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Constraints on the Moment of Inertia of a Proto Neutron Star from the Hyperon Coupling Constants

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Abstract. The influence of the hyperon coupling constants on the moment of inertia of a proto neutron star has been investigated within the framework of relativistic mean field theory for the baryon octet $\{n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0\}$ system. It is found that for a proto neutron star, the mass, the moment of inertia and their own maximum values as a function of radius R or M/R are all more sensitive to the hyperon coupling constants. For all the different hyperon coupling constants mentioned, the case of no hyperons corresponds to the largest moment of inertia.

Key words. The moment of inertia—the relativistic mean field theory—proto neutron star.

1. Introduction

After the core of a massive star collapses to supra-nuclear densities, a proto-neutron star (PNS) is born (Burrows & Lattier 1986). PNS has highly rotating velocity, high temperature and strong magnetic field and it is very important for the understanding of the evolution of celestial bodies.

To describe the rotating properties of the neutron star, Hartle derived the equation of the moment of inertia of a slowly rotating neutron star from the general relativity theory (Hartle 1967). From then on, many people have done research on it (Hartle 1968; Link *et al.* 1999; Li *et al.* 2001).

The relativistic mean field (RMF) theory is one of the more successful methods to calculate the neutron stars matter (Glendenning 1985). As the RMF theory is used to describe the neutron stars, several hyperon coupling constants must be determined. The results show that hyperon coupling constants would greatly affect the properties of neutron stars (Weissenborn *et al.* 2011).

Recently, the moment of inertia of a PNS was investigated with the quantum hadrodynamic equation of state in the presence of strong magnetic fields. In their calculations, RMF theory is used. The results show that the moments of inertia for magnetized neutron stars exhibit rapid changes with density and a release of magnetic field energy could decrease the moment of inertia leading to an increase in the spin rate of the star (Ryu *et al.* 2012).

In this paper, the effect of hyperon coupling constants on the moment of inertia of a PNS is calculated in the framework of RMF theory for the baryon octet system.

2. The relativistic mean field theory and the moment of inertia of a proto neutron star

The Lagrangian density of hadron matter reads as follows (Glendenning 1997):

$$\begin{aligned}\mathcal{L} = & \sum_{\mathrm{B}} \bar{\Psi}_{\mathrm{B}} \left[i\gamma_\mu \left(\partial^\mu + i g_{\omega \mathrm{B}} \gamma_\mu \omega^\mu + \frac{i}{2} g_{\rho \mathrm{B}} \tau \cdot \rho^\mu \right) - (m_{\mathrm{B}} - g_{\sigma \mathrm{B}} \sigma) \right] \Psi_{\mathrm{B}} \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \left(\frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 \right) \\ & - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \rho_{\mu\nu} \cdot \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu \\ & + \sum_{\lambda = e, \mu} \bar{\Psi}_\lambda (i\gamma_\mu \partial^\mu - m_\lambda) \Psi_\lambda. \end{aligned} \tag{1}$$

The energy density and pressure of a PNS are given by

$$\begin{aligned}\varepsilon = & \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 \\ & + \sum_{\mathrm{B}} \frac{2J_{\mathrm{B}}+1}{2\pi^2} \int_0^\infty \kappa^2 \mathrm{d}\kappa \sqrt{\kappa^2 + m^{*2}} \left(\exp\left[(\varepsilon_{\mathrm{B}}(k) - \mu_{\mathrm{B}})/T\right] + 1\right)^{-1} \\ & + \sum_{\lambda = e, \mu} \frac{2J_\lambda+1}{2\pi^2} \int_0^\infty \kappa^2 \mathrm{d}\kappa \sqrt{\kappa^2 + m_\lambda^2} \left(\exp\left[(\varepsilon_\lambda(k) - \mu_\lambda)/T\right] + 1\right)^{-1}, \end{aligned} \tag{2}$$

