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Stochastic Resonance of Accretion Disk and the Persistent Low-Frequency Quasi-Periodic Oscillations in Black Hole X-ray Binaries

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Abstract. In this paper, we use a Langevin type equation with a damping term and stochastic force to describe the stochastic oscillations on the vertical direction of the accretion disk around a black hole, and calculate the luminosity and power spectral density (PSD) for an oscillating disk. Then we discuss the stochastic resonance (SR) phenomenon in PSD curves for different parameter values of viscosity coefficient, accretion rate, mass of black hole and outer radius of the disk. The results show that our simulated PSD curves of luminosity for disk oscillation have the same profile as the observed PSD of black hole X-ray binaries (BHXBs) in the low-hard state, and the SR of accretion disk oscillation may be an alternative interpretation of the persistent low-frequency quasi-periodic oscillations (LFQPOs).

Key words. Accretion disks—power spectral density—stochastic resonance.

1. Introduction

Since quasi-periodic oscillations (QPOs) have been observed in low mass X-ray binaries where the central sources are neutron stars or black holes, discoseismology becomes one of the most important fields in studying its origin. Especially for 0.1–10 Hz LFQPOs, a number of disk oscillation models have been suggested to explain them. For instance, Chakrabarti & Manickam (2000) have pointed out that the LFQPOs in quiescent states and the duration of the quiescent state can be explained by the radial oscillation of shocks in accretion flows. Milson & Taam (1997) have studied the global inertial-acoustic oscillations in optically thick accretion disk at the maximum epicyclic frequency, and found that this global mode can produce a strong peak (LFQPOs) in the luminosity power spectrum. Giannios & Spruit (2004) have proposed that the LFQPOs in black hole accretion may be excited by p-mode oscillations in a truncated disk. Titarchuk & Osherovich (1999) have used a two-oscillator model to explain the classification of QPOs in frequency range from a few Hz to around a kilohertz (Osherovich & Titarchuk 1999; Titarchuk & Osherovich 1999). Hereafter, they have proposed that the persistent low-frequency ($\sim 10^{-2} - 1$ Hz) oscillations are related to the global disk oscillations under the influence of the gravitational force of the central object (Titarchuk & Osherovich 2000). However, the observed luminosity variation or PSD of LFQPOs have not been simulated in most of the disk oscillation models, it seem to indicate that they are incomplete and not convincing.

In recent years, some researchers have studied the stochastic oscillations of accretion disk subject to stochastic force of the large scales, which represents either internal force of the disk by the nonlinear terms or the interaction with external medium, such as tidal force, shock wave, outbursts, friction force, etc. The stochastic dynamics of Keplerian accretion disks under white force in space and time has been studied firstly by Ioannou & Kakouris (2001). They have calculated the statistical steady state, and found that the stochastic force can maintain accretion to the main body if the force is broadband and adequately distributed. Harko & Mocanu (2012) have studied in detail the vertical stochastic oscillations of the accretion disks in both the static Schwarzschild and rotating Kerr geometries, and explicitly obtained the vertical displacements, velocities and luminosities of the stochastically perturbed disks by integrating it numerically. By studying the stochastic oscillations of an accretion disk around a supermassive black hole, Leung *et al.* (2011) have explained the observations of optical intra-day variability of BLLac objects. In the above research, the oscillation on the vertical direction of the accretion disk resembles the motion of a linear harmonic oscillator, which can be described by using the following Langevin type equation:

$$m\ddot{x} + \beta\dot{x} + kx = F(t), \tag{1}$$

where m and x are the mass and the displacement of the oscillator, and β/m and $\omega_0 = \sqrt{k/m}$ represent the damping rate and the intrinsic frequency, respectively. *F(t)* is a stochastic force, which can be called the noise term. The first and the second moment of a harmonic oscillator subject to different types of noise have been widely studied (Gitterman 2003, 2004; Gitterman & Shapiro 2011; Mankin *et al.* 2008), their results show that the stochastic resonance (SR) can be produced easily in this system. The SR phenomenon was pointed out for the first time in 1981 in connection with the Earth's ice age by Benzi *et al.* (1981). A typical character of SR is that the response of the system is enhanced by the noise due to the cooperative effect of noise and an external action or intrinsic dynamics (Jiang & Xin 2000). Conventional SR is shown by the relation between the signal-to-noise ratios with the noise intensity. Berdichevsky & Gitterman (1999) first found SR in a broad sense, where SR exists as non-monotonic behavior of the other response quantity (such as moment, correlation function, power spectrum, etc.) with input parameters except noise intensities, such as driving frequency, driving amplitude or noise-correlation-time. The SR has been largely studied for more than three decades due to its various applications in different field of science such as biology, physics and chemistry.

