. Not. Roy. Astron. Soc.*, **352**, 1102–1108.
Christensen-Dalsgaard, J. 1980, *Mon. Not. Roy. Astron. Soc.*, **190**, 765–791.
Cowling, T. G. 1976, *Magnetohydrodynamics*. Adam Hilger Ltd., Bristol, p. 145.
Dziembowski, W. A., Goode, P. R. 2005, *Astrophys. J.*, **625**, 548–555.
Edwin, P. M., Roberts, B. 1982, *Solar Phys.*, **76**, 239–259.
Elsworth, Y., Howe, R., Isaak, G. R., McLeod, C. P., New, R. 1990, *Nature*, **345**, 322–324.
Elsworth, Y., Howe, R., Isaak, G. R., McLeod, C. P., Miller, B. A., New, R., Wheeler, S. J. 1995, *Astron. Astrophys. Suppl.*, **113**, 379.
Erdélyi, R. 1997, *Solar Phys.*, **171**, 49–59.
Erdélyi, R. 2006, *Royal Society of London Philosophical Transactions Series A*, **364**, 351–381.
Erdélyi, R. 2006, in: *Proceedings of SOHO 18/GONG 2006/HELAS I, Beyond the spherical Sun*, ESA Special Publication, vol. 624.
Erdélyi, R., Taroyan, Y. 1999, The Influence of a Steady State on p- and f-Modes, in: *Magnetic Fields and Solar Processes*, edited by A. Wilson *et al*., ESA Special Publication, vol. 448, p. 81.
Erdélyi, R., Taroyan, Y. 2001a, Effect of a Steady Flow and an Atmospheric Magnetic Field on the Solar p- and f-Modes, in: *Recent Insights into the Physics of the Sun and Heliosphere: Highlights from SOHO and Other Space Missions*.
Erdélyi, R., Taroyan, Y. 2001b, Helioseismic frequency shifts due to steady states, in: *SOHO 10/GONG 2000 Workshop: Helio- and Asteroseismology at the Dawn of the Millennium*.
Erdélyi, R., Kerekes, A., Mole, N. 2004, Random Flow Effects on Surface Waves, in: *SOHO 13 Waves, Oscillations and Small-Scale Transients Events in the Solar Atmosphere: Joint View from SOHO and TRACE*, edited by H. Lacoste, ESA Special Publication, vol. 547, p. 75.
Erdélyi, R., Kerekes, A., Mole, N. 2005, *Astron. Astrophys.*, **431**, 1083–1088.
Erdélyi, R., Varga, E., Zétényi, M. 1999, Magnetoacoustic-Gravity Surface Waves in Steady Plasmas, in: *Magnetic Fields and Solar Processes*, edited by A. Wilson *et al*., ESA Special Publication, vol. 448, p. 269.
Evans, D. J., Roberts, B. 1990, *Astrophys. J.*, **356**, 704–719.
Evans, D. J., Roberts, B. 1991, *Astrophys. J.*, **371**, 387–395.
Evans, D. J., Roberts, B. 1992, *Nature*, **355**, 230–232.
Foullon, C. 2002, Helioseismology and Diagnostics of Internal Magnetic Layers, PhD thesis, School of Mathematics and Statistics, University of St. Andrews, Scotland, UK.
Foullon, C., Nakariakov, V. M. 2010, *Astrophys. Lett.*, **714**, 68–72.
Giles, P. M., Duvall, Jr. T. L., Kosovichev, A. G., Scherrer, P. H. 1997, Solar Meridional Circulation and Rotation Determined by Time-Distance Helioseismology using MDI Data From SOHO, in: *Bulletin of the American Astronomical Society*, Bulletin of the American Astronomical Society, vol. 29, p. 914.
Goossens, M., Ruderman, M. S. 1995, *Physica Scripta Volume T*, **60**, 171–184.

Goossens, M., Erdélyi, R., Ruderman, M. S. 2010, *Space Sci. Rev.*, 125.
Howard, R., Labonte, B. J. 1980, *Astrophys. Lett.*, **239**, 33–36.
Howe, R., Komm, R., Hill, F. 2000, *Solar Phys.*, **192**, 427–435.
Howe, R., Christensen-Dalsgaard, J., Hill, F., Komm, R. W., Larsen, R. M., Schou, J., Thompson, M. J., Toomre, J. 1999, *Bulletin of the American Astronomical Society*, **31**, 1530.
Howe, R., Haber, D. A., Hindman, B. W., Komm, R., Hill, F., Gonzalez Hernandez, I. 2008, *Astronomical Society of the Pacific Conference Series*, **383**, 305–312.
