J. Astrophys. Astr. (2014) **35,** 639–648

Benford's Law in Astronomy

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Received 22 April 2014; accepted 21 May 2014

Abstract. Benford's law predicts the occurrence of the n-th digit of numbers in datasets originating from various sources all over the world, ranging from financial data to atomic spectra. It is intriguing that although many features of Benford's law have been proven, it is still not fully understood mathematically. In this paper we investigate the distances of galaxies and stars by comparing the first, second and third significant digit probabilities with Benford's predictions. It is found that the distances of galaxies follow the first digit law reasonable well, and that the star distances agree very well with the first, second and third significant digit.

Key words. Benford's law—Universe—stars—galaxies—significant digit.

1. Introduction

In 1881, the astronomer and mathematician Newcomb made a remarkable observation with respect to logarithmic books (Newcomb 1881). He noticed that the first pages were more worn out than the last. This led him to the conclusion that the significant digits of various physical datasets are not distributed with equal probability but the smaller significant digits are favored. In 1938, Benford continued this study and he derived the law of the anomalous numbers.

The general significant digit law (Hill 1995a) for all $k \in \mathbb{N}$, $d_1 \in \{1, 2, \ldots, 9\}$ and $d_k \in \{0, 1, \ldots, 9\}$, for $k \geq 2$ is

$$P(d_1, d_2, \ldots, d_k) = \log_{10}\left[1 + \left(\sum_{i=1}^{k} d_i \times 10^{k-i}\right)^{-1}\right] \tag{1}$$

where d_k is the k-th leftmost digit. For example, the probability to find a number whose first leftmost digit is 2, second digit is 3 and third digit is 5 is $P(d_1 = 2, d_2 = 3, d_3 = 5) = \log_{10}(1 + 1/235) = 0.18\,\%$.

The first significant digit can be written as

$$P(k) = \log_{10}\left(1 + \frac{1}{k}\right), \; k = 1, 2, \ldots, 9 \tag{2}$$

This law has been tested against various datasets ranging from statistics (Shao & Ma 2010) to geophysical sciences (De & Sen 2011) and from financial data (Clippe & Ausloos 2012) to multiple choice exams (Hopee 2013). Studies were also performed in physical data like complex atomic spectra (Pain 2008), full width of hadrons (Shao & Ma 2009) and half life times for alpha and β decays (Buck *et al.* 1993; Dong-Dong *et al.* 2009).

An interesting property of this law is that it is invariant under the choice of units of the dataset (scale invariance) (Wojcik 2013). For example, if the dataset contains lengths, the probability of the first significant digits is invariant in the case that the units are chosen to be meters, feet or miles.

Still, Benford's law is not fully understood mathematically. A great step was done with the extension of scale to base invariance (the dependance of the base in which numbers are written) by Hill (1995b). Combining these features and realising that all the datasets that follow Benford's law are a mixture from different distributions, he made the most complete explanation of the law. Another approach in the explanation of the logarithmic law was examined by Boyle (1994) using the Fourier series method.

2. New numerical sequences and Benford's law

A simple example of Benford's law is performed on numerical sequences. It is already proven that the Fibonacci and Lucas numbers obey the Benford's law (Wlodarksi 1971). The three additional numerical sequences considered in this paper are:

- Jacobsthal numbers (J_n), defined as
  - $J_0 = 0$,
  - $J_1 = 1$,
  - $J_n = J_{n-1} + 2J_{n-2}, \; \forall\, n > 1$,
- Jacobsthal–Lucas numbers (JL_n), defined as
  - $JL_0 = 2$,
  - $JL_1 = 1$,
  - $JL_n = JL_{n-1} + 2JL_{n-2}, \; \forall\, n > 1$,
- and Bernoulli numbers (B_n), defined by the contour interval
  - $B_0 = 1$,
  - $B_n = \frac{n!}{2\pi i} \oint \frac{z}{e^z - 1} \frac{dz}{z^{n+1}}$.

