J. Astrophys. Astr. (2014) **35,** 595–605

Measuring Velocity and Acceleration Using Doppler Shift of a Source with an Example of Jet in SS433

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Received 13 October 2013; accepted 13 June 2014

Abstract. We describe here as to how the Doppler shift of a source needs to be used to measure its velocity and acceleration. We also apply this method, as an example here, to spectral lines of the blue-shifted jet in micro-quasar SS433 and discuss the intricacies of these measurements.

Key words. Doppler effect—measuring velocity and acceleration of the source— jet in SS433.

1. Introduction

Recently, we showed that the Doppler shift of a source must necessarily include contribution from its acceleration, apart from that due to its velocity (Wagh 2013a). We also discussed certain implications of this result (Wagh 2013b, c). Here, we discuss the measurements of velocity and acceleration of the source from its Doppler shift.

For completeness, we first recall the derivation of the Doppler shift from Wagh (2013a). We emphasize that this analysis of Doppler's effect holds in all the possible situations of the relative motion of the source of light and the observer.

Doppler's discovery is that if a source of a (monochromatic) wave is in motion relative to an observer, then the observed wavelength of this wave must be different than that emitted by the source. The Doppler shift results from only the relative motion of the source and the observer. From the Doppler shift of a source, we can infer the nature of its motion with respect to us.

Figure 1(a) shows a stationary source S emitting a monochromatic wave whose crest meets an observer at point O at time $t = 0$, at which instant the subsequent crest is then lagging at point M, located at a distance cT behind the point O along the line SO. Here, T is the period of the wave and c is the speed of the wave (for light, in vacuum).

Let the observer at O move with velocity $\vec{v}$ making an angle θ with the line SO and acceleration $\vec{a}$ at $t = 0$, as shown in Figure 1(a). (In general, velocity and acceleration can be differently directed.) Furthermore, let the wave at M reach the observer at point P after lapse T' of time.

Actually, a ray from S passing through the point Q on the wavefront at M reaches the observer at time T', as is shown in Figure 1(a). But, the angle $\phi = \angle$MSQ

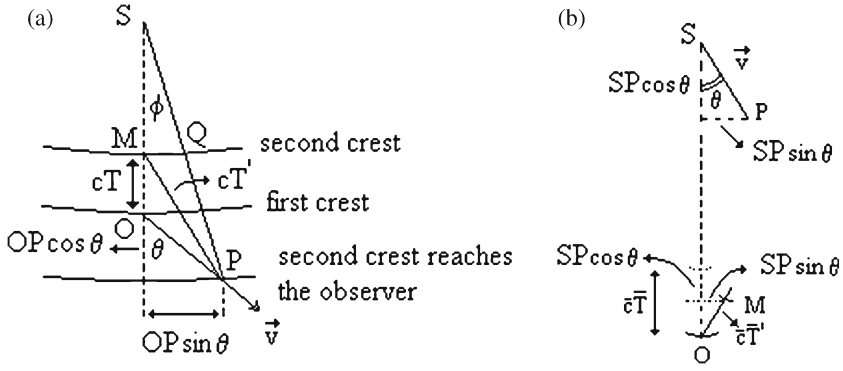

Figure 1 (a, b). Geometry of the Doppler effect.

is exceedingly small when the source is very distant from the observer. Then, MP = QP to a high degree of accuracy; and we may use MP for QP in further computations.

Then, from simple geometrical considerations, we have

$$(\mathrm{MP})^2 \equiv c^2 T'^2 = c^2 T^2 + 2cT\,(\mathrm{OP})\cos\theta + (\mathrm{OP})^2. \tag{1}$$

Figure 1(b) now shows the observer as stationary and the source as moving (towards the observer) with velocity $\vec{v}$ making an angle θ with the line SO and with acceleration $\vec{a}$. Then, we can write

$$(\mathrm{MO})^2 \equiv \bar{c}^2 \bar{T}'^2 = \bar{c}^2 \bar{T}^2 - 2\bar{c}\bar{T}\,(\mathrm{SP})\cos\theta + (\mathrm{SP})^2, \tag{2}$$

where $\bar{c}$ is the speed of wave (for light, in vacuum), $\bar{T}$ is the period of the wave emitted by the source, and $\bar{T}'$ is the measured period, all in a *new* frame of reference.

