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Ion–Cyclotron Resonance Frequency Interval Dependence on the O VI Ion Number Density in the North Polar Coronal Hole 1.5*R*–3*R* Region

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Abstract. The frequency intervals in which O VI ions get in resonance with ion–cyclotron waves are calculated using the kinetic model, for the latest six values found in literature on O VI ion number densities in the $1.5R$–$3R$ region of the NPCH. It is found that the common resonance interval is 1.5 kHz to 3 kHz. The R-variations of wave numbers necessary for the above calculations are evaluated numerically, solving the cubic dispersion relation with the dielectric response derived from the quasi-linear Vlasov equation for the left-circularly polarized ion-cyclotron waves.

Key words. Solar corona—ion–cyclotron waves—coronal heating—Vlasov equation.

1. Introduction

Heating of solar corona is still an active research area. It has recently been suggested that the ion–cyclotron resonance could play a key role in the problem of coronal heating. It is generally argued that various electromagnetic plasma waves including Alfvén waves, slow magneto-acoustic waves and their dissipation at small scales and play an important role in this heating process both in the inner and the outer corona (Deforest & Gurman 1998; Ofman *et al.* 1999; Bemporad & Abbo 2012). Most of the waves responsible for the fluctuations in the solar corona can be described using the magnetohydrodynamics (MHD) formulation, which is usually based on a collisional closure (Chmielewski *et al.* 2013). However especially in the outer corona (i.e. $r > 1.5R$) and for the scales responsible for coronal heating, where resonances such as the ion cyclotron resonance are thought to be important, a collisional closure is not well justified. In addition, predictions for various physical observables seem to be in better agreement with observations when the interactions between MHD waves propagating from solar base and the particles in the North Polar Coronal Hole

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(NPCH) is considered in the context of a kinetic model. Vocks & Marsch utilized this success of the kinetic model in confronting the heating problem of solar coronal plasma (Vocks & Marsch 2002). Solar and Heliospheric Observatory (SOHO) data also showed an important temperature anisotropy for the ions in the NPCH region, that is, much bigger values of $T_\perp$ were measured compared to those of $T_\parallel$ for the O VI, Mg X and Si VIII ions (Marsch 1999). These facts suggest a kinetic approach for the problem of coronal heating and in particular the ion cyclotron resonance interval dependence may be justified.

It is usually argued that solar corona can be divided into two main regions, the inner corona or the coronal base, where all species are collisionally coupled via Coulomb collisions, and the outer corona or the extended corona, which includes the supersonic wind and is nearly collisionless. The difference between the two regions is also visible in the structure of the magnetic fields, with dynamic loop-like structures in the coronal base and a uniform flux expansion in the extended corona (Parker 1991; Spruit *et al.* 1991; Cranmer 2002). The first work on SOHO/UVCS spectral data on the Mg X and O VI ions argue that the plasma properties in NPCH in the 1.75–2.1R regions (Doyle *et al.* 1999; Cranmer *et al.* 1999) correspond to collisionless plasmas. However after 1999, Doyle *et al.* started this region at 1.5R. This means that beyond 1.5R, plasma transport processes cannot be explained by classical collisional processes.

The consideration of the low value of plasma beta together with temperature anisotropy in NPCH means that the cause of higher $T_\perp$ is the ion–cyclotron resonance, which acts primarily in the perpendicular direction. On the other hand, when the perpendicular temperature values for various ion species are compared, it is seen that heavy ions are heated more (Wilhelm *et al.* 1998). Besides the temperature anisotropy for each ion, the priority of heavy ions in heating is another important observational data obtained from SOHO which suggests the ion–cyclotron resonance (Marsch & Tu 1997; Doyle *et al.* 1999; Cranmer *et al.* 1999).

In the kinetic model, Vlasov equation governs the time evolution of the distribution function of the plasma as a result of the energy transfer from ion–cyclotron waves to particles. Vocks & Marsch (2001) studied the distribution function in the solar corona up to 0.57R ($R = r/R_\odot$) by solving the Vlasov equation in the context of quasilinear approximation. They showed that $T_\perp$ rises up to 3×10^7 K for O VI ions.