A sample of the first 1000 numbers of these sequences is used to extract the probabilities of the first significant digit to take the values 1,2, . . . ,9 and the second and

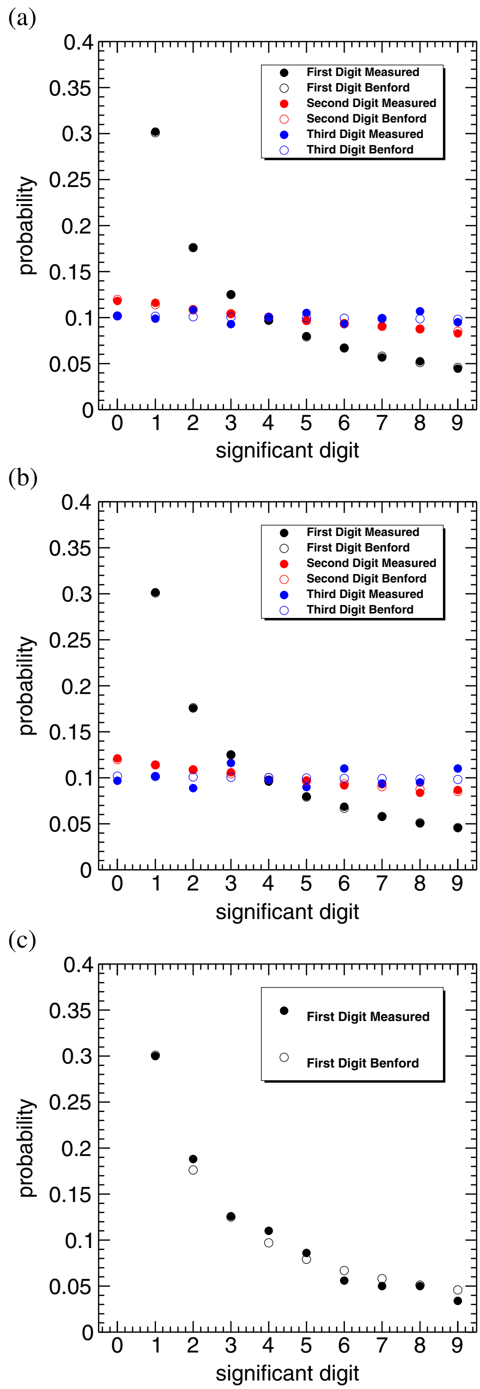

Figure 1. Comparison of Benford's law (open circles) predictions and the distribution of the first, second and third significant digits of the (**a**) Jacobsthal, (**b**) Jacobsthal–Lucas and (**c**) Bernoulli sequences (full circles). The probabilities for the first digit is plotted with black, the second with red and the third with blue circles according to Benford's law.

third significant digits to be 0,1, . . . ,9. The results can be seen in Fig. 1. Filled circles represent the result from the analysis of the Jacobsthal and Jacobsthal–Lucas numbers and the open circles indicate the probabilities calculated from Benford's formula (equation (1)). It is clear that all three sequences follow Benford's law for the first (black), second (red) and third (blue) significant digit.

In the following sections we examine the distances of stars and galaxies and compare the probabilities of occurrence of the first, second and third significant digit with Benford's logarithmic law. If the location of the galaxies in our Universe and the stars in our galaxy are caused by uncorrelated random processes, Benford's law might not be followed because each digit would be equiprobable to appear. To our knowledge this is the first paper that attempts to correlate cosmological observables with Benford's law.

3. Applications to astronomy

Cosmological data with accurate measurements of celestial objects are available since the 1970s. We examine if the frequencies of occurrence of the first digits of the distances of galaxies and stars follow Benford's law.