We are not required to 'specify' as to how the observer has traveled the distance (OP) to write eq. (1) and how the source has traveled the distance (SP) to write eq. (2). These equations thus hold very generally. (Difference in the signs of cosine terms of these equations is because observer and source are shown moving away from each other in Fig. 1(a) and are shown moving towards each other in Figure 1(b).

Now, if the speed of the wave in the two frames of reference considered above is the same, as is to be for light, then we have $c = \bar{c}$. Then, when motion of source in Fig. 1(b) is 'identical' to that of the observer in Fig. 1(a), we need the equality of $(\mathrm{OP})/c$ and $(\mathrm{SP})/c$. Otherwise, the observer will be able to measure self-motion using the difference between T' and $\bar{T}'$, that is, using Doppler effect in light.

Then, general principle that no observer 'measures' self-motion using physical effects, which is a form of the general principle of relativity, demands $T = \bar{T}$ and $T' = \bar{T}'$. Then, time runs at the same rate for the observer and the source. Thus, Doppler effect demands that time is universal, and not relative. We emphasize then that, we do not require Lorentz-type transformations to obtain the Doppler shift assuming, additionally to the constancy of speed of light, certain special (inertial) kind of observers.

We also stress that only the constancy of the speed of light for all observers is being used in the general solution, and assumptions like the constancy of the

acceleration, as adopted in the following, are incidental to obtaining manageable equations.

Now, for time interval equal to the period of the wave, constancy of the acceleration may be a good approximation in situations of our interest.

For constant acceleration $\vec{a}$ and initial velocity $\vec{u}$, the geometry now dictates that

$$(cT')^2 = \left(uT' + \frac{1}{2}aT'^2\right)^2 \sin^2\theta + \left[cT + \left(uT' + \frac{1}{2}aT'^2\right)\cos\theta\right]^2. \quad (3)$$

On manipulating this equation, in terms of the observed frequency $\nu_o = 1/T'$ and the emitted frequency $\nu_e = 1/T$, we have

$$\nu_o^4 + (2\beta\nu_e\cos\theta)\nu_o^3 - \left[(1-\beta^2)\nu_e^2 - \alpha\nu_e\cos\theta\right]\nu_o^2 + \alpha\beta\nu_e^2\nu_o + \frac{\alpha^2}{4}\nu_e^2 = 0, \quad (4)$$

where we have set $\alpha = a/c$ and $\beta = u/c$.

To first order in α and neglecting the term with product $\alpha\beta$, the observed frequency is obtained using equation (4) as

$$\begin{aligned}\nu_o &= \nu_e\left[\sqrt{1-\beta^2\sin^2\theta - \left(\frac{\alpha}{\nu_e}\right)\cos\theta} - \beta\cos\theta\right] \\ &\approx \nu_e\left[1 - \frac{\beta^2}{2}\sin^2\theta - \beta\cos\theta\right] - \frac{\alpha}{2}\cos\theta. \end{aligned} \quad (5)$$

In section 2, we will focus on formula (5) for further analysis. Also, refer to the analysis in Appendix corresponding to the case of uniform velocity, and note that the standard Doppler shift expression (A1) disagrees with equation (5) when $\alpha = 0$.

2. Strategy underlying measurement: Constant acceleration

Consider that the source under considerations is emitting two spectral lines at frequencies $\nu_{e(1)}$ and $\nu_{e(2)}$ and the corresponding observed frequencies are $\nu_{o(1)}$ and $\nu_{o(2)}$, respectively. Then, using equation (5), we obtain

$$\begin{aligned} f(\beta,\theta) \equiv 1 - \frac{\beta^2}{2}\sin^2\theta - \beta\cos\theta &= \frac{\nu_{o(2)} - \nu_{o(1)}}{\nu_{e(2)} - \nu_{e(1)}} \\ &= \left[\frac{\lambda_{o(1)} - \lambda_{o(2)}}{\lambda_{e(1)} - \lambda_{e(2)}}\right] \times \frac{\lambda_{e(1)}\lambda_{e(2)}}{\lambda_{o(1)}\lambda_{o(2)}}, \end{aligned} \quad (6)$$

where the λ's are the corresponding wavelengths.

The function $f(\beta,\theta) = 1 - (\beta^2\sin^2\theta/2) - \beta\cos\theta$ is then the quantity measured here in terms of the observed and emitted frequencies or wavelengths, and this function is plotted in Fig. 2 as a surface.