Pekünlü *et al.* (2004) solved dispersion relation by using the Vlasov equation in the NPCH 1.5R–3.5R region by assuming that the numerical density of O VI ions equals $10^{-3} N_\mathrm{p}$; N_e varies with R as given by Feldman *et al.* (1997) and solar coronal plasma is neutral. With their results they supported the proposal that in the 2.5 kHz–10 kHz frequency interval, the ion–cyclotron resonance is one of the basic alternatives which can explain heating of solar coronal plasma.

It is known that in NPCH the plasma parameters remain the same only for a few years (Kohl *et al.* 2006). To reduce this indefiniteness in the plasma parameters, one must use their newest values. The frequency interval in which dispersion occurs in the ion–cyclotron waves also changes with these parameters.

In this work, we calculate the frequency intervals in which O VI ions get in resonance with ion–cyclotron waves for six different O VI ion number density values. For this we solve the dispersion relation in the NPCH 1.5R–3.5R region under the following assumptions: (i) NPCH plasma is neutral (Wilhelm *et al.* 1998); (ii) the relations connecting the lower and upper limits of O VI to proton number density are as given by Cranmer *et al.* (2008) and (iii) the analytical models of Doyle *et al.* (1999), Feldman *et al.* (1997) and Esser *et al.* (1999) determine the electron number density.

2. Properties of NPCH plasma

We start by presenting the physical properties of the NPCH plasma that are important for our work. The ion plasma and ion–cyclotron frequencies are $\omega_{\rm ip} = \sqrt{N_{\rm OVI} q_{\rm i}^2/\varepsilon_0 m_{\rm i}}$ and $\omega_{\rm ic} = (q_{\rm i} B/m_{\rm i})$ respectively, where $q_{\rm i}$ is the ion charge, ε_0 is the vacuum permittivity, $m_{\rm i}$ is the mass of O VI ions and B is the magnetic field strength (Seshadri 1973). In the region that we are going to work, the magnetic field strength is given by Hollweg (1999) as $B(R) = 1.5(f_{\max} - 1)R^{-3.5} + 1.5R^{-2}$ in Gauss, $f_{\max} = 9$. Moreover we need the relations connecting the lower and upper limits of O VI to proton number density. For these limits we use Cranmer's values: $8 \times 10^{-7} N_{\rm p}$ and $2.4 \times 10^{-6} N_{\rm p}$. In the literature three analytical models for electron number densities given by Doyle, Feldman and Esser exist. We shall use all of these models for comparison. They are as given below:

$$N_{\rm e} = 10^6 \left(\frac{1 \times 10^8}{R^8} + \frac{2.5 \times 10^3}{R^4} + \frac{2.9 \times 10^5}{R^2} \right) \mathrm{m}^{-3}, \tag{1}$$

$$N_{\rm e} = 10^6 \left(\frac{3.2 \times 10^8}{R^{15.6}} + \frac{2.5 \times 10^6}{R^{3.76}} + \frac{1.4 \times 10^5}{R^2} \right) \mathrm{m}^{-3}, \tag{2}$$

$$N_{\rm e} = 10^6 \left(\frac{3.7 \times 10^8}{R^{16.86}} + \frac{1 \times 10^7}{R^{9.64}} + \frac{2.4 \times 10^6}{R^{3.76}} \right) \mathrm{m}^{-3}. \tag{3}$$

To find the R dependence of $T_\parallel$ and $T_\perp$ for O VI ions we used the work of Cranmer (2009). By using the numerical analysis techniques from that work, we generated the following polynomial representations of $T_\parallel$ and $T_\perp$ as

$$T_\perp(R) = -aR^2 + bR - c, \tag{4}$$

$$T_\parallel(R) = dR^6 - eR^5 + fR^4 - gR^3 + hR^2 - kR + l \tag{5}$$

with the coefficients

$$\begin{aligned} &a = 3089565, \quad b = 112140917, \quad c = 165236923;\\ &d = 346836459, \quad e = 5417086469, \quad f = 34628809623,\\ &g = 115913467671, \quad h = 214129741099,\\ &k = 206844054714, \quad l = 81605668892. \end{aligned} \tag{6}$$

