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Fractal Property in the Light Curve of BL Lac Object S5 0716 + 714

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Abstract. In this paper, we compile the historical R-band data of S5 0716 + 714 from literature and obtain its fractal dimension by using a fractal method and then simulate the data with the Weierstrass–Mandelbrot (W–M) function. It is considered that the light curve has a fractal property.

Key words. Galaxies: active—BL Lacertae objects: individual (S5 0716 + 714)—methods: statistical.

1. Introduction

A fractal is a mathematical set that has a fractal dimension that usually exceeds its topological dimension and may fall between the integers, so it is typically self-similar patterns (Mandelbrot 1982). In recent years, fractal theory is also used to solve the astrophysical problems, e.g., Cristiano (2007) confirmed that the galaxy distribution has multi-fractal property in large scales. Based on the fractal model, Elmegreen *et al.* (2006) found a rule of the mass distribution in the thin accretion disk of NGC 628. The infrared observation indicated that the interstellar clouds have a fractal structure with dimension 1.36 (Falgakone *et al.* 1991). Mocanu *et al.* (2012) analysed the energy spectrum noise properties of S5 0716 + 714 by structure function and fractal dimension. Leung *et al.* (2011) analysed the fractal dimension of the Intra-Day Variability (IDV) light curves of the BL Lac objects in optical, radio and X-ray bands, respectively, and they found found a high confidence correlation between spectral index and fractal dimension. Due to the abundant optical observational data, here, we obtain the long-term light curve of S5 0716 + 714 and fractal dimension and then simulate the data with Weierstrass–Mandelbrot (W–M) function.

2. Fractal dimension of light curve

We compile the historical R-band data of S5 0716 + 714 from literature (Zhang & Zhang 2007). In Fig. 1 (top panel), the light curve from 1994 to 2006 is shown. There are no data on the flat areas of the light curves. For these flat areas, it is necessary to be filled by some interpolation methods (Tang & Zhang 2010). Considering fractal to be a self-similar system, these areas do not affect the fractal dimension.

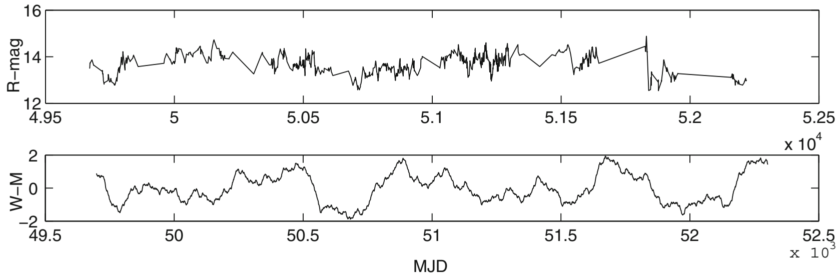

Figure 1. The observational data of the R-band (top panel) and simulation data with W–M function (bottom panel).

The fractal dimension contains the Hausdorff, correlation and information dimensions and so on (Zhang 1995). It is an important parameter that reveals the order of fractal phenomenon and internal rules. Actually, there are many ways to calculate this fractal dimension. A sample method for the box dimension can be described by the equation

$$D = \lim_{\tau \to 0} \frac{\lg N(\tau)}{\lg \tau}, \tag{1}$$

where τ is the side length of a square box, and $N(\tau)$ is the total number of box covering the fractal object. Using this method, we get the fractal dimension of the R-band data and the linear correlation coefficient, the values $D = 1.214$, $r = 0.98$ (see Fig. 2). These results indicate that the light curve has a fractal property.

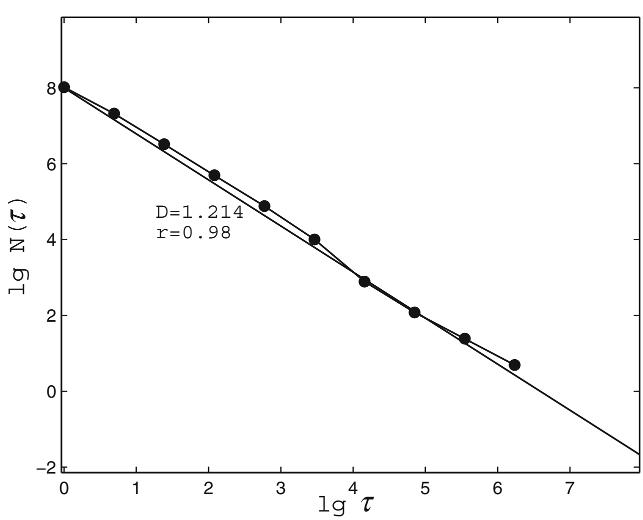

Figure 2. The $\lg \tau - \lg N(\tau)$ map of the R-band light curve.

3. W–M function simulation

The W–M function is a continuous non-derivable function. From a simple algorithm, we can deduce complex results. The definition of W–M function is written as follows:

$$W(x) = A^{(D-1)} \sum_{n=n_1}^{\infty} \frac{2\pi \gamma^n x}{\gamma^{(2-D)n}}, \text{ with } 1 < D < 2 \text{ and } \gamma > 1, \quad (2)$$

where A is the characteristic scale that reflects the volatility of the W-M function, D is the fractal dimension that describes the complexity of the system and γ is the frequency that reflects the randomness of the function. Based on the above analysis, we get the fractal dimension $D = 1.214$, the characteristic scale $A = 2.0$ (e.g., Zhang & Zhang 2007) and frequency $\gamma = 1.5$ (Majumdar *et al.* 1990) to simulate the light curve (equation (2)) as shown in Fig. 1 (bottom panel).

Overall we found that the W–M function is consistent with R-band data. Thus, we can use the W–M function to quantitatively analyse the variation tendency and the complexity of the light curve. The fractal dimension of the light curve is lower ($D = 1.214$), but its amplitude is high up to 2–3 mag, which indicates that the internal energy of the source varied fiercely; the frequency $\gamma = 1.5$, which demonstrates that the source changed frequently in the last 12 years. So far, although the intrinsic mechanism of the fractal phenomena is not clear (Nishizuka & Shibata 2013), using the W–M function to quantify the optical variability of BL Lac is a good method.

4. Conclusions

As seen from the above results, we conclude that the light curve has a fractal property and R-band data can be simulated with the W–M function.

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