At this point, notice that for angular ranges $\sim$[0, 110°] and $\sim$[240°, 360°], we have $f(\beta,\theta) < 1$; and within the angular range $\sim$[110°, 240°], we have $2 \geq f(\beta,\theta) > 1$. We stress that $f > 1$ does not necessarily mean relativistic velocity. These ranges also overlap: for [90°, 110°] and [250°, 270°], $f > 1$ and $f < 1$.

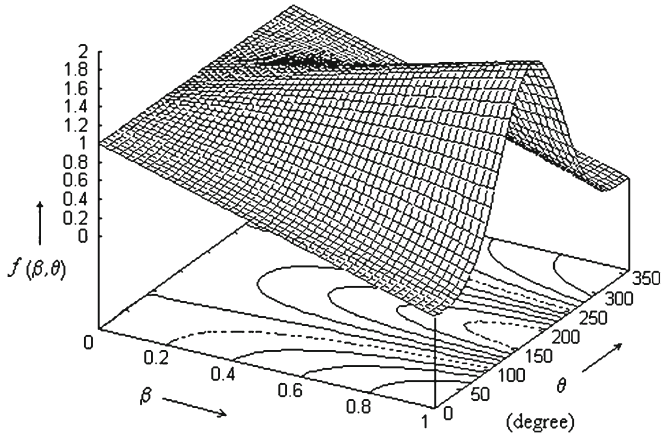

Figure 2. Function $f(\beta, \theta)$ in equation (6) as a surface.

Notice also that when $f(\beta, \theta) \neq 1$, there exists a 'non-zero' lower bound, $\beta_{\min}$, for velocity β. For $f < 1$, $\beta_{\min} = 1 - f$ and for $f > 1$, $\beta_{\min} = f - 1$. For convenience, we then plot the contours of $f(\beta, \theta) =$ constant in Fig. 3. For $f(\beta, \theta) = 1$, $\beta_{\min} = 0$ because $\beta = 0$ for all values of the angle θ. Importantly, for $f(\beta, \theta) \approx 1$, velocity can be non-relativistic over a large angular range. Then, from the observational data of $\left(\lambda_{e(1)}, \lambda_{e(2)}, \lambda_{o(1)}, \lambda_{o(2)}\right)$, we obtain the 'observed' value of $f(\beta, \theta)$, which we denote by $\{f\}$.

Now, from equation (5), we have

$$\frac{\alpha}{2c} \cos\theta = \frac{\{f\}}{\lambda_e} - \frac{1}{\lambda_o} \tag{7}$$

and, as we are using the 'observed' quantities, this is then the 'observed' value. We denote this observed value by $\{\alpha \cos\theta/2c\}$.

From the data related to just a pair of spectral lines, we can then infer the values of f, $\beta_{\min}$ and $(\alpha \cos\theta)/2c$. Furthermore, from $f > 1$ or $f < 1$, we can also infer as to how the material emitting that pair of lines is, likely to be, angularly situated.

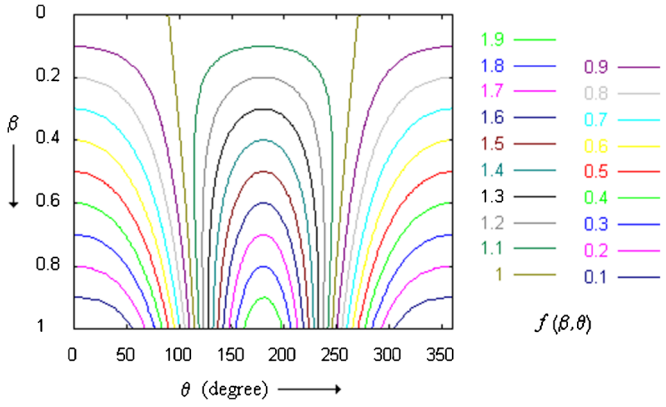

Figure 3. β as a function of θ for various values of $f(\beta, \theta)$.

Of course, this information can be considered to be 'reliable' only if that pair of spectral lines is emitted by the 'same' material, with the reality of this situation not being decidable on the basis of that information.

For more than a pair of spectral lines, we note that error in the observed value of $f(\beta, \theta)$ obtained for a selected pair of lines 'naturally' provides us the basis for further analysis in the following manner.