3.1 *Galaxies*

We use the measured distances of the galaxies from references (Gurugubelli *et al.* 2008; Bartel 1985a, b, 1988; Chugaj 1987; Hoeflich 1987; Wagoner *et al.* 1988; Chilukuri *et al.* 1987; Schmidt 1992; Schmidt-Kaler 1992; Schmidt *et al.* 1992, 1993a, b, 1994a, b; Fernley *et al.* 1998; Panagia 1998, 1999; Garnavich *et al.* 1999; Leonard *et al.* 2002a, b, 2003; Dessart *et al.* 2006, 2008; Takats 2012; Sonnenorn *et al.* 1997; Gould *et al.* 1998; Romaniello *et al.* 2000; Hamuy *et al.* 2001; Mitchell *et al.* 2002; Bartel *et al.* 2003, 2007; Nugent *et al.* 2006; Baron *et al.* 1993, 1994, 1995, 1996, 2007; Poaznanski *et al.* 2009; Jones *et al.* 2009; Kasen *et al.* 2009; D'Andrea *et al.* 2010; Olivares *et al.* 2010; Roy *et al.* 2011; Kirshner *et al.* 1974;

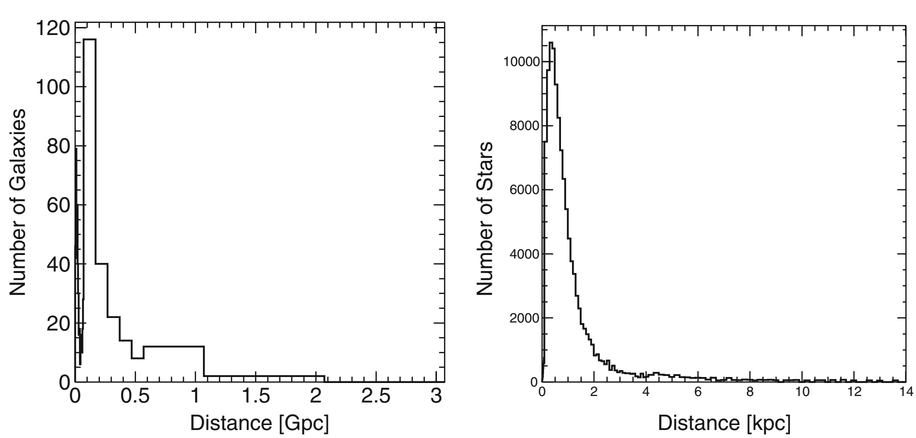

Figure 2. Complete dataset from where the measurements for the galaxies (left) and stars (right) are shown.

Branch *et al.* 1981; Panagia *et al.* 1986, 1991; Eastman *et al.* 1989, 1996; Schmutz *et al.* 1990; McCall 1993; Gould 1994, 1995; Sparks 1994; Branchini *et al.* 1994; Iwamoto *et al.* 1994; Crotts *et al.* 1995; Clocchiatti *et al.* 1995; Richmond *et al.* 1996; Botticella *et al.* 2010; Van Dyk *et al.* 2006; Walker 1998; Hanuschik *et al.* 1991; Hendry *et al.* 2006; Nadyozhin 2003; Vinko *et al.* 2006; Takats *et al.* 2006; Fraser *et al.* 2011; Inserra *et al.* 2012; Elmhamdi *et al.* 2003). The distances considered on this dataset are based on measurements from Type II Supernova and all the units are chosen to be Mpc. The Type-II supernova (SNII) radio standard candle is based on the maximum absolute radio magnitude reached by these explosions, which is 5.5×10^{23} ergs/s/Hz (Fig. 2).