The purpose of the present paper is to study the SR phenomenon of stochastic oscillation in accretion disk, and explain the observed LFQPOs of low mass X-ray binaries. When accretion disk is subjected to a stochastic force, the amplitude of the disk oscillation may appear SR, and it must influence the oscillating luminosity. The QPOs, a significant character of luminosity variability, may relate to the stochastic oscillation of accretion disk. Therefore, in this paper, based on the research of Titarchuk & Osherovich (2000), we have calculated the PSD of vertical oscillating luminosity of accretion disk subject to viscidity force and a white noise type stochastic force, and discussed in detail the SR phenomenon of PSD and proposed an alternative explanation of the observed persistent LFQPOs ($\sim 10^{-2} - 1$ Hz) in X-ray binaries.

2. The PSD of vertical stochastic oscillating luminosity of accretion disk

We consider a standard thin disk as the whole body under the influence of gravity of central object, viscidity force and a stochastic force. The vertical motion of disk may be described by the Langevin equation, which can be written as (Leung *et al.* 2011)

$$M_{\rm d}\ddot{z} + \beta\dot{z} + kz = F(t). \tag{2}$$

The quantities $M_{\rm d}$ and z denote the mass and vertical oscillating displacement of disk, respectively. *F(t)* is a white noise type stochastic force, whose statistical properties are

$$\langle F(t)\rangle = 0, \langle F(t)F(s)\rangle = D\delta(t-s) = CM_{\rm d}\delta(t-s), \tag{3}$$

where $\delta(t-s)$ is the Kronecker delta function. We assume that the intensity of the stochastic force D and C are constants, and let $\gamma = \beta/2M_{\rm d}$, $\omega_0 = \sqrt{k/M_{\rm d}}$, $\omega = \sqrt{\omega_0^2 - \gamma^2}(\gamma < \omega_0)$. Then equation (2) can be changed to

$$\ddot{z} + 2\gamma\dot{z} + \omega_0^2 z = \frac{F(t)}{M_{\rm d}}. \tag{4}$$

If we choose the initial displacement and velocity of the disk to be

$$z(0) = z_0, \quad \dot{z}(0) = v_0, \tag{5}$$

then we can solve equation (4) formally as

$$\begin{aligned} z(t) &= {\rm e}^{-\gamma t}\left[z_0\cos(\omega t) + \frac{\gamma z_0 + v_0}{\omega}\sin(\omega t)\right] \\ &\quad + \frac{1}{M_{\rm d}\omega}\int_0^t F(s){\rm e}^{-\gamma(t-s)}\sin[\omega(t-s)]{\rm d}s. \end{aligned} \tag{6}$$

The velocity can be obtained as

$$\begin{aligned} \dot{z}(t) &= {\rm e}^{-\gamma t}\left[v_0\cos(\omega t) - \frac{\gamma v_0 + \omega_0^2 z_0}{\omega}\sin(\omega t)\right] \\ &\quad + \frac{1}{M_{\rm d}\omega}\int_0^t F(s){\rm e}^{-\gamma(t-s)}\gamma\sin[\omega(t-s)] - \omega\cos[\omega(t-s)]{\rm d}s. \end{aligned} \tag{7}$$

The total energy of the disk oscillation can be given by

$$E = \frac{1}{2}M_{\rm d}\dot{z}^2 + \frac{1}{2}M_{\rm d}\omega_0^2 z^2. \tag{8}$$

The luminosity of disk oscillation represents the energy lost in the disk due to viscous dissipation and the presence of the stochastic force, and can be given by (Harko & Mocanu 2012)

$$L(t) = -\frac{\mathrm{d}E}{\mathrm{d}t} = 2M_{\mathrm{d}}\gamma\dot{z}^2 - F(t)\dot{z}. \tag{9}$$

The steady-state mean autocorrelation function of luminosity can be described as

$$\begin{aligned} C_{LL}(\tau) &= \lim_{t\to\infty}\overline{\langle L(t+\tau)L(t)\rangle} \\ &= \lim_{t\to\infty}[4M_{\mathrm{d}}^2\gamma^2\langle\dot{z}^2(t)\dot{z}^2(t+\tau)\rangle + \langle F(t)F(t+\tau)\dot{z}(t)\dot{z}(t+\tau)\rangle \\ &\quad -2M_{\mathrm{d}}\gamma\langle F(t)\dot{z}(t)\dot{z}(t+\tau)^2\rangle - 2M_{\mathrm{d}}\gamma\langle F(t+\tau)\dot{z}(t+\tau)\dot{z}(t)^2\rangle]. \end{aligned} \tag{10}$$