To fix ideas, let

$$f = f_O \pm \delta f \tag{8}$$

for a 'specific' pair of spectral lines $(\lambda_{e(1)}, \lambda_{e(2)})$.

Now, all other spectral lines whose function f, evaluated using the line of wavelength $\lambda_{e(1)}$, has values in the range $[f_O + \delta f, f_O - \delta f]$ can be considered to be emitted by the material moving 'similarly' to that emitting the first spectral line $\lambda_{e(1)}$. These lines possess 'differing' values of $\beta_{\min}$, in general. Each $\beta_{\min}$ will have associated error of $\pm\delta f$. For consistency with the existence of the minimum value of velocity $\beta_{\min}$ for $f(\beta, \theta) \neq 1$, it is required that the material emitting these lines to be moving with speed corresponding to the 'maximum' value $(\max \beta_{\min})$, of $\beta_{\min}$; and will then have corresponding error.

Here, $(\max \beta_{\min})$ is the 'smallest' value of the speed β with which the material can be moving and emitting the spectral lines under considerations. We may then use it as 'the value' of β (with corresponding error) in the expression for the function $f(\beta, \theta)$ to infer the angle of motion of the material emitting the line as

$$\cos\theta = \frac{1}{(\max \beta_{\min})}\left(1 - \sqrt{1 + (\max \beta_{\min})^2 + 2(f-1)}\right). \tag{9}$$

The observed value of $(\alpha/2c)\cos\theta$ as in eq. (7) then provides us the value of acceleration of the material emitting the line. Corresponding values of angle θ then provide us the angular spread of the material emitting these lines.

Of course, the analysis is certainly much more complicated than that presented above. In reality, different pairs of lines from a source will yield drastically differing results for the above analysis. In particular, the angles obtained with one pair of lines will be seen to be quite different when another pair of lines is used. Nevertheless, the following part of the strategy emerges on relevant considerations.

Then, a pair of lines that accommodates maximum number of other lines from the source within its error limits for function $f(\beta, \theta)$ will, most likely, be representing the source. This is so because lines from a multi-temperature plasma moving at bulk speed β and accelerating/decelerating at rate α can be expected to show this feature. Note also that results for angles obtained with the acceleration term need to be in 'general conformity' with those obtained without it.

We now turn to the example of the blue-shifted jet in micro-quasar SS433.

3. Example of the blue-shifted jet in micro-quasar SS433

SS433 shows (Margon 1984; Mirabel 1999; Margon & Anderson 1989; Fabrika 2004) emission lines from ionized gas in a highly collimated jet ejected by a compact object and is a member of the 'micro-quasar' class of X-ray binaries. The 'kinematic model' of SS433 (Margon & Anderson 1989) describes Doppler shifts of the blue- and red-shifted H_α lines as a result of a pair of oppositely directed precessing jets

with speed, $v_J = 0.26c$. As per this analysis, the jet in SS433 precesses with a 162.5 day period in a cone with half-angle of 19.85° about an axis which is 78.83° to the line-of-sight. Marshall *et al*. (2013) also reported multi-wavelength observations of the Jet in SS433.

In what follows, we show how the acceleration dependence of the Doppler shift changes the estimate of the speed of the jet.

From Table 4 in Marshall *et al*. (2013), consider the line Fe XXV of rest wavelength $\lambda_{e(1)} = 1.855$ Å and observed wavelength $\lambda_{o(1)} = 1.86 \pm 0.001$ Å, and line Fe XXVI of rest wavelength $\lambda_{e(2)} = 1.78$ Å and observed wavelength $\lambda_{o(2)} = 1.786 \pm 0.001$ Å.

Then, for the pair of these lines, we have

$$\{f\} = 0.981 \pm 0.001, \quad \beta_{\min} = 0.019 \pm 0.001, \quad \beta_{\perp} = 0.195 \pm 0.005$$
$$\beta_{78.83^\circ} = 0.082 \pm 0.004, \quad \left\{\frac{\alpha}{2}\cos\theta\right\} = -0.0088 \pm 0.0001.$$

We may then conclude that the material emitting the Fe XXV and Fe XXIV lines in the blue-shifted jet of SS433 is moving with speed $\sim 0.08c$ and is decelerating.

