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Hawking Temperature of an Arbitrarily Accelerating Black Hole

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Abstract. Hawking temperature of an arbitrarily accelerating black hole with electric and magnetic charges are obtained based on the Klein–Gordon equation with a correct-dimension new tortoise coordinate transformation.

Key words. Hawking temperature: black hole: correct-dimension new tortoise coordinate transformation.

1. Introduction

In 1974, Hawking (1974) made a striking discovery that black holes could produce thermal radiation. In this paper, we will obtain Hawking temperature of an arbitrarily accelerating black hole based on the Klein–Gordon equation, which is identical to the one obtained by the Hamilton–Jacobi equation under the same correct-dimension new tortoise coordinate transformation (Xie *et al.* 2013).

2. Hawking temperature of an arbitrarily accelerating black hole

The line element of an arbitrarily accelerating black hole with electric and magnetic charges can be expressed as (Meng *et al.* 2010)

$$\begin{aligned} \mathrm{d}s^2 = &-\left[1 - \frac{2m}{r} + \frac{E^2+Q^2}{r^2} - 2ar\cos\theta - \frac{4a\left(E^2+Q^2\right)\cos\theta}{r}\right.\\ &\left.-r^2\left(f^2 + h^2\sin^2\theta\right) - \frac{1}{3}\lambda r^2\right]\mathrm{d}\upsilon^2\\ &+2\mathrm{d}\upsilon\mathrm{d}r + 2r^2 f\,\mathrm{d}\upsilon\mathrm{d}\theta + 2r^2 h\sin^2\theta\mathrm{d}\upsilon\mathrm{d}\varphi + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

where

$$f = -a\sin\theta + b\sin\varphi + c\cos\varphi,\ h = \cot\theta\left(b\cos\varphi - c\sin\varphi\right).$$

In the curve space-time, the Klein–Gordon equation of the particle with mass μ, electric charge e and magnetic charge q is (Yang 2008)

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial x^{\mu}} - ieA_{\mu} - iqB_{\mu}\right)\left[\sqrt{-g}\cdot g^{\mu\nu}\left(\frac{\partial}{\partial x^{\nu}} - ieA_{\nu} - iqB_{\nu}\right)\Phi\right] - \mu^2\Phi = 0. \tag{2}$$

Using the condition of Lorentz $\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\left[\sqrt{-g}\cdot g^{\mu\nu}\left(eA_{\nu}+qB_{\nu}\right)\right] = 0$, a new tortoise coordinate will be as follows (Zhao *et al.* 2010):

$$r_* = r + \frac{1}{2\kappa\left(\upsilon_0,\theta_0,\varphi_0\right)}\ln\left[\frac{r - r_H\left(\upsilon,\theta,\varphi\right)}{r_H\left(\upsilon,\theta,\varphi\right)}\right], \upsilon_* = \upsilon - \upsilon_0, \theta_* = \theta - \theta_0, \varphi_* = \varphi - \varphi_0. \tag{3}$$

When $r \to r_H, \upsilon \to \upsilon_0, \theta \to \theta_0, \varphi \to \varphi_0$, we select the adjustable parameter κ as

$$\kappa = \frac{\frac{2m}{r_H^2} - \frac{r_H'^2}{2r_H^3} - \frac{r_{H\varphi}'^2}{2r_H^3\sin^2\theta_0} - \frac{1}{2r_H} - \frac{3\left(E^2+Q^2\right)}{2r_H^3} + \frac{8a\left(E^2+Q^2\right)\cos\theta_0}{2r_H^2} - \frac{1}{6}\lambda r_H}{2r_H\left(a\cos\theta_0 + \frac{m}{r_H^2} + \frac{r_H'^2}{2r_H^3} + \frac{r_{H\varphi}'^2}{2r_H^3\sin^2\theta_0} - \frac{E^2+Q^2}{2r_H^3} + \frac{2a\left(E^2+Q^2\right)\cos\theta_0}{r_H^2} + \frac{1}{6}\lambda r_H\right)}. \tag{4}$$

The Klein–Gordon equation becomes a standard wave equation:

$$\frac{\partial^2\Phi}{\partial r_*^2} + 2\frac{\partial^2\Phi}{\partial r_*\partial\upsilon_*} + B\frac{\partial^2\Phi}{\partial r_*\partial\theta_*} + C\frac{\partial^2\Phi}{\partial r_*\partial\varphi_*} + \left(D + i2\omega_0\right)\frac{\partial\Phi}{\partial r_*} = 0, \tag{5}$$

where

$$\begin{aligned}\omega_0 = {} & \lim_{r\to r_H,\upsilon\to\upsilon_0,\theta\to\theta_0,\varphi\to\varphi_0} \\ & \times\left\{\left(eA_1+qB_1\right)g^{10}\dot{r}_H + \left[g^{12}\left(eA_1+qB_1\right) + g^{22}\left(eA_2+qB_2\right)\right]r_H'\right. \\ & +\left[g^{13}\left(eA_1+qB_1\right) + g^{33}\left(eA_3+qB_3\right)\right]r_{H\varphi}' - g^{10}\left(eA_0+qB_0\right) \\ & \left.-g^{11}\left(eA_1+qB_1\right) - g^{12}\left(eA_2+qB_2\right) - g^{13}\left(eA_3+qB_3\right)\right\},\end{aligned}$$

and B, C and D are all constants. The κ of equation (4) is just the event horizon surface gravity. According to the relation $T = \kappa/2\pi\kappa_B$, we can obtain the Hawking temperature of the black hole on the event horizon.

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