The total number of galaxies selected is 702 with distances reaching 1660 Mpc (see Fig. 2(left)). The results can be seen in Fig. 3(a) where we denote Benford's law

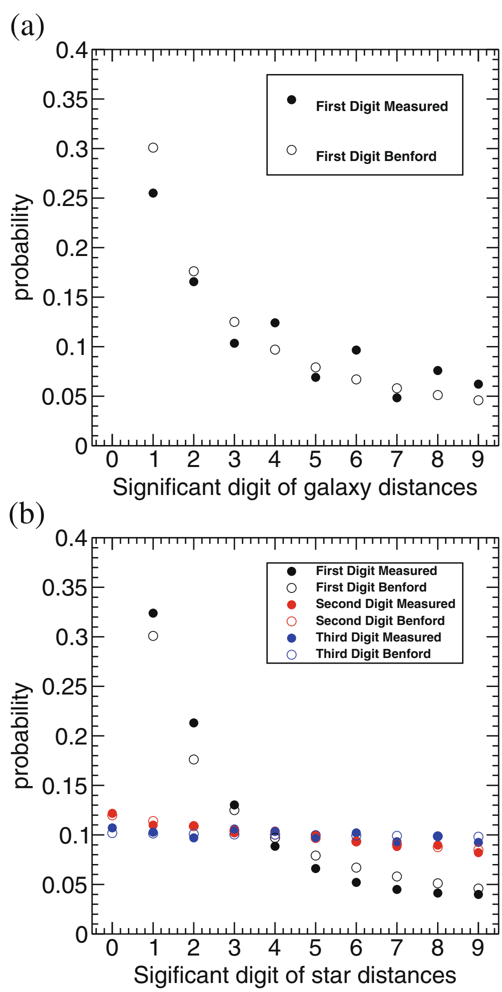

Figure 3. Comparisons of Benford's law (open circles) and the distribution of the first (black), second (red) and third (blue) significant digit of the distances of the (**a**) galaxies and (**b**) stars (full circles).

predictions with open circles and the measurements with filled circles. Unfortunately due to lack of statistics the second and the third significant digits cannot be analysed. The trend of the distribution tends to follow Benford's prediction reasonably well.

3.2 *Stars*

The information for the distances of the stars is taken from the HYG database (2011). In this list, 115 256 stars are included with distances reaching up to 14 kpc. The full dataset used for the extraction of the result can be seen in Fig. 2(right). The result after analysing this dataset can be seen in Fig. 3(b). The first (filled circles) and especially the second (filled red circles) and the third (filled blue circles) significant digits follow well the probabilities predicted by Benford's law (open circles).

4. Summary

The Benford law of significant digits was applied for the first time to astronomical measurements. It is shown that the stellar distances in the HYG database follow this law quite well for the first, second and third significant digits. Also, the probabilities of the first significant digit of galactic distances using the Type II supernova photosphere method is in good agreement with the Benford distribution; however, the errors are sufficiently large so that additional digits cannot be analysed. We note, however, that the plots in Fig. 3(a) indicate that selection effects due to the magnitude limits of both samples may be responsible for this behaviour and so it is not firmly established. Therefore it is necessary to repeat this study using different galactic distance measures and larger catalogs of both galaxies and stars to see if the Benford law is still followed when larger distances are probed. Such larger samples of galaxies would also allow the examination of second and perhaps third significant digits.

Acknowledgements

We would like to thank I. P. Karananas for lengthy discussions on this subject. We would also like to thank Emeritus Professor Anastasios Filippas, and the reviewer for valuable comments and suggestions. The present work was co-funded by the European Union (European Social Fund ESF) and Greek national funds through the Operational Program Education and Lifelong Learning of the National Strategic Reference Framework (NSRF) 2007-1013 ARISTEIA-1893-ATLAS MICROMEGAS.

References

Baron, E. *et al.* 1993, Interpretation of the Early Spectra of SN 1993J in M81, *The Astrophys. J.*, **416**, 21–23.

Baron, E. *et al.* 1994, Modeling and Interpretation of the Optical and HST UV Spectrum of SN 1993J, *The Astrophys. J.*, **426**, 334–339.

Baron, E. *et al.* 1995, Non-LTE Spectral Analysis and Model Constraints on SN 1993J, *The Astrophys. J.*, **441**, 170–181.

Baron, E. *et al.* 1996, Prelimianry spectral analysis of SN 1994I, *Mon. Not. R. Astron. Soc.*, **279**, 799–803.