Considering the next two emission lines S XVI ($\lambda_{e(1)} = 4.729$ Å, $\lambda_{o(1)} = 4.735 \pm 0.001$ Å) and S XV ($\lambda_{e(2)} = 5.055$ Å, $\lambda_{o(2)} = 5.050 \pm 0.002$ Å) of sulphur S, we have

$$\{f\} = 0.966 \pm 0.003, \quad \beta_{\min} = 0.034 \mp 0.003, \quad \beta_{\perp} = 0.261 \pm 0.044$$
$$\beta_{78.83^\circ} = 0.132 \pm 0.001, \quad \left\{\frac{\alpha}{2}\cos\theta\right\} = -0.007 \pm 0.001.$$

We may then conclude that the material emitting the S XVI and S XV lines in the blue-shifted jet of SS433 is moving with speed ∼0.13c and is decelerating. However, for the same angle, it is moving somewhat faster and decelerating somewhat slower than the material of the jet emitting the Fe XXV and Fe XXIV lines.

But, aforementioned conclusions are entirely premature, for they have been obtained using only a pair of spectral lines. There is no way we can be sure about the commonness of the material emitting the pair under considerations!

A proper analysis then uses (Wagh & Sarwe 2013) the bounds on function $f(\beta, \theta)$ from a pair of lines to determine which lines have their f-value within these bounds, checks for the consistency of the results with the angular distribution obtained without the acceleration-dependence of the Doppler shifts, etc.

In this analysis, it is however seen (Wagh & Sarwe 2013) that the value of acceleration obtained for the material of the jet in SS433 is unreasonably large. This is an indication that the constancy of acceleration as is used to obtain eq. (5) is not a good approximation in the situation under consideration. Consequently, it is then necessary to consider variable acceleration. The next section therefore describes the relevant strategy.

4. Strategy with temporally variable acceleration

Let us consider the simple situation of acceleration varying sinusoidally with time. This is not any unreasonable assumption, for a train carriage being pulled or pushed by an engine exhibits this feature. As the coupling between the engine and the

carriage slackens, friction at the rails decelerates the carriage; and when it gets tout, carriage accelerates due to the pull/push of the engine. This process repeats periodically, as the engine pulls/pushes the train. We expect it also in systems like SS433, wherein the mechanisms of acceleration and deceleration of the material are operating simultaneously. The material of the jet is being pushed from behind by the material ejected by the prime mover at a later time and is being obstructed by material the jet is ramming into.

Then, let the temporal rate of acceleration be given by

$$\frac{\mathrm{d}a}{\mathrm{d}t} = a_0 \sin(\omega t), \quad a_0 = \text{constant}. \tag{10}$$

Then, on integration, we obtain

$$(\mathrm{SP}) = \frac{a_0}{\omega^3}\cos(\omega t) + k_1\frac{t^2}{2} + k_2 t + k_3, \tag{11}$$

where k_1, k_2, k_3 are integration constants. When the frequency ω is appropriately small, we may approximate the cosine term and obtain

$$(\mathrm{SP}) = \frac{a_0}{\omega^3} + \left(k_1 - \frac{a_0}{\omega}\right)\frac{t^2}{2} + k_2 t + k_3. \tag{12}$$

Substituting this in eq. (2), we then obtain after suitable manipulations:

$$\nu_\mathrm{o} \approx f(\beta,\theta)\nu_\mathrm{e} - \frac{1}{c}\left(k_3 + \frac{a_0}{\omega^3}\right)\nu_\mathrm{e}^2\cos\theta - \frac{1}{2c}\left(k_1 - \frac{a_0}{\omega}\right)\cos\theta, \tag{13}$$

where $f(\beta,\theta)$ is as defined earlier with $k_2 = \beta c$, and have retained only the first order terms in acceleration. We set $k_1 = 0$ and $k_3 = 0$. (Notice that we recover the analysis with constant acceleration when $k_1 \neq 0$ and in the (obvious) situation of $a_0 = 0$.)