3. Dispersion relation in NPCH 1.5*R*–3*R* interval for O VI ions

The wave equation

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2} \boldsymbol{\varepsilon} \mathbf{E} = 0 \tag{7}$$

obtained under the cold plasma approximation governs the interaction between the ion cyclotron waves and plasma particles in NPCH (Stix 1962). The dielectric tensor $\boldsymbol{\varepsilon}$ appearing in the wave equation is calculated using the following quasi-linearized Vlasov equation:

$$-i\left(\mathbf{k}\cdot\mathbf{v}-\omega\right)f+\frac{q}{m}\left(\mathbf{v}\times\mathbf{B}_0\right)\cdot\frac{\partial f}{\partial\mathbf{v}}=-\frac{q}{m}\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f_0}{\partial\mathbf{v}} \tag{8}$$

which defines the deviation of the distribution function f from the Maxwellian distribution function f_0. Here $\mathbf{B}_0$ is the solar coronal magnetic field and ω is the frequency of the plane wave. In this case, calculations show that both J_x/E_x and J_y/E_y ratios take the same constant value (Schmidt 1966). Hence σ is a scalar and $\boldsymbol{\varepsilon}=\varepsilon\mathbf{I}$, where $\mathbf{I}$ is the unit matrix and

$$\varepsilon\equiv1+\frac{\sigma}{i\omega\varepsilon_0}=1+\frac{q^2\pi}{m\omega^2\varepsilon_0}\int_{-\infty}^{\infty}\int_{0}^{\infty}\frac{\left[(\omega-kv_\parallel)\frac{\partial f_0}{\partial v_\perp}+kv_\perp\left(\frac{\partial f_0}{\partial v_\parallel}\right)\right]}{\omega-\omega_{\mathrm{ic}}-kv_\parallel}v_\perp^2\,\mathrm{d}v_\perp\mathrm{d}v_\parallel. \tag{9}$$

Since for the left circularly polarized ion–cyclotron waves

$$n^2=\varepsilon=\frac{c^2}{\omega^2}k^2, \tag{10}$$

the dispersion relation becomes

$$k^2\left(c^2-\frac{\omega^2}{k^2}\right)=\int_{-\infty}^{\infty}\frac{\frac{q^2\pi}{m\varepsilon_0}\int_0^\infty\left[\left(\frac{\omega}{k}-v_\parallel\right)\frac{\partial f_0}{\partial v_\perp}v_\perp^2+\frac{\partial f_0}{\partial v_\parallel}v_\perp^3\right]\mathrm{d}v_\perp}{\frac{\omega}{k}-v_\parallel-\frac{\omega_{\mathrm{ic}}}{k}}\mathrm{d}v_\parallel. \tag{11}$$

The exact value of this integral contains a residual contribution; however since this contribution diminishes at $\omega=\omega_{\mathrm{ic}}$, in this work we omit it. For simplification, we assume that Maxwell velocity distribution is valid in both directions (parallel and perpendicular to the magnetic field) at the first approximation. That is

$$f_0=N_{\mathrm{OVI}}\alpha_\perp^2\alpha_\parallel\pi^{\frac{3}{2}}\exp\left[-\left(\alpha_\perp^2v_\perp^2+\alpha_\parallel^2v_\parallel^2\right)\right]. \tag{12}$$

Here $\alpha_\perp=(2k_{\mathrm{B}}T_\perp/m)^{-\frac{1}{2}}$ and $\alpha_\parallel=(2k_{\mathrm{B}}T_\parallel/m)^{-\frac{1}{2}}$. Expanding

$$\frac{1}{\omega-kv_\parallel-\omega_{\mathrm{ic}}} \tag{13}$$

in power series up to third order and using this into the integral, we get the following cubic dispersion relation:

