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Error Analysis on Plane-to-Plane Linear Approximate Coordinate Transformation

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Abstract. In this paper, the error analysis has been done for the linear approximate transformation between two tangent planes in celestial sphere in a simple case. The results demonstrate that the error from the linear transformation does not meet the requirement of high-precision astrometry under some conditions, so the linear approximate transformation should be taken seriously.

Key words. Error analysis—coordinate transformation—celestial sphere—tangent plane.

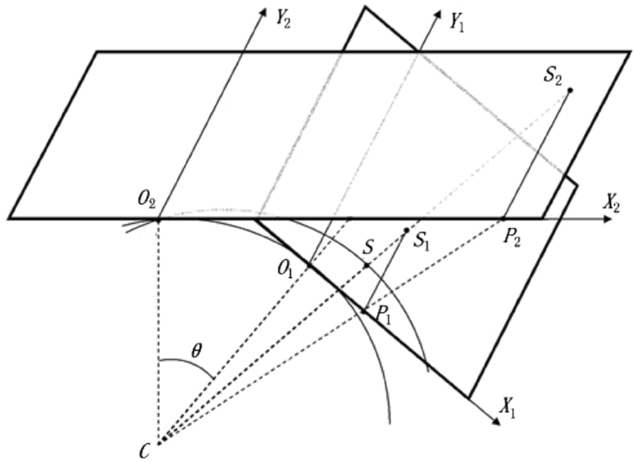
1. Introduction

In astronomy, some tasks require performing the coordinate transformation between two tangent planes in celestial sphere that are project planes. Normally, such transformations involve several steps (Calabretta & Greisen 2002), and are computationally intensive, so Makovoz (2004) gives a set of fast and exact transformation formula. Sometimes linear approximate transformation with four or six constants is also applied as in Anderson & King (2003), but how much error will it bring?

2. Error analysis and discussion

Consider the following case. A celestial sphere of a unit radius has a celestial object S located on it. The point S is projected onto two 'image' planes P_1 and P_2, tangent to the sphere at points O_1 and O_2. Each plane has a local Cartesian coordinate system defined as $(X_1\ Y_1)$ and $(X_2\ Y_2)$. The tangent points O_1 and O_2 are the origins of the local coordinate systems, and their Y-axes lie in the longitudinal planes of the celestial spherical coordinate system (see Fig. 1(a)). Let θ denote the angle between CO_1 and CO_2. Based on the exact transformation formulae given by Makovoz (2004) and their first-order Taylor expansions about $x = 0$ and $y = 0$, we get the error formulas of linear approximate transformation as follows:

$$\Delta x_2 = \frac{x_1^2 \tan\theta}{\cos^2\theta - x_1 \sin\theta\cos\theta}, \qquad \Delta y_2 = \frac{x_1 y_1 \sin\theta}{\cos^2\theta - x_1 \sin\theta\cos\theta}, \tag{1}$$



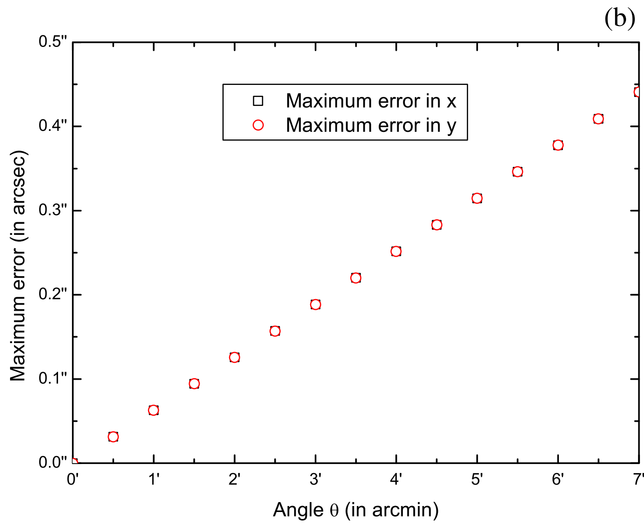


Figure 1. (**a**) Figure shows two tangent planes in celestial sphere and their origins and axes. (**b**) Figure shows the maximum errors of linear approximate transformation with different θ for 1 m telescope at Yunnan Observatory.

where $x_1, y_1 \in [-1, 1]$. For 1 m telescope at Yunnan Observatory, its focus is 13.3 m, its CCD resolution is 2048×2048, the size of pixel is $13.5 \times 13.5\ \mu\text{m}^2$ and the size of field-of-view is about $7' \times 7'$. Based on these parameters, we get maximum errors in the x and y directions with different θ, as shown in Fig. 1(b). It can be seen inches that there are almost same errors in both x and y and the error is $0.441''$ when $\theta = 7'$. To high-precision astrometry, it should be taken seriously and not be neglected.

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