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Black Hole Analogue in Bose–Einstein Condensation

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Abstract. We have proposed a black hole analogue in a Bose–Einstein condensation. By introducing the Painlevé co-ordinates and using K–G equations, we have obtained the critical temperature of the black hole analogue in a Bose–Einstein condensation.

Key words. Black hole analogue—Bose–Einstein condensation.

1. Introduction

In 1975, Hawking published his astonishing discovery that the black hole has temperature inversely proportional to its mass and is of the order of about 10^{-8} K, far below 3 K cosmic microwave background radiation. So, to study the black hole directly in space is a delusion. Therefore, some research teams turned to envisage creating black hole analogues in a laboratory, such as Bose–Einstein condensation. The basic idea of Bose–Einstein condensation was originally suggested by Bose (1924) and Einstein (1924). When the temperature of the system is below the critical temperature, Bose–Einstein condensation takes place. In recent years, great progress in this respect has been made, especially about the sonic black-hole analogues in which the sound waves cannot escape. In this paper, we discuss K–G particles, rather than waves.

2. Critical temperature

We introduce the modified thin film model and assume the film near the event horizon is a Bose–Einstein condensation system. The system is composed of K–G particles that are identical and nearly dependent.

The quantum theory of black hole thermodynamics should take the general uncertainty principle into account, which has the form as $\Delta q_1 \cdots \Delta q_r \Delta p_1 \cdots \Delta p_r \approx h^r(1+\alpha p^2)$. The volume of μ-space $\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}p_x\mathrm{d}p_y\mathrm{d}p_z$ divided by the size of the phase cell $h^3(1+\alpha p^2)^3$ is the number of quantum states of K–G particles: $\mathrm{d}n = \frac{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}p_x\mathrm{d}p_y\mathrm{d}p_z}{h^3(1+\alpha p^2)^3}$. If describing it in the spherical co-ordinates, the number of quantum states is then expressed as

$$\mathrm{d}n = \frac{(4\pi)^2}{h^3}\frac{r^2\mathrm{d}r p^2\mathrm{d}p}{(1+\alpha p^2)^3}. \tag{1}$$

According to Bose distribution, the number of particles staying at the energy level of ε_l is $a_l = \frac{\omega_l}{e^{\frac{\varepsilon_l}{KT}}-1}$. Obviously, since the number of particles at any energy level cannot be negative, it requires $e^{\frac{\varepsilon_l}{KT}} > 1$. If ε_0 represents the lowest energy level, $\varepsilon_0 > 0$, then the total number of the particles can be expressed as $N = \sum \frac{\omega_l}{e^{\frac{\varepsilon_l}{KT}}-1}$. Substituting the sum with integration, and taking the expression (1) into account, we have

$$N = \frac{(4\pi)^2}{h^3} \int \frac{r^2 \mathrm{d}r p^2 \mathrm{d}p}{(1+\alpha p^2)^3}. \tag{2}$$

Using the Klein–Gordon equations of the spherically static symmetric-black hole analogue in the Painlevé co-ordinates (Zhang & Zhao 2005), we can obtain the definite expression that $P^2 = \frac{4\omega^2}{f^2}$, in the vicinity of the event horizon. Substituting it into equation (2), we have

$$N = \frac{(4\pi)^2}{h^3} \int r^2 \mathrm{d}r \int \frac{8\omega^2 \mathrm{d}\omega}{f^3 \frac{h\omega}{KT_C}\left(1+\frac{4\alpha\omega^2)}{f^2}\right)^3}.$$

Finally, we obtain the critical temperature T_C as

$$T_\mathrm{C} = \frac{64\pi^5\alpha^2}{S_A^2} \frac{1}{(\sqrt{\varepsilon+\delta}-\sqrt{\varepsilon})^2} N^2. \tag{3}$$

3. Discussion

When $T < T_\mathrm{C}$, the large number of particles $N_0(T)$ at the lowest energy level ε_0 cannot be ignored, so we have

$$N_0(T) + \frac{(4\pi)^2}{h^3} \int \frac{r^2 \mathrm{d}r p^2 \mathrm{d}p}{\left(e^{\frac{\varepsilon_l}{KT}}\right)(1+\alpha p^2)^3} = N.$$

Taking equation (3) into account, we have

$$N_0(T) = N\left(1-\frac{T}{T_\mathrm{C}}\right). \tag{4}$$

It is obvious that, below the critical temperature, $N_0(T)$ and N are in the same magnitude.

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