Chaotic Behaviour of Intra-Day Variability of BL Lac Object S5 0716+714

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Abstract. An optical monitoring shows Blazar object S5 0716+714 has complex Intra-Day Variability (IDV) behaviour. In this paper, we introduce a method of non-linear time series analysis, and calculate the correlation dimension of the IDV light curves of S5 0716+714 over seven nights in the December of 2006. According to our calculations, the correlation dimension \( D \approx 1.993 - 5.178 \) for all of the observed data, it is implied that the IDV behaviour may be a chaotic system with some additional noise.

Key words. BL Lac object: individual (S5 0716+714)—intra-day variability—chaos: correlation dimension.

1. Introduction

Blazars represent the most violently variable objects among all active galactic nucleus. They show rapid and strong variability, high and variable polarization, and a non-thermal continuum. If the variable time scales ranging from a few hours to a few days, a varying phenomenon termed Intra-Day Variability, simply IDV. There are many observations for the IDV in different bands (Wagner & Witzel 1995; Villata et al. 2002; Ciprini et al. 2004; Gabányi et al. 2007). Many theoretical models explain this interesting phenomena, ranging from source-intrinsic (e.g., shock-in-jet, Spada et al. 2001; Larionov et al. 2013) to source-extrinsic (e.g., scintillation due to electron density fluctuations in the interstellar medium, Rickett et al. 2001; Savolainen & Kovalev 2008) have been proposed. Although the origin of variability is not yet clear, some arguments are really useful to understand the intrinsic nature of the emitting regions and the accretion process (Beckert et al. 2002; Bach et al. 2008; Sarma et al. 2010).

In fact, the variation behavior of IDV is a subject of much debate. In most analysis, it is implicitly assumed that these fluctuations are stochastic in nature (Fan et al. 2008; Leung et al. 2011). However, some studies have suggested that the variation is a deterministic non-linear system (e.g. chaotic, self-organized structure)
(Karak et al. 2010; Mineshige et al. 1994). The Non-Linear Time Series (NLTS) analysis is the most convenient tool for checking whether the origin of the variability is chaotic, stochastic, or a mixture of the two and is often adopted to study complex systems (e.g., weather, epidemiology, economics). This technique has also been used to analyse X-ray data of astrophysical sources (Misra et al. 2004, 2006).

In the optical regime, S5 0716+714 is one of the most violently variable objects with the amplitude of variations up to 2 mag (range 13.4 to 15.7 mag, Zhang & Zhang 2007). Recently, very rapid brightness increase by 0.611 magnitudes over 3.6 min has been found by Fan et al. (2011). The rapid variability and its brightness have made this source to be one of the ideal examples for detecting important clues about the mass of the black hole and the size of the emitting region (Fan 2005). In this paper, our motivation is to determine the IDV behaviour of this source by using NLTS analysis.

The present paper is organized as follows. In section 2, we briefly review the NLTS analysis method and IDV data of S5 0716+714. The results and conclusions are presented in section 3.

2. NLTS analysis and observation data

Based on Takens’ theorem (Takens 1981) on studies of chaotic time series, we can reconstruct a smooth attractor from the observations by using a generic function. The reconstruction space preserves the properties of the dynamical system, thus making it easy to obtain the nature and the internal laws of the chaotic system (Sugihara & May 1990; Theiler et al. 1992). The time series can be described as \( x(t) \) with \( t = 1, 2, \ldots, N \). By choosing an appropriate delay time \( \tau \) and an embedding dimension \( M \), a new series of \( M \)-dimensional vector can be reconstructed as

\[
X_m(t) = \{x(t), x(t + \tau), \ldots, x(t + (m - 1)\tau)\},
\]

where \( t = 1, 2, \ldots, M \) and \( M = N - (m - 1)\tau \). In the \( M \) dimensional space, we compute the correlation function:

\[
C_M(R) = \lim_{M \to \infty} \frac{1}{M(M - 1)} \sum_{i}^{M} \sum_{j, j \neq i}^{M} H(R - |X_i - X_j|).
\]

The correlation function is the average number of data points within a distance \( R \) from a data point. In eq. (2), \( X \) is the position vector of a point, \( H(\bullet) \) is a Heaviside function, and its expression can be given by

\[
H(x) = \begin{cases} 
0, & x \leq 0, \\
1, & x > 0.
\end{cases}
\]