Substituting equations (3) and (7) into the above equation, we can obtain

$$C_{LL}(\tau) = \frac{3C^2\mathrm{e}^{-2\gamma\tau}}{2\omega^2}\left[(\omega^2-\gamma^2)\cos(\omega\tau) - 2\omega\gamma\sin(\omega t)\cos(\omega t) + \gamma^2\right]. \tag{11}$$

By Fourier transform, equation (11) may be translated into the power spectrum density (PSD) of the oscillating luminosity of the accretion disk as

$$P(\nu) = \frac{3C^2\gamma\left(\omega_0^4+4\gamma^2\pi^2\nu^2+4\pi^4\nu^4-3\pi^2\nu^2\omega_0^2\right)}{\sqrt{32}(\gamma^2+\pi^2\nu^2)\left[\gamma^2+\left(\pi\nu-\sqrt{\omega_0^2-\gamma^2}\right)^2\right]\left[\gamma^2+\left(\pi\nu+\sqrt{\omega_0^2-\gamma^2}\right)^2\right]}. \tag{12}$$

3. Stochastic resonance in the PSD of oscillating luminosity of accretion disk

For a standard thin disk, we assume that the center of the disk is a compact object of mass M, and the surface density distribution is approximated by the formulae (Shakura & Sunyaev 1973; Titarchuk & Osherovich 2000)

$$\begin{aligned} \Sigma &= \Sigma_0 = \text{constant for } R_{\mathrm{in}} \leq R \leq R_{\mathrm{adj}}, \\ \Sigma &= \Sigma_0\left(\frac{R}{R_{\mathrm{adj}}}\right)^{-\mu} \text{ for } R_{\mathrm{adj}} \leq R \leq R_{\mathrm{out}}, \end{aligned} \tag{13}$$

where R_{in} and R_{out} are the innermost and outer radius of disk, respectively. R_{adj} is an adjustment radius in the disk, and its value depends on the effective Reynolds number which is within (2–3)R_{in}. The index μ of the surface density is either 3/5 or 3/4.

According to equation (13), we can obtain the mass of the whole disk

$$M_{\mathrm{d}} = 2\pi\int_{R_{\mathrm{in}}}^{R_{\mathrm{out}}}\Sigma R\mathrm{d}R = 5.6\times10^{11}\frac{\Sigma_0 m^2 r_{\mathrm{adj}}^2}{2-\mu}\left(\frac{r_{\mathrm{out}}}{r_{\mathrm{adj}}}\right)^{2-\mu}, \tag{14}$$

where we have $R_{\rm in} = 3R_g$, $r_{\rm out} = R_{\rm out}/R_{\rm in}$, $r_{\rm adj} = R_{\rm adj}/R_{\rm in}$, $m = M/M_\odot$ and $R_g = 2GM/c^2$ is the Schwarzschild radius. Surface density constant is given by (Shakura & Sunyaev 1973)

$$\Sigma_0 = 4.6\alpha^{-1}\dot{m}^{-1} r_{\rm adj}^{3/2}(1 - r_{\rm adj}^{-1/2})^{-1}. \tag{15}$$

The quantities α and $\dot{m}$ denote viscidity coefficient and mass accretion rate with critical accretion rate unit.

According to literatures (Leung *et al.* 2011; Titarchuk & Osherovich 2000), we consider that the damping rate in equation (4) is associated with the viscidity coefficient α for accretion disk. Then we assume the expression of γ to be

$$2\gamma = 4.2 \times 10^{26}\alpha/M_{\rm d}, \tag{16}$$

and we can obtain the intrinsic frequency as

$$\omega_0 = \frac{4.4\pi \times 10^3 \text{ Hz}}{m} \frac{2-\mu}{r_{\rm out}^{2-\mu} r_{\rm adj}^{\mu}} \left[1 - \frac{\mu}{(\mu+1) r_{\rm adj}}\right]^{1/2}. \tag{17}$$

Substituting equations (16), (17) into equation (12), we can obtain the relation between the PSD of stochastic oscillating luminosity of accretion disk and frequency. We have plotted the PSD curves of several accretion disk models for various viscosity parameters α, mass accretion rates $\dot{m}$, mass of centre compact object m, and radii of disk out boundary $r_{\rm out}$ (shown in Fig. 1). In all sequences, the index μ of the surface density is chosen to be 3/5, the adjustment radius of the disk is $R_{\rm adj} = 3R_{\rm in}$, and the intensity of the stochastic force is taken as $C = 1$.

