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Entropy Spectrum of Modified Schwarzschild Black Hole via an Action Invariance

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Abstract. The entropy spectrum of a modified Schwarzschild black hole in the gravity's rainbow are investigated. By utilizing an action invariance of the black hole with the help of Bohr–Sommerfield quantization rule, the entropy spectrum for the modified black hole are calculated. The result of the equally spaced-entropy spectrum being consistent to the original Bekenstein's spectra is derived.

Key words. Entropy spectrum—modified black hole—action invariance.

The spectroscopy of black holes is an important aspect with regard to black hole quantum property and has been studied since long. Majhi & Vagenas (2011) considered that the outgoing particles pass through the event horizon, and an adiabatic invariant action variable for black holes was proposed and written by them as

$$I = \int p_i \mathrm{d}q_i = \int \int_0^{p_i} \mathrm{d}p_i' \mathrm{d}q_i = \int \int_0^{p_0} \mathrm{d}p_0' \mathrm{d}q_0 + \int \int_0^{p_r} \mathrm{d}p_r' \mathrm{d}q_r, \tag{1}$$

where p_i is the conjugate momentum of the coordinate q_i, $q_0 = \tau$ where τ is the Euclidean time. By applying the Bohr–Sommerfeld quantization rule to the adiabatic invariant, the quantum spectrum for static non-charged black hole was given (Majhi & Vagenas 2011). Then, by showing an action invariance, a method for studying black hole quantization was put forward. In Liu (2012), Majhi and Vagenas's method was extended to static charged black hole and the adiabatic invariant was expressed as

$$I = \oint p_i \mathrm{d}q_i = \oint \int_0^{p_i} \mathrm{d}p_i' \mathrm{d}q_i, \tag{2}$$

where q_i is the dynamic degree freedom in the Euclidean coordinate, the integral path encircle and is very close to the event horizon.

Using the action variable formula (2), we extend Majhi and Vagenas's work to a modified Schwarzschild black hole in the gravity's rainbow as (Magueijo & Smolin 2004):

$$\mathrm{d}s^2 = -\frac{\left(1-\frac{2M_t}{r}\right)}{f_1^2}\mathrm{d}t^2 + \frac{1}{f_2^2\left(1-\frac{2M_t}{r}\right)}\mathrm{d}r^2 + \frac{r^2}{f_2^2}\mathrm{d}\Omega^2, \tag{3}$$

where f_1 and f_2 are two energy functions and by this the present modified Schwarzschild spacetime is endowed with Planck scale modifications.

For the non-charged spherically static symmetric spacetime, the only dynamic degree freedom can be written as q_r, the Hamilton's equation $\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}\tau} = \frac{\mathrm{d}H'_\tau}{\mathrm{d}p'_r}$ can be used. Then the adiabatic invariant quantity (2) is

$$I = \oint p_r \mathrm{d}q_r = \oint \int_0^{p_r} \mathrm{d}p'_r \mathrm{d}q_r = \oint \int_0^{H_\tau} \frac{\mathrm{d}H'_\tau}{\mathrm{d}\dot{r}} \mathrm{d}r = \oint \int_0^{H_\tau} \mathrm{d}H'_\tau \mathrm{d}\tau. \tag{4}$$

Considering the period of the Euclidean time of a loop about the event horizon is equal to the inverse of the temperature of black holes (Gibbons & Hawking 1977; Ropotenko 2009), we have

$$I = \int_0^{H_\tau} \frac{\mathrm{d}H'_\tau}{T'_\tau}. \tag{5}$$

For the modified black hole (3), using the first law of black hole thermodynamics $\frac{\mathrm{d}H'_\tau}{T'_\tau} = \mathrm{d}S'_{\mathrm{bh}}$, the adiabatic invariant (5) can be written as

$$I = \hbar \int_0^{S_{\mathrm{bh}}} dS'_{\mathrm{bh}} = \hbar S_{\mathrm{bh}}. \tag{6}$$

For the present canonical invariant corresponding to one dynamic degree freedom q_r, using the Bohr–Sommerfeld quantization rule $I = 2\pi n\hbar$, where $n = 1, 2, 3, \ldots$, the entropy spectrum of the Schwarzschild modified black hole in gravity's rainbow can be obtained as

$$S_{\mathrm{bh}(n)} = \frac{I_n}{\hbar} = 2\pi n, \tag{7}$$

Thus, the equally spaced entropy spectrum is derived.

It is found that the entropy spectrum of the modified black hole is the same as the usual Schwarzschild black hole (Liu 2012). That is, the obtained entropy spectrum is independent of the energies of test particles and is not dependent on the Planck scale modifications of the spacetime. In this sense, in supporting the universality of the black hole spectroscopy, an illustration via the action invariance of black holes is provided.

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