$$\frac{\omega_{\mathrm{ip}}^2}{(\omega-\omega_{\mathrm{ic}})^3\,\alpha_\parallel^3\sqrt{\pi}}\left[\frac{T_\perp}{T_\parallel}-1\right]k^3+\left[\frac{\omega_{\mathrm{ip}}^2}{2(\omega-\omega_{\mathrm{ic}})^2\alpha_\parallel^2}\left(\frac{\omega}{\omega-\omega_{\mathrm{ic}}}+\frac{T_\perp}{T_\parallel}-1\right)+c^2\right]k^2$$
$$+\left[\frac{\omega_{\mathrm{ip}}^2}{\sqrt{\pi}\,(\omega-\omega_{\mathrm{ic}})\,\alpha_\parallel}\left(\frac{\omega}{\omega-\omega_{\mathrm{ic}}}+\frac{T_\perp}{T_\parallel}-1\right)\right]k+\frac{\omega_{\mathrm{ip}}^2\omega}{(\omega-\omega_{\mathrm{ic}})}-\omega^2=0, \tag{14}$$

by integrating term by term.

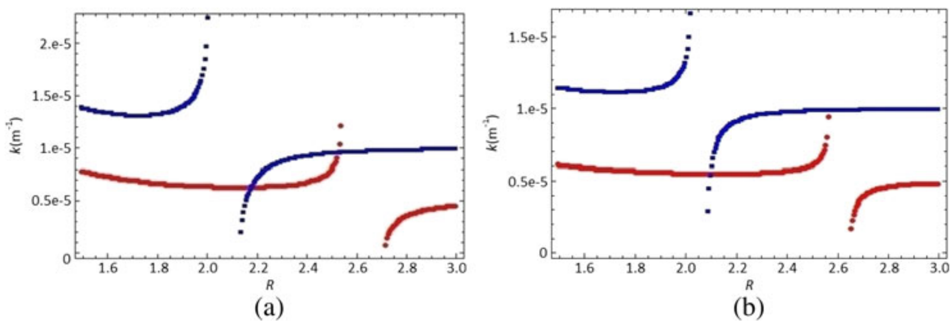

Figure 1. Variation of wave number k versus R. Red line indicates $\omega = 1.5$ kHz and blue line indicates $\omega = 3$ kHz. Here Doyle's electron number density formula and the upper and lower limits of O IV to proton number densities (**a**) 2.4×10^{-6} and (**b**) 8×10^{-7} by Cranmer are used.

4. Results and discussion

Pekünlü *et al.* (2004), in their paper showed that ion–cyclotron resonances may be an important mechanism to heat corona in the NPCH $1.5R$–$3R$ region. Their results were based on the quasi-neutrality of the plasma; two O VI number density values, recorded in 1997 and 1998, $6.8 \times 10^{-5} N_P$ (Raymond *et al.* 1997) and $10^{-3} N_P$ (Wilhelm *et al.* 1998) respectively and the Feldman's electron number density. They determined the frequency interval in which O VI ions get in resonance with ion–cyclotron waves as 2.5–10 kHz.

Since after 2008 new data on N_{OVI}/N_p are available, we wish to see how these will affect the frequency intervals in which O VI ions get in resonance with ion–cyclotron waves. We also want to search the other two electron number density analytic models, Doyle's and Esser's models on the resonance interval. We calculated six different number densities for O VI ions. For each number density we solved the dispersion relation using the data given in section 2. Solutions of the dispersion relation give us the frequency intervals in which resonance occurs.

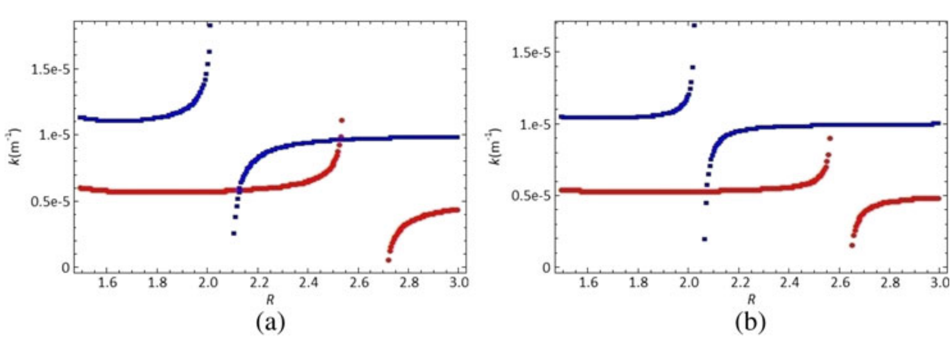

Figure 2. Variation of wave number k versus R. Red line indicates $\omega = 1.5$ kHz and blue line indicates $\omega = 3$ kHz. Here Feldman's electron number density formula and the upper and lower limits of O IV to proton number densities (**a**) 2.4×10^{-6} and (**b**) 8×10^{-7} by Cranmer are used.