It is found that in Fig. 1 the PSD is not decreasing monotonously with frequency, one resonance peak appears when these parameters satisfy a certain condition. We consider this is the SR phenomenon in a broad sense.

Figures 1(a) and (b) show that α and $\dot{m}$ have same influence on the PSD curves. According to equations (14), (15) and (16), we can see that α and $\dot{m}$ determine the value of damping of the accretion disk oscillation, and do not influence the intrinsic frequency. Therefore, as the parameters α and $\dot{m}$ are decreased, the damping coefficient is also decreased. In this case, the resonance peak of PSD curve becomes higher and acute, the resonance phenomenon is more prominent, but the position of peak hardly changes. On the contrary, the resonance becomes weak as α and $\dot{m}$ increase, and when they are taken to a higher value, the resonance peak disappears.

From Figures 1(c) and 1(d), both the parameters m and $R_{\rm out}$ have the same influence on the PSD too. And when they are taken different values, we can find that the SR with one peak exists in all curves of the PSD. As m and $R_{\rm out}$ increase, both the damping rate and the intrinsic frequency of accretion disk oscillation decreases. Therefore, as the intrinsic frequency decreases, the resonance peak of the PSD curves becomes higher and acute, and its position moves towards the decreased frequency, so the SR phenomenon is more obvious.

According to the above discussion, we can find that the intensity of stochastic force does not change the profile of the PSD, but the damping term decided by the parameters α, $\dot{m}$, m and $R_{\rm out}$ influence the height of resonance peak, and both m and $R_{\rm out}$ deciding the intrinsic frequency of oscillation influence the central frequency