Since \( R \) is a small quantity, the relationship between \( C_M(R) \) and \( R \) can be written as \( \lim_{R \to 0} C_M(R) \propto R^D \). After getting logarithm in the both sides of the equation, we can obtain the correlation dimension \( D \), as follows:

\[
D(M) = \lim_{R \to 0} \frac{d \log C_M(R)}{d \log R}.
\]
The correlation dimension $D(M)$ can be used to differentiate between different time series behavior, since for a stochastic system, $D \approx M$, while for a chaotic system, $D(M) \approx \text{constant}$ for $M$ greater than a certain value $M_{\text{max}}$. Figure 1 shows the correlation dimension $D$ as a function of the embedding dimension $M$ for random noise (crosses), Lorenz attractor (filled circles) and Lorenz system added noise (dashed line). As expected, the correlation dimension plot $D$ for the random data is consistent with the $D \approx M$ curve, while for the Lorenz system it shows a significant deviation and saturates at $M \approx 7$ to $D \approx 2.266$. The dashed line indicates the case that we have added noise into the chaotic system. Clearly, the effect of random noise in the data is to increase the $D$ values.

In order to analyse the chaotic properties of IDV light curves, we consider an individual source S5 0716+714. This source has been observed in simultaneous multi-wavelength monitoring by a number of researchers (Raiteri et al. 2003; Gupta et al. 2008; Wu et al. 2012; Rani et al. 2013). The monitoring covered the period from 18 to 27 December 2006 (from JD 2,454,088 to 2,454,097), with no observation on December 19, 25, and 26 due to bad weather, see Fig. 2. The details of the monitoring can be found in Wu et al. (2012).

3. Results

Taking the case of JD 2,454,088 in the B-band as an example, the observation data series are divided into $M$-dimensional vector space, and then the correlation function $C_M(R)$ is calculated. As shown in Fig 3(a), the area between the two green lines is the linear scaling range. The slopes of these lines in the linear scaling range express the correlation dimension $D$. The slope value of 1.140, $\ldots$, 1.716, $\ldots$, 2.109 corresponds to the $M$ of 2, $\ldots$, 5, $\ldots$, 10, respectively. The corresponding relationship between the $D$ and $M$ is depicted in Fig 3(b).

![Figure 1](image-url)  
*Figure 1. D vs. M for random noise (crosses), Lorenz system (filled circles) and Lorenz system with added noise (dashed line).*
Figure 2(a–g). IDV light curves of the entire monitoring period. The B-, V- and R-band points are marked by squares, circles and triangles, respectively. For clarity, the B- and R-band light curves are shifted by $-0.4$ and $+0.3$ mag, respectively (Wu et al. 2012).

The saturation value of $D$ can be deduced from Fig. 3(a). The big blue circle includes $M$ from 2 to 9. The lines lying in it are sparse and the slopes increase gradually, which indicates that the $D$ does not reach the saturation state. After the $M$ beyond 10, the lines lying in the small blue circle become dense and the slopes remain stable, which indicates that the correlation dimension $D$ has reached saturation. Specific calculations are done by MATLAB program (GP arithmetic, Grassberger & Procaccia 1983).

The same analysis is carried out to the remaining data. Figure 4(a–g) shows the $D$ vs. $M$ curves for the data points on different nights. The details are presented in Table 1. Column 1 is the observation ID, columns 2 to 7 are the correlation dimension $D$ and the embedding dimension $M$ for B-, V- and R-band data, respectively.

Figure 3. Variation of $\log C_M(R)$ as a function of $\log R$ for different embedding dimensions (a) and the correlation dimension $D$ vs. embedding dimension $M$ curves (b), for JD 2,454,088.
4. Conclusions

Wu et al. (2012) performed a quantitative assessment on whether there are variations on these observational data using analysis of variance. Their conclusions have pointed out that there are variations on JDs 2,454,088, 2,454,090, 2,454,092 and 2,454,097, and no variations on JDs 2,454,093 and 2,454,094. On the remaining date, JD 2,454,091, the variation in the V-, R-bands were not obvious and no variation was noticed in the B-band.

According to our calculations, the saturation values of the correlation dimension $D \approx 1.993 - 5.178$ for all the observed data in this period are quite different from the cases for the stochastic system and the pure chaotic system, in fact, were just between them. As indicated by simulations of the Lorenz system with noise, the effect of additional noise in the data is to increase the $D$ values. Hence, it is reasonable to infer that the IDV behavior of S5 0716+714 may arise from a chaotic system with some additional noise.
References