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Hawking Temperature of Acoustic Black Hole

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Abstract. Using a new tortoise coordinate transformation, the Hawking radiation of the acoustic black hole was discussed by studying the Klein–Gordon equation of scalar particles in the curve space-time. It was found that the Hawking temperature is connected with time and position on the event horizon.

Key words. Hawking temperature—acoustic black holes—tortoise coordinate transformation.

1. Introduction

In order to observe the Hawking radiation of black holes in a laboratory environment, Unruh (1981) first proposed the basic idea of the acoustic black hole in 1981. It was found that the fluid inside the sonic horizon surface is equivalent to the black hole of general relativity. So we can get properties of the actual black hole by studying these acoustic black holes, if this kind of acoustic black holes can be produced within the conditions of a laboratory.

2. Hawking temperature of acoustic black hole

In the advanced Eddington–Finkelstein time coordinate, the line element of a acoustic black hole is as follows:

$$\mathrm{d}s^2 = -\left(1 - \frac{r_\mathrm{H}^4}{r^4}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2). \tag{1}$$

With parameter $r = r_\mathrm{H}(v)$, it is the local event horizon of the acoustic black hole.

The new tortoise coordinate can be expressed as (Zheng *et al*. 2010; Zhikun & Xie 2011; Weizhen *et al.* 2011)

$$r_* = \frac{1}{2\kappa}\ln\left[\frac{r - r_\mathrm{H}}{r_\mathrm{H}}\right], \quad v_* = v - v_0, \quad \theta_* = \theta - \theta_0, \quad \varphi_* = \varphi - \varphi_0, \tag{2}$$

where κ is an adjustable parameter, and κ, v_0, θ_0 and φ_0 are all constants under tortoise coordinate transformation. According to the coordinate transformation, the

Klein–Gordon equation can be rewritten as

$$\frac{r^4 - r_{\rm H}^4}{2\kappa r^4(r - r_{\rm H})}\frac{\partial^2\Phi}{\partial^2 r_*^2} + 2\frac{\partial^2\Phi}{\partial r_*\partial \nu_*} + \frac{4\kappa(r - r_{\rm H})}{r}\frac{\partial\Phi}{\partial\nu_*}$$
$$+2\left[\frac{r^4 + r_{\rm H}^4}{r^5} - \frac{r^4 - r_{\rm H}^4}{2r^4(r - r_{\rm H})}\right]\frac{\partial\Phi}{\partial r_*} + 2\kappa(r - r_{\rm H})\frac{\cot\theta}{r^2}\frac{\partial\Phi}{\partial\theta_*}$$
$$+2\kappa(r - r_{\rm H})\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta_*^2} + 2\kappa(r - r_{\rm H})\frac{1}{r^2\sin^\theta}\frac{\partial^2\Phi}{\partial\varphi_*^2}$$
$$-2\kappa(r - r_{\rm H})\mu^2\Phi = 0. \tag{3}$$

The numerator and denominator of the coefficient on the term $\frac{\partial^2\Phi}{\partial r_*^2}$ both approach zero at the horizon $r_{\rm H}$. Therefore, we can calculate the limit of the coefficient by using L' Hospital law. Let the limit be equal to an undetermined constant A, and κ as

$$\kappa = \frac{2}{r_{\rm H}}. \tag{4}$$

If we have $A = 1$, the Klein–Gordon equation can be changed into the standard form at the event horizon. In this case, κ is just the event horizon surface gravity of the acoustic black hole.

According to $T = \frac{\kappa}{2\pi k_{\rm B}}$, now we can get the Hawking temperature as

$$T = \frac{1}{\pi r_{\rm H} k_{\rm B}}. \tag{5}$$

3. Conclusion

By using a new tortoise coordinate transformation to solve the Klein–Gordon equation, the event horizon surface gravity κ has been obtained by taking the limitation of the coefficient of $\frac{\partial^2\Phi}{\partial r_*^2}$, and the Hawking temperature T has also been obtained. It is found that the event horizon surface gravity and the Hawking temperature is connected with time and position on the event horizon.

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