Ishikawa, R., Tsuneta, S. 2010, *Astrophys. Lett.*, **718**, 171–175.
Ishikawa, R., Tsuneta, S., Jurčák, J. 2010, *Astrophys. J.*, **713**, 1310–1321.
Jain, R., Roberts, B. 1991, *Solar Phys.*, **133**, 263–280.
Jain, R., Roberts, B. 1993, *Astrophys. J.*, **414**, 898–907.
Jain, R., Roberts, B. 1994, *Solar Phys.*, **152**, 261–266.
Jefferies, S. M., Duvall, Jr. T. L., Harvey, J. W., Pomerantz, M. A. 1990, Helioseismology from the South Pole: Results from the 1987 Campaign, in: *Progress of Seismology of the Sun and Stars*, edited by Y. Osaki & H. Shibahashi, Lecture Notes in Physics, Berlin Springer Verlag, vol. 367, pp. 135–143.
Jess, D. B., Shelyag, S., Mathioudakis, M., Keys, P. H., Christian, D. J., Keenan, F. P. 2012, *Astrophys. J.*, **746**, 183.
Kerekes, A., Erdélyi, R., Mole, N. 2008a, *Astrophys. J.*, **683**, 527–535.
Kerekes, A., Erdélyi, R., Mole, N. 2008b, *Solar Phys.*, **251**, 469–489.
Komm, R. W., Howard, R. F., Harvey, J. W. 1993, *Solar Phys.*, **147**, 207–223.
Komm, R. W., Howe, R., Hill, F. 2000, *Astrophys. J.*, **531**, 1094–1108.
Lamb, H. 1932, *Hydrodynamics*.
Leibacher, J. W., Stein, R. F. 1971, *Astrophys. Lett.*, **7**, 191–192.
Leighton, R. B., Noyes, R. W., Simon, G. W. 1962, *Astrophys. J.*, **135**, 474–482.
Libbrecht, K. G., Woodard, M. F. 1990, *Lecture Notes in Physics*, Berlin Springer Verlag, vol. 367, pp. 145–156.
Libbrecht, K. G., Woodard, M. F. 1990, *Nature*, **345**, 779–782.
Libbrecht, K. G., Woodard, M. F., Kaufman, J. M. 1990, *Astrophys. J., Supplement*, **74**, 1129–1149.
Lighthill, M. J. 1960, *Royal Society of London Philosophical Transactions Series A*, **252**, 397–430.
Lites, B. W., Kubo, M., Socas-Navarro, H., Berger, T., Frank, Z., Shine, R., Tarbell, T., Title, A., Ichimoto, K., Katsukawa, Y., Tsuneta, S., Suematsu, Y., Shimizu, T., Nagata, S. 2008, *Astrophys. J.*, **672**, 1237–1253.
Mędrek, M., Murawski, K., Roberts, B. 1999, *Astron. Astrophys.*, **349**, 312–316.
McIntosh, S. W., de Pontieu, B., Carlsson, M., Hansteen, V., Berner, P., Goossens, M. 2011, *Nature*, **475**, 477–480.
Miles, A. J., Roberts, B. 1989, *Solar Phys.*, **119**, 257–278.
Miles, A. J., Roberts, B. 1992, *Solar Phys.*, **141**, 205–234.
Miles, A. J., Allen, H. R., Roberts, B. 1992, *Solar Phys.*, **141**, 235–251.
Mole, N., Erdélyi, R., Kerekes, A. 2007, *World Scientific*, vol. 368, pp. 13–23.
Mole, N., Kerekes, A., Erdélyi, R. 2008, *Solar Phys.*, **251**, 453–468.
Murawski, K., Roberts, B. 1993a, *Astron. Astrophys.*, **272**, 595.
Murawski, K., Roberts, B. 1993b, *Astron. Astrophys.*, **272**, 601.
Okamoto, T. J., De Pontieu, B. 2011, *Astrophys. Lett.*, **736**, 24.
Petrovay, K., Erdélyi, R., Thompson, M. J. 2007, *Solar Phys.*, **240**, 197–209.
Petrovay, K., Kerekes, A., Erdelyi, R. 2010, *Geophysical and Astrophysical Fluid Dynamics*, **104**, 619–630.
Pintér, B. 1999, The influence of the atmospheric magnetic field on solar oscillation modes, Ph.D. thesis, Katholieke Universiteit Leuven, Belgium.
