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Hawking Radiations from an Arbitrarily Accelerating Kerr Black Hole

Xue-Jun Yang[1,*], Yan-Nv Zhang[1] & Wei Liu[2]
[1]*Department of Physics, Shaoxing University, Shaoxing 312000, China.*
[2]*Department of Mathematics, Shaoxing University, Shaoxing 312000, China.*
**e-mail: yang_xue_jun@163.com*

Abstract. The Hawking radiation from an arbitrarily accelerating Kerr black hole is calculated by using a new tortoise coordinate transformation and the improved Damour–Ruffini method.

Key words. Hawking radiation: arbitrarily accelerating Kerr black hole: new tortoise coordinate transformation.

1. Introduction

In 1976, Damour & Ruffini (1976) presented a method proving Hawking radiation. Zhao *et al.* (1994) improved the method of Damour and Ruffini, and calculated the location of the event horizon and the temperature of the varying black holes at the same time. It is a pivotal step to introduce a tortoise coordinate transformation in these methods. There is a problem of dimension in the usual tortoise coordinate transformation. In this paper, we will use a new tortiose coordinate in which the dimension is correct.

2. Hawking effect

The line element of an arbitrarily accelerating Kerr black hole (Jing *et al.* 1992) is

$$\begin{aligned} \mathrm{d}s^2 = & -(1-C)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r - 2f\rho^2\mathrm{d}v\mathrm{d}\theta - 2\sin^2\theta(CA+\rho^2 g)\mathrm{d}v\mathrm{d}\varphi \\ & - 2A\sin^2\theta\mathrm{d}r\mathrm{d}\varphi + \rho^2\mathrm{d}\theta^2 + 2Af\rho^2\sin^2\theta\mathrm{d}\theta\mathrm{d}\varphi \\ & + \sin^2\theta[(A^2C+2gA\rho^2)\sin^2\theta + A^2 + r^2]\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

where

$$f = -a\sin\theta + b\sin\varphi + c\cos\varphi, \quad g = ctg\theta(b\cos\varphi - c\sin\varphi),$$

$$C = 2ar\cos\theta + \frac{2mr}{\rho^2} + \rho^2(f^2 + g^2\sin^2\theta), \quad \rho^2 = r^2 + A^2\cos^2\theta,$$

and a, b, c, m are the functions of the advanced Eddington coordinate υ. The quantities m and A are the mass and specific angular momentum of the black hole, respectively.

We introduce a new tortiose coordinate transformation (Zhao *et al.* 2010)

$$\begin{aligned} r_* &= \frac{1}{2\kappa\,(\upsilon_0, \theta_0, \varphi_0)} \ln\left[\frac{r - r_{\mathrm{H}}\,(\upsilon, \theta, \varphi)}{r_{\mathrm{H}}\,(\upsilon, \theta, \varphi)}\right], \\ \upsilon_* &= \upsilon - \upsilon_0, \quad \theta_* = \theta - \theta_0, \\ &\quad \varphi_* = \varphi - \varphi_0, \end{aligned} \tag{2}$$

when $r \to r_{\mathrm{H}}, \upsilon \to \upsilon_0, \theta \to \theta_0$ and $\varphi \to \varphi_0$. We select the adjustable parameter κ as (Zhao *et al.* 1994)

$$\begin{aligned} \kappa = {} & [A^2\dot{r}_h^2 \sin^2\theta_0 - A^2\dot{r}_h - 3\dot{r}_h r_h^3 - gA\rho_h^2 \sin^2\theta_0 \dot{r}_h + 2A\dot{r}_h r'_{h\varphi} - ar_h\rho_h^2 \cos\theta_0 \\ & -mr_h + r_h^2 - f_h\rho_h^2 r'_h + r_h'^2 + r'_{h\varphi}\sin^{-2}\theta_0 - (g\rho_h^2 + A)r'_{h\varphi}]/ \\ & [A^2 r_h + r_h^3 + g_h A r_h \rho_h^2 \sin^2\theta_0 - A^2 r_h \dot{r}_h \sin^2\theta_0 - A r_h r'_{h\varphi}]. \end{aligned} \tag{3}$$

The Klein–Gordon equation becomes a standard wave equation as follows:

$$\frac{\partial^2\Psi}{\partial r_*^2} + 2\frac{\partial^2\Psi}{\partial r_*\partial\upsilon_*} + H\frac{\partial\Psi}{\partial r_*} + B\frac{\partial^2\Psi}{\partial r_*\partial\theta_*} + J\frac{\partial^2\Psi}{\partial r_*\partial\varphi_*} = 0, \tag{4}$$

where H, B and J are constants.

According to the method suggested by Damour & Ruffini (1976) and from equation (4), we get the distribution function of the out-going energy flux:

$$N_\omega^2 = \frac{1}{\mathrm{e}^{(\omega - \omega_\theta - \omega_\varphi)/K_{\mathrm{B}}T} - 1}, \tag{5}$$

where K_{B} is the Boltzman's constant, $T = \frac{\kappa}{2\pi K_{\mathrm{B}}}$ is the radiation temperature. Equation (5) is the quasi-black-body's thermal spectrum, as we know from equation (3) that T is a distribution of local temperatures.

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