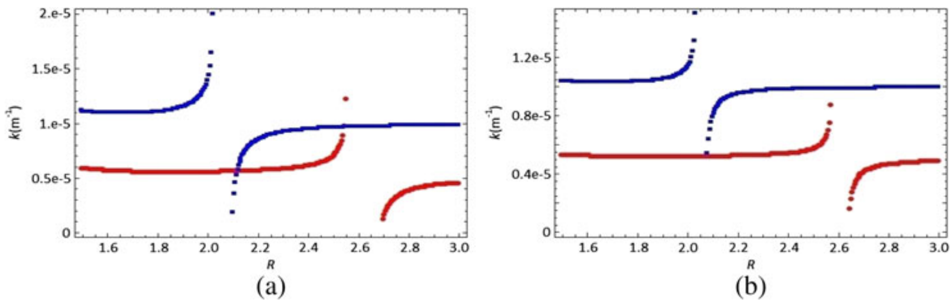

Figure 3. Variation of wave number k versus R. Red line indicates $\omega = 1.5$ kHz and blue line indicates $\omega = 3$ kHz. Here Esser's electron number density formula and the upper and lower limits of O IV to proton number densities (**a**) 2.4×10^{-6} and (**b**) 8×10^{-7} by Cranmer are used.

Substituting the numerical values of ω_{ip} and ω_{ic}, the upper and lower limits of $N_{\text{OVI}}/N_{\text{p}}$, the R-variation formulas for Hollweg's magnetic field strength, $T_{\perp}$ and $T_{\|}$ temperatures and $N_{\text{e}}(R)$ given by Doyle, Feldman and Esser, respectively we numerically solved the k variation versus R from the dispersion relation. The k curves show infinite discontinuities at certain R values where left-circularly polarized ion-cyclotron waves propagating along the magnetic field line direction from the base of solar corona get in resonance with O VI ions at $\omega = \omega_{\text{ic}}$. The results are shown in Figures 1–3(a) and (b). For ω values less than 1.5 kHz and for those values larger than 3 kHz we could not obtain infinite discontinuity in the NPCH $1.5R$–$3R$ region for all O VI ion number densities. Therefore we say that the common resonance frequency interval is 1.5 kHz to 3 kHz. In all the figures the red curves indicate $\omega =$ 1.5 kHz and the blue curves indicate $\omega = 3$ kHz. The R values where ion–cyclotron wave resonance occurs naturally change from figure to figure.

The Doyle's, Feldman's and Esser's electron number density models differ from each other in the $1.5R$–$2.5R$ region although they have almost the same in the $2.5R$–$3R$ region. To see if this deviation would have any effect on the ion–cyclotron resonance interval we solved the dispersion relation for each one and saw that they did not have any significant effect as we had expected (Figures 1a, 2a, 3a). However, the upper and lower $N_{\text{OVI}}/N_{\text{p}}$ ratio values yielded reasonable results ((a) and (b) in all figures).

5. Conclusions

We find the common frequency interval that resonance occurs for the six different O VI ion number densities in the $1.5R$–$3R$ region of NPCH by solving numerically the k variation versus R from the dispersion relation. Our results also depend on the observed $T_{\perp}$ and $T_{\|}$ temperature values. The Doyle's, Feldman's and Esser's electron number density models are almost equivalent to each other in calculating the frequency interval in which O VI ions get in resonance with ion–cyclotron waves even if they differ in the $1.5R$–$2.5R$ region. However, the newest upper and lower limits of $N_{\text{OVI}}/N_{\text{p}}$ given by Cranmer *et al.* (2008) play a role in the numerical values related to this interval. This work improves the limits of ion–cyclotron resonance according to the new data available. In future this work may be reconsidered according to the newest data.

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