Now, let $\Delta\nu_\mathrm{o} = \nu_{\mathrm{o}(1)} - \nu_{\mathrm{o}(2)}$, $\Delta\nu_\mathrm{e} = \nu_{\mathrm{e}(1)} - \nu_{\mathrm{e}(2)}$, $\Delta(\nu_\mathrm{e}^2) = (\nu_{\mathrm{e}(1)})^2 - (\nu_{\mathrm{e}(2)})^2$. Then, from eq. (13), we have

$$\left(\frac{\Delta\nu_\mathrm{o}}{\Delta\nu_\mathrm{e}}\right) = f(\beta,\theta) + h(a_0,\omega,\theta)\left(\frac{\Delta(\nu_\mathrm{e}^2)}{\Delta\nu_\mathrm{e}}\right), \tag{14}$$

where we have set $h(a_0,\omega,\theta) = -(a_0/\omega^3 c)\cos\theta$. This equation is 'linear' in $(\Delta\nu_\mathrm{o}/\Delta\nu_\mathrm{e})$ and $(\Delta(\nu_\mathrm{e}^2)/\Delta\nu_\mathrm{e}) \equiv \nu_{\mathrm{e}(1)} + \nu_{\mathrm{e}(2)}$. From the observational data $(\nu_\mathrm{e}, \nu_\mathrm{o})$ or, equivalently, $(\lambda_\mathrm{e}, \lambda_\mathrm{o})$, we can then obtain $f(\beta,\theta)$ and $h(a_\mathrm{o},\omega,\theta)$ by linear regression. The acceleration a_0/ω of the emitter can then be estimated.

Velocity β_min, corresponding to the above $f(\beta,\theta)$, provides us the minimum velocity with which the material emitting frequency $\nu_{\mathrm{e}(1)}$ of the jet is moving. For the mean jet speed, we may select the average of β_min or (max β_min), and this value is 'added' to each β_min. Then, angle θ can be obtained from the f-values as before. We thus obtain the 'observational parameters' of the kinematics of the material of the jet. Of course, details depend on the parameter ω and further work is necessary to arrive at a satisfactory model.

Nevertheless, we note the following: The order of the term $(\Delta\nu_\mathrm{o}/\Delta\nu_\mathrm{e})$ is unity and so is that of $f(\beta,\theta)$. Then, as the order of the quantity $(\Delta(\nu_\mathrm{e}^2)/\Delta\nu_\mathrm{e})$ is $10^8 c$,

the order of the quantity $h(a_0, \omega, \theta)$ is $10^{-8}c^{-1}$. Thus, the acceleration a_0/ω is of order $10^{-8}\omega^2$, which is a reasonable value. Then, the last term of eq. (13) will be negligible. We may note that the time-scale of change in velocity is now of the order of $\beta c/(a_0/\omega) \sim \beta c 10^8/\omega^2$.

5. Concluding remarks

In summary, Doppler shifts of spectral lines lead to measurements of the velocity and acceleration of the material emitting them. Noticeably, our method of deriving the Doppler shift accommodates situations of constant velocity, constant acceleration and that of variable acceleration by way of eq. (2), as shown. We have also discussed an astrophysical example of the blue-shifted jet from a micro-quasar SS433, including the intricacies of these measurements when the acceleration is variable. The method is, of course, more general than just this application to the jet in SS433.

Since our purpose was to discuss method(s) of measuring velocity and acceleration of a source using the Doppler shift, we do not provide here the details of jet in SS433, which will be found in Wagh and Sarwe (2013).

Acknowledgements

The author would like to thank the anonymous referee for raising the issues of the Appendix and for other useful as well as helpful suggestions.

Appendix

The 'usual' Doppler shift formula is (see, for example, French 1966):

$$\nu_o = \nu_e \frac{\sqrt{1-\beta^2}}{1+\beta\cos\theta}. \tag{A1}$$

This formula has also received experimental validation, for example, in many cosmic ray experiments involving relativistically moving charged particles emitting radiation. We recall the derivation of this formula that is based on the following diagram:

Figure A1(a) shows a stationary source emitting 'first' pulse at time t_1 and the 'second' pulse at time t_2, the time measured in the frame of the source. The frequency of emission is then $1/(t_2 - t_1)$. The 'initial' line-of-sight (SO), is shown making an angle $\bar{\theta}$ with the Y-axis. When the first pulse reaches the observer at O, time elapsed is $T_1 = t_1 + (\mathrm{SO})/c$.