Baron, E. *et al.* 2007, Reddening, Abundances and Line Formation in SNe II, *The Astrophys. J.*, **662**, 1148–1155.

Bartel, N. 1985a, Angular diameter determinations of radio supernovae and the distance scale, Supernovae as distance indicators; Proceedings of the Workshop, Cambridge, MA, September 27, 28, 1984 (A86-38101 17-90), Berlin and New York, Springer-Verlag, 107–122.

Bartel, N. 1985b, Hubble's constant determined using very-long baseline interferometry of a supernova, *Nature*, **318**, 25–30.

Bartel, N. 1988 Determinations of distances to radio sources with VLBI, The impact of VLBI on astrophysics and geophysics; Proceedings of the 129th IAU Symposium, Cambridge, MA, May 10-15, 1987 (A89-13726 03-90), Dordrecht, Kluwer Academic Publishers, 175–184.

Bartel, N. *et al.* 2003, SN 1979C VLBI: 22 Years of Almost Free Expansion, *The Astrophys. J.*, **591**, 301–315.

Bartel, N. *et al.* 2007, SN 1993J VLBI. IV. A Geometric Distance to M81 with the Expanding Shock Front Method, *The Astrophys. J.*, **668**, 924–940.

Benford, F. 1938, The law of anomalous numbers, *Proc. Am. Phil. Soc.*, **78**, 551.

Botticella, M. T. *et al.* 2010, Supernova 2009kf: An Ultraviolet Bright Type IIP Supernova Discovered with Pan-Starrs 1 and GALEX, *The Astrophys. J. Lett.*, **717**, 52–56.

Boyle, J. 1994, An application of Fourier series to the most significant digit problem, *The Amer. Math. Mon.*, **101(9)**, 879–886.

Branch, D. *et al.* 1981, The Type II Supernova 1979c in M100 and the Distance to the Virgo Cluster, *The Astrophys. J.*, **244**, 780–804.

Branchini, E. *et al.* 1994, Testing the Lease Action Principle in an $\Omega_0 = 1$ Universe, *The Astrophys. J.*, **434**, 37–45.

Buck, B. *et al.* 1993, An illustration of Benford's first digit law using alpha decay half lives, *Eur. J. Phys.*, **14**, 59.

Chilukuri, M. *et al.* 1987, Type-II Supernova Photospheres and the Distance to Supernova 1987A, Atmospheric Diagnostics of Stellar Evolution. Chemical Peculiarity, Mass Loss, and Explosion. Proceedings of the 108th Colloquium of the International Astronomical Union, held at the University of Tokyo, Japan, September 1–4, 1987, Lecture Notes in Physics, Volume 305, ed. K. Nomoto; Publisher, Springer-Verlag, Berlin, New York, 1988. ISBN # 3-540-19478-9. LC # QB806. I18 1987, p. 295.

Chugaj, N. N. 1987, Supernova 1987A - Ha Profile and the Distance of the Large Magellanic Cloud, Astronomicheskii Tsirkulyar No. 1494/May.

Clippe, P., Ausloos, M. 2012, Benford's law and Theil transform on financial data, arXiv:1208.5896v1.

Clocchiatti, A. *et al.* 1995, Spectrophotometric Study of SN 1993J at First Maximum Light, *The Astrophys. J.*, **446**, 167–176.

Crotts, A. P. S. *et al.* 1995, The Circumstellar Envelope of SN 1987A. I, The Shape of the Double-Lobed Nebula and its Rings and the Distance to the Large Maggelanic Cloud, *The Astrophys. J.*, **438**, 724–734.

De, A. S., Sen, U. 2011, Benford's law detects quantum phase transitions similarly as earthquakes, *Europhys. Lett.*, **95**, 50008.

Dessart, L. *et al.* 2006, Quantitative spectroscopic analysis of and distance to SN1999em, *Astron. Astrophys.*, **447(2)**, 691–707.