Pintér, B. 2008a, *Astronomische Nachrichten*, **329**, 503.

Pintér, B. 2008b, *Solar Phys.*, **251**, 329–340.
Pintér, B., Erdélyi, R. 2011, *Space Sci. Rev.*, **158**, 471–504.
Pintér, B., Goossens, M. 1999, *Astron. Astrophys.*, **347**, 321–334.
Pintér, B., Erdélyi, R., Goossens, M. 2007, *Astron. Astrophys.*, **466**, 377–388.
Pintér, B., Erdélyi, R., New, R. 2001a, *Astron. Astrophys.*, **372**, 17–20.
Pintér, B., New, R., Erdélyi, R. 2001b, *Astron. Astrophys.*, **378**, 1–4.
Pintér, B., Erdélyi, R., New, R., Goossens, M. 2001c, Influence of equilibrium flows and the atmospheric magnetic field on solar oscillation modes, in: *SOHO 10/GONG 2000 Workshop: Helio- and Asteroseismology at the Dawn of the Millennium*, edited by A. Wilson & P. L. Pallé, ESA Special Publication, vol. 464, pp. 227–230.
Rabello-Soares, M. C. 2012, *Astrophys. J.*, **745**, 184.
Roberts, B. 1981a, *Solar Phys.*, **69**, 39–56.
Roberts, B. 1981b, *Solar Phys.*, **69**, 27–38.
Roberts, B., Campbell, W. R. 1986, *Nature*, **323**, 603–605.
Rosenthal, C. S., Christensen-Dalsgaard, J., Nordlund, Å., Stein, R. F., Trampedach, R. 1999, *Astron. Astrophys.*, **351**, 689–700.
Sakurai, T., Goossens, M., Hollweg, J. V. 1991a, *Solar Phys.*, **133**, 247–262.
Sakurai, T., Goossens, M., Hollweg, J. V. 1991b, *Solar Phys.*, **133**, 227–245.
Salabert, D., García, R. A., Pallé, P. L., Jiménez-Reyes, S. J. 2010, *Astronomical Society of the Pacific Conference Series*, vol. 428, pp. 51–56.
Salabert, D., García, R. A., Pallé, P. L., Jiménez, A. 2011, *Journal of Physics Conference Series*, **271(1)**, 012030–012033.
Shelyag, S., Keys, P., Mathioudakis, M., Keenan, F. P. 2011, *Astron. Astrophys.*, **526**, 5.
Taroyan, Y. 2003, Global Resonant MHD Waves in the Magnetosphere, *Publications of the Astronomy Department of the Eotvos Lorand University*, **13**, 177–180.
Tirry, W. J., Goossens, M., Pintér, B., Čadež, V., Vanlommel, P. 1998, *Astrophys. J.*, **503**, 422.
Toomre, J., Christensen-Dalsgaard, J., Howe, R., Larsen, R. M., Schou, J., Thompson, M. J. 2000, *Solar Phys.*, **192**, 437–448.
Tripathy, S. C., Hill, F., Jain, K., Leibacher, J. W. 2006, *Beyond the Spherical Sun*, ESA Special Publication, vol. 624, pp. 93–99.
Tsuneta, S., Ichimoto, K., Katsukawa, Y., Nagata, S., Otsubo, M., Shimizu, T., Suematsu, Y., Nakagiri, M., Noguchi, M., Tarbell, T., Title, A., Shine, R., Rosenberg, W., Hoffmann, C., Jurcevich, B., Kushner, G., Levay, M., Lites, B., Elmore, D., Matsushita, T., Kawaguchi, N., Saito, H., Mikami, I., Hill, L. D., Owens, J. K. 2008, *Solar Phys.*, **249**, 167–196.
Turova, I. P. 2014, *Astronomy Letters*, **40**, 145–160.
Ulrich, R. K. 1970, *Astrophys. J.*, **162**, 993–1002.
Vanlommel, P., Čadež, V. M. 2000, *Solar Phys.*, **196**, 227–244.
Vanlommel, P., Debosscher, A., Andries, J., Goossens, M. 2002, *Solar Phys.*, **205**, 1–24.
Varga, E., Erdélyi, R. 2001, *Publications of the Astronomy Deparment of the Eotvos Lorand University*, **11**, 83.
Vögler, A., Shelyag, S., Schüssler, M., Cattaneo, F., Emonet, T., Linde, T. 2005, *Astron. Astrophys.*, **429**, 335–351.
Woodard, M. F., Noyes, R. W. 1985, *Nature*, **318**, 449–450.