Let the observer move along the line OP making an angle $\bar{\phi}$ with the X-axis, with P being the point at which the second pulse reaches the observer at time $T_2 = t_2 + (\mathrm{SP})/c$. Then,

$$T_2 - T_1 = t_2 + (\mathrm{SP})/c - t_1 - (\mathrm{SO})/c \tag{A2}$$

With $(\mathrm{SO}) = \sqrt{x_1^2 + y_1^2}$, $(\mathrm{SP})^2 = \sqrt{(x_1+\delta x)^2 + (y_1+\delta y)^2}$, $x_1 >> \delta x$, $y_1 >> \delta y$, we get

$$T_2 - T_1 \approx t_2 - t_1 + \cos\bar{\theta}\delta x + \sin\bar{\theta}\delta y \tag{A3}$$

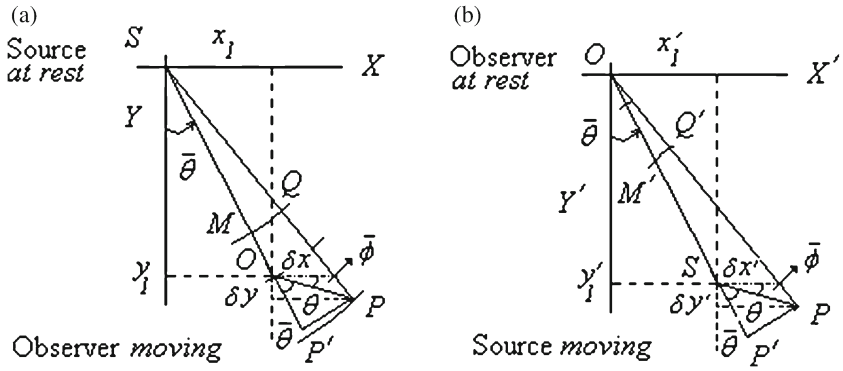

Figure A1(a, b). Geometry for 'usual' formula of Doppler shift.

But, $\delta x = (\mathrm{OP})\cos\bar{\phi}$ and $\delta y = (\mathrm{OP})\sin\bar{\phi}$; and we get to first order

$$T_2 - T_1 \approx t_2 - t_1 + \frac{(\mathrm{OP})}{c}\cos(\bar{\theta} + \bar{\phi}) \equiv t_2 - t_1 + \frac{(\mathrm{OP})}{c}\cos\theta$$
$$c\Delta T = c\Delta t + (\mathrm{OP})\cos\theta, \tag{A4}$$

where θ is the angle line OP makes with line SO.

If we consider the observer to be stationary and source to be moving, considerations similar to those of the above applied to of Fig. A1(b) then yield to first order:

$$T_2' - T_1' \approx t_2' - t_1' + \frac{(\mathrm{SP})}{c}\cos(\bar{\theta} + \bar{\phi}) \equiv t_2' - t_1' + \frac{(\mathrm{SP})}{c}\cos\theta$$
$$c\Delta T' = c\Delta t' + (\mathrm{SP})\cos\theta. \tag{A5}$$

We distinguish the time measured by the observer with an overhead prime.

Notice that we are not required to specify how the observer has moved along line OP or the source has moved along the line SP, that is to say, with uniform velocity or with acceleration. Equations (A4) and (A5) hold generally.

Now, as described on page 142 in French (1966), we can approximate

$$(\mathrm{SP}) = u\Delta t' \tag{A6}$$

for u as 'uniform' velocity. On using equation (A6) in equation (A5), we then obtain

$$\Delta T' = (1 + \beta\cos\theta)\,\Delta t' \quad\Longrightarrow\quad \frac{1}{\Delta T'} \equiv \nu_{\mathrm{o}} = \nu_{\mathrm{e}}\frac{\sqrt{1-\beta^2}}{1+\beta\cos\theta}, \tag{A7}$$

where we have used $\beta = u/c$ and that $\Delta t' = 1/\nu_{\mathrm{e}}\sqrt{1-\beta^2}$ with ν_{e} being the frequency of the emitted wave in the rest frame of the source. This is the standard formula of equation (A1) or equation (5)–(17) on page 143 of French (1966).

But, eq. (A6) *has an error. Distance* (SP) *is 'not'* $u\Delta t'$, *but* $(\mathrm{SP}) = u\Delta T'$. *The source, if at all it does so, moves only during the period* $\Delta T' = T_2' - T_1'$, *and not during* $\Delta t = t_2 - t_1$.