Dessart, L. *et al.* 2008, Using Quantitative Spectroscopic Analysis to Determine the Properties and Distances of Type II Plateau Supernovae: SN 2005cs and SN 2006bp, *The Astrophys. J.*, **675**, 644–669.

Dong-Dong, N I *et al.* 2009, Benford's law and β-decay half-lives, *Commun. Theor. Phys.*, **51(4)**, 713–716.

D'Andrea, C. B. *et al.* 2010, Type II-P Supernovae from the SDSS-II Supernova Survey and the Standardized Candle Method, *The Astrophys. J.*, **708**, 661–674.

Eastman, R. G. *et al.* 1989, Model Atmospheres for SN 1987A and the Distance to the Large Magellanic Cloud, *The Astrophys. J.*, **347**, 771–793.

Eastman, R. G. *et al.* 1996, The Atmospheres of Type II Supernovae and the Expanding Photosphere Method, *The Astrophys. J.*, **466**, 911–937.

Elmhamdi, A. *et al.* 2003, Photometry and Spectroscopy of the Type IIP SN 1999em from Outburst to Dust Formation, *Mon. Not. R. Astron. Soc.*, **338**, 939–956.

Fernley, J. *et al.* 1998, The absolute magnitudes of RR Lyraes from HIPPARCOS parallaxes and proper motions, *Astron. Astrophys.*, **330**, 515–520.

Fraser, M. *et al.* 2011, SN 2009md: Another Faint Supernova from a Low-Mass Progenitor, *Mon. Not. R. Astron. Soc.*, **417**, 1417–1433.

Garnavich, P. *et al.* 1999, Supernova 1987A in the Large Magellanic Cloud, *IAU Circ.*, **7102**, 1.

Gould, A. 1994, The Ring Around Supernova 1987A Revisited. I. Ellipticity of the Ring, *The Astrophys. J.*, **425**, 51–56.

Gould, A. 1995, Supernova Ring Revisited. II. Distance to the Large Magellanic Cloud, *The Astrophys. J.*, **452**, 189–1994.

Gould, A. *et al.* 1998, Upper Limit to the Distance to the Large Magellanic Cloud, *The Astrophys. J.*, **494**, 118–124.

Gurugubelli *et al.* 2008, Photometric and spectroscopic evolution of type II-P supernova SN 2004A, *Bull. Astron. Soc. India*, **36(2-3)**, 79–97.

Hamuy, M. *et al.* 2001, The Distance to SN 1999em from the Expanding Photosphere Method, *The Astrophys. J.*, **558**, 615–642.

Hanuschik, R. W. *et al.* 1991, Absorption Line Velocities and the Distance to Supernova 1987A, *Astron. Astrophys.*, **249**, 36–42.

Hendry, M. A. *et al.* 2006, SN 2004A: Another Type II-P Supernova with a Red Supergiant Progenitor, *Mon. Not. R. Astron. Soc.*, **369**, 1303–1320.

Hill, T. P. 1995a, Base-Invariance Implies Benford's Law, *Proc. Amer. Math. Soc.*, **123.3**, 887–895.

Hill, Theodore P. 1995b, The significant-digit phenomenon, *The Amer. Math. Mon.*, **102(4)**, 322–327.

Hoeflich, P. 1987, Model calculations for scattering dominated atmospheres and the use of supernovae as distance indicators, Nuclear astrophysics; Proceedings of the Workshop, Tegernsee, Federal Republic of Germany, Apr. 21–24, 1987 (A89-10202 01-90), Berlin and New York, Springer-Verlag, 307–315.

Hopee, F. M. 2013, Benford's law and distractors in multiple choice exams, arXiv:1311.7606.

Inserra, C. *et al.* 2012, Quantitative Photospheric Spectral Analysis of the Type IIP Supernova 2007od, *Mon. Not. R. Astron. Soc.*, **422**, 1178–1185.

Iwamoto, K. *et al.* 1994, Theoretical Light Curves for the Type Ic Supernova SN 1994I, *The Astrophys. J.*, **437**, 115–118.

