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Scalar Condensation of Holographic Superconductors using Ginzburg–Landau Density with Quartic Term

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Abstract. We study holographic superconductors analytically by using the Ginzburg–Landau action with the γ-quartic term $|\Psi|^4$. Our results show that γ-term plays a role in the scalar condensation. It is found that the system displays two kinds of critical temperatures. One is independent of γ. But the other increases with increasing γ, which suggests the onset of easier condensation.

Key words. Holographic superconductor—scalar condensation—Ginzburg–Landau action—AdS/CFT correspondence.

1. Introduction

Holographic superconductors are strongly coupled-field theories which undergo a superconducting phase transition below a critical temperature $T_{\rm c}$, and which have a dual gravity (Maldacena 1998). It was suggested to construct gravitational duals of the transition from normal to superconducting states in the boundary theory (Hartnoll *et al.* 2008). In recent years, AdS/CFT correspondence has been applied to condensed matter physics, and in particular to the scalar condensation and superconductivity. However, most theoretical studies focused on the Ginzburg–Landau theory only with the quadratic term (Gregory *et al.* 2009; Pan *et al.* 2010; Ge *et al.* 2010). They did not discuss the effect of the quartic term. In this paper, we aim to investigate the scalar condensation of holographic superconductors by the Ginzburg–Landau density with the quartic term.

2. Model

To begin with, we consider 4-dimensional Schwarzschild AdS black hole with the metric

$$\mathrm{d}s^2 = \frac{L^2\alpha^2}{z^2}[-f(z)\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2] + \frac{L^2}{z^2 f(z)}\mathrm{d}z^2, \tag{1}$$

with $f(z) = 1 - z^3$ and $\alpha = \frac{r_+}{L^2} = \frac{4\pi T}{3}$. Here L is the AdS radius. In this background, we now consider a Ginzburg–Landau density

$$\mathcal{L} = -\frac{1}{4}F^{\rm ab}F_{\rm ab} - |\partial\Psi - iA\Psi|^2 + \frac{2\,|\Psi|^2}{L^2} - \frac{\gamma\,|\Psi|^4}{2L^4} \tag{2}$$

for a Maxwell field and a charged complex scalar field. Here γ is a parameter for the quartic term.

Making the assumption for static solutions ($\Psi = \Psi(z)$, $A_t = \phi(z)$ and $A_z = A_x = A_y = 0$), we can get the equations of motion for Ψ and ϕ as

$$z^2 \partial_z \left(\frac{f(z)}{z^2} \partial_z \Psi \right) + \frac{\phi^2}{\alpha^2 f(z)} \Psi + \frac{1}{z^2} \left(2\Psi - \frac{\gamma}{L^2} |\Psi|^2 \Psi \right) = 0, \tag{3}$$

$$\alpha^2 f(z) \partial_z^2 \phi - \frac{2L^2 \alpha^2}{z^2} |\Psi|^2 \phi = 0, \tag{4}$$

where we have used equations (1) and (2). At the horizon $z = 1$, we can obtain $\phi(1) = 0$, and equations (3) and (4) lead to

$$\Psi'(1) = \frac{2}{3} \Psi(1) - \frac{\gamma}{3L^2} \Psi^3(1). \tag{5}$$

Then near the horizon $z = 1$, we have

$$X(z) = X(1) + X'(1)(z-1) + \frac{1}{2} X''(1)(z-1)^2, \tag{6}$$

for $X = \Psi, \phi$, respectively. And near the asymptotic AdS boundary $z \to 0$, we find $\Psi(z) = \Psi^{(2)} z^2$ and

$$\phi(z) = \mu - \frac{\rho}{r_+} z. \tag{7}$$

In the dual field theory, μ and ρ are the chemical potential and charge density, respectively. The condensate of the scalar operator O in the field theory dual to the field Ψ is given by $\langle O_2 \rangle = \sqrt{2} \Psi^{(2)} r_+^2 / L^3$ (Hartnoll *et al.* 2008).

3. Results and discussion

In the following, we derive the spatially independent condensate solution that corresponds to the superconductor phase below some critical temperature by employing the analytic method (Gregory *et al.* 2009). Matching the asymptotic solutions and their first derivatives (5)–(7) near the horizon and the AdS boundary at the point, say z_m, we obtain a set of equations for $\Psi^{(2)}$ and $\Psi(1)$:

$$\begin{aligned}
&[20L^4 - 4\Psi^2(1) L^2 \gamma - 3\Psi^4(1) \gamma^2] z_m + 4L^4 + 10\Psi^2(1) L^2 \gamma + 3\Psi^4(1) \gamma^2 \\
&\quad = \frac{9\rho^2 (1 - z_m)}{\alpha^4 [3 + 2\Psi^2(1) L^2 (1 - z_m)]^2},
\end{aligned} \tag{8}$$

$$\Psi^{(2)} = \frac{\Psi(1)}{z_m^2} \left\{ 1 - \frac{1}{3}(1 - z_m)[2 - \Psi^2(1) L^{-2} \gamma] - \frac{C(1 - z_m)^2}{36 \Psi(1) \alpha^2 L^4} \right\}, \tag{9}$$

$$\frac{C}{\Psi(1)\alpha^2} = \frac{9\rho^2}{\alpha^4 [3 + 2(1 - z_m)\Psi^2(1) L^2]^2} + 8L^4 + 2\Psi^2(1) L^2 \gamma - 3\Psi^4(1) \gamma^2. \tag{10}$$

Then, we can get the expectation value $\langle O_2 \rangle$ analytically with z_m as a parameter. When the temperature T arrives at the critical temperature T_c, $\langle O_2 \rangle$ becomes zero,

Table 1. The values of the critical temperature T_{2c}. Here we have set $L = \rho = 1$.

γ	0.01	0.1	0.3	0.5	0.7	0.9
T_{2c}	0.0091	0.0282	0.0472	0.0587	0.0700	0.0732

and the condensation occurs for $T < T_c$. Using the Hawking temperature $T = 3\alpha/(4\pi)$, $\langle O_2 \rangle$ yields the asymptotic behaviour

$$\langle O_2 \rangle \sim |T_{2c}^2 - T^2| \sqrt{1 - \frac{T}{T_{1c}}}, \tag{11}$$

which is obtained from equations (8)–(10). Here the critical temperature T_{1c}, which is decided by $\Psi(1) = 0$, is independent of γ. For $\Psi(1) \neq 0$, T_{2c} is γ dependent. At $z_m = 1/2$, T_{1c} in equation (11) is found to be

$$T_c \equiv \frac{3\sqrt{\rho}}{4\pi L},$$

which agrees with the findings for $\gamma = 0$ (Ge *et al.* 2010). The values of the critical temperature T_{2c} are shown in Table 1 for $L = \rho = 1$. It is found that T_{2c} increases with increasing γ.

4. Conclusion

In conclusion, we have studied the effects of the γ-quartic term $|\Psi|^4$ on the scalar condensation for holographic superconductors. We match the asymptotic solutions and their first derivative near the horizon and the AdS boundary at the point z_m. The γ dependence of the critical temperatures is found. The critical temperature T_{2c} increases with increasing γ, which suggests the onset of easier condensation.

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