Then, using the 'correct' expression $(\mathrm{SP}) = u\Delta T'$, we obtain

$$\Delta T' = \frac{\Delta t'}{1 - \beta\cos\theta} \implies \nu_o = \nu_e\,(1 - \beta\cos\theta)\sqrt{1-\beta^2}. \tag{A8}$$

However, it is easy to see that this 'correct' special relativistic formula contradicts the results of the Ives–Stilwell experiment.

On calculations for $\theta = 0$ and $\theta = \pi$, we obtain for the first- and second-order wavelength shifts:

$$\Delta^{(1)}\lambda = \lambda_e\beta, \quad \Delta^{(2)}\lambda = \frac{3}{2}\lambda_e\beta^2 \implies \Delta^{(2)}\lambda = \frac{3}{2\lambda_e}[\Delta^{(1)}\lambda]^2. \tag{A9}$$

Equation (A9) provides the parabolic relation between the first order and second order wavelength shifts, but with 'three times' the coefficient as implied by equation (A1). Equation (A9) is therefore ruled out by the Ives–Stilwell experiment (Ives & Stilwell 1938).

Now, the above method of deriving the Doppler shift can be seen to be related to that described in section 1. For this purpose, consider Fig. A2 providing the details needed as in Fig. 1(a).

Referring to Fig. A2, we have $(\mathrm{MP}) \geq (\mathrm{M'P}) = (\mathrm{MQ}) \geq (\mathrm{MP'})$. Then, the derivation of equation (A1) as shown above uses the part $(\mathrm{MQ}) \approx (\mathrm{MP'})$, while the method in Section 1 of obtaining the Doppler shift uses the part $(\mathrm{MP}) \approx \sqrt{(\mathrm{MP'})^2 + (\mathrm{OP})^2\sin^2\theta}$ of the above inequality relation. Then, $(\mathrm{MP'})$ underestimates (MQ), the actual distance travelled by light during time ΔT, and (MP) overestimates it.

Replacing $(\mathrm{OP})\sin\theta$ by $(\mathrm{OP})\sin\theta - (\mathrm{P'P''})$, with $\mathrm{P'P''} = (\mathrm{SM'})\sin\phi \sim (\mathrm{SO})\sin\phi$, we convert the formula of the first method to that of the second method. Then experiments will not distinguish between the two methods.

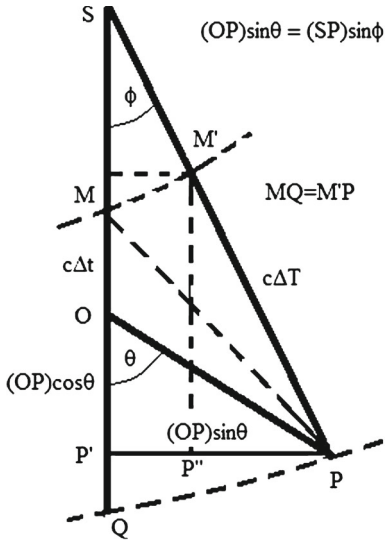

Figure A2. Geometry for connecting two methods of deriving formula for Doppler shift.

Now, we notice that equation (5) reduces to $1 \pm \beta$, the non-special relativistic expressions of the Doppler shift, for $\theta = 0, \pi$. On the other hand, the standard formula, equation (A1), reduces to $\sqrt{\frac{1-\beta}{1+\beta}}$ for $\theta = 0$ and to $\sqrt{\frac{1+\beta}{1-\beta}}$ for $\theta = \pi$. Equation (5) will not correspond with equation (A1), which is validated by a variety of experiments (Ashby 2003).

Therefore, we have the situation that the standard formula, equation (A1) is well-supported by experiments, but is theoretically inappropriate. Theoretically correct special relativistic formula of equation (A9) gets ruled out by the Ives–Stilwell experiment. However, it is still unclear as to why equation (A1) correspond well with the experimental results.

On the other hand, equation (5) is obtained for the case of constant acceleration, and it disagrees with the standard formula and, hence, with experiments. Further investigations are thus essential for an appropriate resolution of this situation.

We may nevertheless use variable acceleration to express distance (SP) in equation (A5) in terms of appropriate quantities, or as described in section 4, to gain further understanding of the ways of measuring velocity and acceleration of the source from its Doppler shift. This is described in the main body of this paper.

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