Jones, M. I. *et al.* 2009, Distance Determination to 12 Type II Supernovae Using the Expanding Photosphere Method, *The Astrophys. J.*, **696**, 1176–1194.

Kasen, D. *et al.* 2009, Type II Supernovae: Model Light Curves and Standard Candle Relationships, *The Astrophys. J.*, **703**, 2205–2216.

Kirshner, R. P. *et al.* 1974, Distances to Extragalactic Supernovae, *The Astrophys. J.*, **193**, 27–36.

Leonard, Douglas C. *et al.* 2002a, A Study of the Type II-Plateau Supernova 1999gi and the Distance to its Host Galaxy, NGC 3184, *The Astron. J.*, **124(5)**, 2490–2505.

Leonard, D. C. *et al.* 2002b, The Distance to SN 1999em in NGC 1637 from the Expanding Photosphere Method, *Publ. Astron. Soc. Pacific*, **114**, 35–64.

Leonard, D. C. *et al.* 2003, The Cepheid Distance to NGC 1637: A Direct Test of the Expanding Photosphere Method Distance to SN 1999em, *The Astrophys. J.*, **594**, 247–278.

McCall, M. L. 1993, The Distance to the Large Magellanic Cloud from SN 1987A, *The Astrophys. J.*, **417**, 75–77.

Mitchell, R. C. *et al.* 2002, Detailed Spectroscopic Analysis of SN 1987A: The Distance to the Large Magellanic Cloud Using the Spectral-Fitting Expanding Atmosphere Method, *The Astrophys. J.*, **574**, 293–305.

Nadyozhin, D. K. 2003, Explosion Energies, Nickel Masses and Distances of Type II Plateau Supernovae, *Mon. Not. R. Astron. Soc.*, **346**, 97–104.

Newcomb, S. 1881, Note on the frequency of use of the different digits in natural numbers, *Am. J. Math.*, **4(1/4)**, 39–40.

Nugent, P. *et al.* 2006, Toward a Cosmological Hubble Diagram for Type II-P Supernovae, *The Astrophys. J.*, **645**, 841–850.

Olivares, F. E. *et al.* 2010, The Standardized Candle Method for type II Plateau Supernovae, *The Astrophys. J.*, **715**, 833–853.

Pain, J. C. 2008, Benford's law and complex atomic spectra, *Phys. Rev. E*, **77**, 012102.

Panagia, N. 1998, New Distance Determination to the LMC, *Memorie della Societa Astronomia Italiana*, **69**, 225.

Panagia, N. 1999, Distance to SN 1987 A and the LMC, New Views of the Magellanic Clouds, IAU Symposium #190, (eds) Y.-H. Chu, N. Suntzeff, J. Hesser & D. Bohlender, ISBN: 1-58381-021-8, p. 549.

Panagia, N. *et al.* 1986, Subluminous, Radio Emitting Type I Supernovae, *The Astrophys. J.*, **300**, 55–58.

Panagia, N. *et al.* 1991, Properties of the SN 1987A Circumstellar Ring and the Distance to the Large Magellanic Cloud, *The Astrophys. J.*, **380**, 23–26.

Poaznanski, D. *et al.* 2009, Improved Standardization of Type II-P Supernovae: Application to an Expanded Sample, *The Astrophys. J.*, **694**, 1067–1079.

Richmond, M. W. *et al.* 1996, $UBVRI$ Photometry of the Type Ic SN 1994I in M51, *The Astron. J.*, **111(1)**.

Romaniello, M. *et al.* 2000, Hubble Space Telescope Observations of the Large Magellanic Cloud Field Around SN 1987A: Distance Detetrmination with Red Clump and Tup of the Red Giant Branch Stars, *The Astrophys. J.*, **530**, 738–743.

Roy, R. *et al.* 2011, SN 2008in – Bridging the Gap Between Normal and Faint Supernovae of Type IIP, *The Astrophys. J.*, **736**, 76.