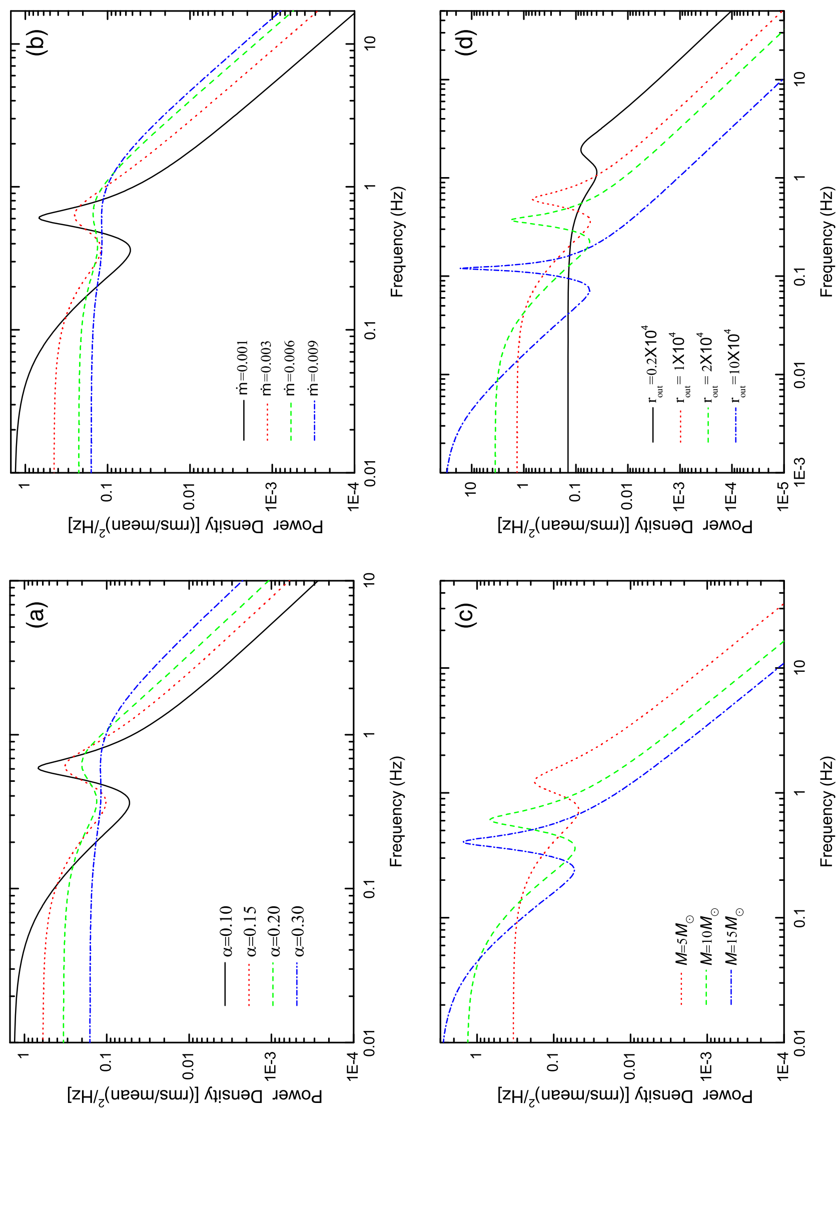

Figure 1. The PSD of the luminosity of the stochastically oscillating accretion disk. $C = 1.0$, **(a)** $\dot{m} = 0.001$, $M = 10M_\odot$, $r_{\rm out} = 10^4$; **(b)** $\alpha = 0.1$, $M = 10M_\odot$, $r_{\rm out} = 10^4$; **(c)** $\dot{m} = 0.001$, $\alpha = 0.1$, $r_{\rm out} = 10^4$; **(d)** $\dot{m} = 0.001$, $\alpha = 0.1$, $M = 10M_\odot$.

corresponding to the resonance peak. Inspecting equation (9), we consider that the action of stochastic force is the dominating reason producing stochastic oscillating luminosity in accretion disk, which is due to both viscous dissipation and the work of stochastic force. When viscidity and intrinsic frequency satisfy a certain condition, a periodical oscillation due to the SR appears in the PSD of luminosity.

4. Conclusions and discussion

In this present paper, we have introduced a stochastic oscillation model of a thin accretion disk, and obtained the PSD of disk luminosity. The results show that the profile of PSD simulated by our model is the same as the observed results, and the SR phenomenon appearing in the PSD curves is similar to the observed QPOs in low-mass X-ray binaries.

First, the expected relation between the PSD and the frequency occurs in our simulated PSD of black hole binaries. The continuum of PSD in the low-hard state can usually be approximated as a broken power law with a flat power spectrum below ~1 Hz. Above the break, the PSD decreases with frequency, and is characterized by $P(\nu) \propto \nu^{-\beta}$, where the power-law index β presents a steep slope. For example, the PSDs of GRO J1655-40 and GRS1915+105 are well described with $\beta = 0$ in the frequency range $0 \sim 1$ Hz, and above 1 Hz, the power-law index $\beta \approx 2$ (McClintock & Remillard 2006). It is well known with our simulated results (shown in Fig. 1).

Second, the resonance peak in our simulated PSD explain the origin of the observed LFQPOs of black hole binaries. The PSD in the low hard state is characterized by strong LFQPOs (0.1–10 Hz). We consider that the reason LFQPOs are produced in PSD curves is because of the SR of the vertical oscillation of accretion disk subject to stochastic force. Therefore, the LFQPO frequency can be represented as the central frequency of resonance peak.

$$\nu_{\rm QPO} = \nu|_{\frac{\partial P(\nu)}{\partial \nu}=0}. \tag{18}$$

This QPO frequency is approximate to the intrinsic frequency adopted by the oscillation frequency of the whole disk, which was used to explain the persistent LFQPOs by Titarchuk & Osherovich (2000). Therefore, the range of LFQPO frequency in our model is only from 0.001 Hz to a few Hz. For example, for the source of LMC X-1, we adopt the mass of black hole $m = 16$, and $r_{\rm adj} = 2$, and $\mu = 5/3$ (Titarchuk & Osherovich 2000), and assume $\alpha = 0.16$, $\dot{m} = 0.001$, $r_{\rm out} = 10^4$. Then we can obtain that the value of $\nu_{\rm QPO}$ is in the range of 0.077–0.093 Hz. It is very close to the LFQPO frequency 0.08 ± 0.009 Hz found by Ebisawa *et al.* (1989).

In summary, using the model of stochastic oscillation of accretion disk, we can simulate successfully the PSD of black hole binaries in low hard state, and explain well the physical mechanism of LFQPOs with SR phenomenon. However, it seems difficult to explain high frequency QPOs because we could not obtain a high intrinsic frequency of disk oscillation. In addition, our model does not consider the influence of the general relativistic factor, magnetic force and so on, also does not agree the case of a truncated disk and ADAF disk. We will improve our model considering all complex factors in our forthcoming work, and believe that the stochastic oscillation of different type disk in relativistic frame should be more significant to explain the astrophysical observation.

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