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Conductivity of Holographic Superconductor within Ginzburg–Landau Theory

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Abstract. The frequency-dependent conductivity is obtained for the holographic superconductor by using the Ginzburg–Landau theory with a $|\Psi|^4$ term. Our results show that $|\Psi|^4$ term plays a role in the low-temperature behaviour of the conductivity.

Key words. Holographic superconductor—conductivity—Ginzburg–Landau action—AdS/CFT correspondence.

AdS/CFT (Maldacena 1998) relates the highly quantum dynamics of the boundary operator O to simple classical dynamics of the bulk scalar field Ψ (Gubser *et al.* 1998). The holographic superconductors were built in the sense of AdS/CFT correspondence (Hartnoll *et al.* 2008). The holographic phase transition was investigated in an AdS soliton background (Cai *et al.* 2011). In this paper, we will study the conductivity by means of AdS/CFT correspondence.

The matter sector of the model is described by the Ginzburg–Landau density:

$$\mathcal{L} = -\frac{1}{4}F^{ab}F_{ab} - |\partial\Psi - iA\Psi|^2 + \frac{2|\Psi|^2}{L^2} - \frac{\alpha|\Psi|^4}{2L^4} \tag{1}$$

for a Maxwell field and a charged complex scalar field. Here α is a parameter. The gravity sector is $\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}x^2 + \mathrm{d}y^2)$ with $f(r) = \frac{r^2}{L} - \frac{M}{r}$. L is the AdS radius, and M is the mass of the black hole.

The large r limit gives $\Psi = \frac{\Psi^{(1)}}{r} + \frac{\Psi^{(2)}}{r^2}$ and $A_t = \mu + \frac{\rho}{r}$. For $\Psi^{(1)} = 0$ (or $\Psi^{(2)} = 0$), the condensate of the scalar operator O is given by $\langle O_2\rangle = \sqrt{2}\Psi^{(2)}$ (or $\langle O_1\rangle = \sqrt{2}\Psi^{(1)}$) (Hartnoll *et al.* 2008). At the horizon $r = r_0$, $A_t = 0$ is found.

To compute the conductivity in the dual CFT, we need to solve the equation

$$\partial_r^2 A_x + \frac{f'}{f}\partial_r A_x + \left(\frac{\omega^2}{f^2} - \frac{2|\Psi|^2}{f} + \frac{\alpha|\Psi|^4}{2f}\right)A_x = 0 \tag{2}$$

for the fluctuations of the vector potential A_x in the bulk. The asymptotic large r behaviour of A_x is $A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r}$. At the horizon, we set the ingoing

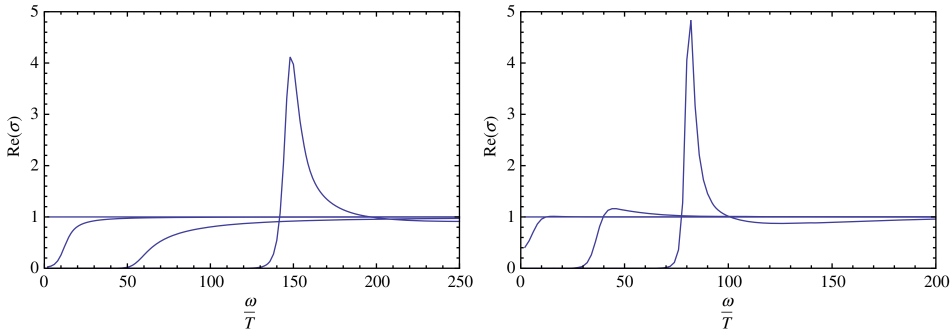

Figure 1. The frequency dependent conductivity σ for O_1 operator (left) and O_2 operator (right). Here we set $\alpha = 1/300$.

boundary conditions: $A_x \propto f^{-i\omega/3r_0}$. AdS/CFT yields the dual source $A_x = A_x^{(0)}$ and the expectation value $\langle J_x \rangle = A_x^{(1)}$ for the current scenario. Then the conductivity $\sigma(\omega) = \frac{\langle J_x \rangle}{E_x}$ is given by

$$\sigma(\omega) = -\frac{\langle J_x \rangle}{\dot{A}_x} = -\frac{i\langle J_x \rangle}{\omega A_x} = -\frac{i A_x^{(1)}}{\omega A_x^{(0)}}, \tag{3}$$

which can be solved numerically.

The frequency-dependant conductivity is shown in Fig. 1. The real part of the conductivity $\mathrm{Re}(\sigma) = 1$ suggests that the conductivity is in the normal phase, which denotes no condensate at or above the critical temperature T_c. As the Hawking temperature $T = \frac{3M^{1/3}}{4\pi L^{4/3}}$ decreases, a gap in $\mathrm{Re}(\sigma)$ appears and becomes increasingly deep for $T < T_c$. When $\mathrm{Re}(\sigma) > 1$, the superfluid density in O_2 system is reduced more than the O_1 system. When compared to the case without α (Hartnoll *et al.* 2008), the larger α makes the gap deeper and the superfluid density smaller. When the temperature is very low, there exists a very high peak in the O_1 system, which is absent in the case of $\alpha = 0$. When the system includes $|\Psi|^4$ term, the superconductivity is easier to exhibit. Below T_c, $|\Psi|^4$ term helps a condensate to develop, and makes the gap deeper for charged excitations.

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