Schmidt, B. P. 1992, Expanding Photospheres of Type II Supernovae and the Extragalactic Distance Scale, American Astronomical Society, 181st AAS Meeting, #107.04D, *Bull. Amer. Astron. Soc.*, **24**, 1292.

Schmidt-Kaler, T. 1992, The Distance to the Large Magellanic Cloud from Observations of SN1987A, Variable Stars and Galaxies, in honour of M. W. Feast on his retirement, ASP Conference Series, (ed.) Warner B., vol. 30 p. 195.

Schmidt, B. P. *et al.* 1992, Expanding Photospheres of Type II Supernovae and the Extragalactic Distance Scale, *The Astrophys. J.*, **395**, 366–386.

Schmidt, Brian P. *et al.* 1993a, The unusual supernova SN1993J in the galaxy M81, *Nature*, **364**, 600–602.

Schmidt, Brian P. *et al.* 1993b, Type II Supernovae, Expanding Photospheres, and the Extragalactic Distance Scale, Thesis (Ph.D.) – Harvard University, 1993, Source: Dissertation Abstracts International, Volume 54–11, Section B, 5717.

Schmidt, B. P. *et al.* 1994a, The Expandind Photosphere Method Applied to SN 1992am At $cz = 14600\,\mathrm{km/s}$, *The Astron. J.*, **107**, 4.

Schmidt, B. P. *et al.* 1994b, The Distance to Five Type II Supernovae Using the Expanding Photosphere Method and the Value of H_0, *The Astrophys. J.*, **432**, 42–48.

Schmutz, W. *et al.* 1990, NON-LTE Model Calculations for SN 1987A and the Extragalactic Distance Scale, *The Astrophys. J.*, **355**, 255–270.

Shao, L., Ma, B. Q. 2009, First digit distribution of hadron full width, *Mod. Phys. Lett. A*, **24**, 3275–3282.

Shao, L., Ma, B. Q. 2010, First-digit law in nonextensive statistics, *Phys. Rev. E*, **82(041110)**.

Sparks, W. B. 1994, A Direct Way to Measure the Distances of Galaxies, *The Astrophys. J.*, **433**, 19–28.

Sonnenorn, G. *et al.* 1997, The Evolution of Ultraviolet Emission Lines from Circumstellar Material Surrounding SN 1987A, *The Astrophys. J.*, **477**, 848–864.

Takats, V. J. 2012, Improved distance determination to M 51 from supernovae 2011dh and 2005cs, *Astron. Astrophys.*, **540(A93)**, 7.

Takats, K. *et al.* 2006, Distance Estimate and Progenitor Characteristics of SN 2005cs in M51, *Mon. Not. R. Astron. Soc.*, **372**, 1735–1740.

Takats, K. *et al.* 2012, Measuring Expansion Velocities in Type II-P Supernovae, *Mon. Not. R. Astron. Soc.*, **419**, 2783–2796.

The HYG Database 2011, http://www.astronexus.com.

Van Dyk, S. D. *et al.* 2006, The Light Echo Around Supernova 2003gd in Messier 74, *Publ. Astron. Soc. Pacific*, **118**, 351–357.

Vinko, J. *et al.* 2006, The First Year of SN 2004dj in NGC 2403, *Mon. Not. R. Astron. Soc.*, **369**, 1780–1796.

Wagoner, R. V. *et al.* 1988, Supernova 1987A: A Test of the Improved Baade Method of Distance Determination, *BAAS 08/1988*, **20**, 985.

Walker, A. R. 1998, The Distances of the Maggelanic Clouds, arXiv:astro-ph/9808336v1.

Wlodarksi, J. 1971, Fibonacci and Lucas numbers tend to obey Benford's law, Westhoven, Federal Republic of Germany.

Wojcik, M. R. 2013, Notes on scale-invariance and base-invariance for Benford's law, arXiv:1307.3620.