$$\begin{aligned} p = & -\frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 \\ & + \frac{1}{3} \sum_{\mathrm{B}} \frac{2J_{\mathrm{B}}+1}{2\pi^2} \int_0^\infty \frac{\kappa^4}{\sqrt{\kappa^2 + m^{*2}}} \mathrm{d}\kappa \left(\exp\left[(\varepsilon_{\mathrm{B}}(k) - \mu_{\mathrm{B}})/T\right] + 1\right)^{-1} \\ & + \frac{1}{3} \sum_{\lambda = e, \mu} \frac{2J_\lambda+1}{2\pi^2} \int_0^\infty \frac{\kappa^4}{\sqrt{\kappa^2 + m_\lambda^2}} \mathrm{d}\kappa \left(\exp\left[(\varepsilon_\lambda(k) - \mu_\lambda)/T\right] + 1\right)^{-1}, \end{aligned} \tag{3}$$

where m^* is the effective mass of baryons

$$m^* = m_{\mathrm{B}} - g_{\sigma \mathrm{B}} \sigma. \tag{4}$$

We use the Oppenheimer–Volkoff equation (O–V equation) to obtain the mass and the radius of a PNS

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{(p+\varepsilon)(M+4\pi r^3 p)}{r(r-2M)}, \tag{5}$$

$$M = 4\pi \int_0^r \varepsilon r^2 \mathrm{d}r. \tag{6}$$

The moment of inertia of a slowly rotating netron star is given by (Glendenning 1997)

$$I = \frac{8\pi}{3}\int_0^R \mathrm{d}r r^4 \frac{\varepsilon + p}{\sqrt{1-2M(r)/r}} \frac{[\Omega - \omega(r)]}{\Omega} \mathrm{e}^{-\nu}. \tag{7}$$

Here, ν is given by

$$-\frac{\mathrm{d}\nu(r)}{\mathrm{d}r} = \frac{1}{\varepsilon + p}\frac{\mathrm{d}p}{\mathrm{d}r}, \tag{8}$$

and the angular velocity is given by

$$-\frac{1}{r^4}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4 j \frac{\mathrm{d}\bar{\omega}}{\mathrm{d}r}\right) + \frac{4}{r}\frac{\mathrm{d}j}{\mathrm{d}r}\bar{\omega} = 0. \tag{9}$$

The $j(r)$ is

$$j(r) = \mathrm{e}^{-(\nu+\lambda)} = \mathrm{e}^{-\nu}\sqrt{1-2M(r)/r}, \qquad r < R. \tag{10}$$

The boundary condition are given by

$$\frac{\mathrm{d}\bar{\omega}}{\mathrm{d}r}_{|r=0} = 0, \tag{11}$$

$$\nu(\infty) = 0, \tag{12}$$

$$\overline{\omega}(R) = \Omega - \frac{R}{3}\frac{\mathrm{d}\bar{\omega}}{\mathrm{d}r}_{|r=R}. \tag{13}$$

3. Parameters

In our calculations, the coupling constants of the nucleons are chosen as the GL97 constants (Glendenning 1997) listed in Table 1. The temperature of the PNS, in which the interior of the neutrinos are still trapped, is in the range of ~10 to 30 MeV (Burrows & Lattier 1986; Prakash *et al.* 1997). So the PNS temperature is chosen as $T = 15$ MeV in this work.

Table 1. The coupling constants of the nucleons GL97.

m	m_σ	m_ω	m_ρ	g_σ	g_ω	g_ρ	g_2	g_3	C_3	ρ_0	B/A	K	a_{sym}	m^*/m
939	500	782	770	7.9835	8.7	8.5411	20.966	−9.835	0	0.153	16.3	240	32.5	0.78

We define the ratios: $x_\sigma = x_{\sigma h} = \frac{g_{\sigma h}}{g_{\sigma N}}$, $x_\omega = x_{\omega h} = \frac{g_{\omega h}}{g_{\omega N}}$, $x_\rho = x_{\rho h} = \frac{g_{\rho h}}{g_{\rho N}}$, with h denoting hyperons. The research results show that the ratio of hyperon coupling constant to nucleon coupling constant is in the range of ∼1/3 to 1 (Glendenning & Moszkowski 1991). So the parameters in our calculations can be chosen as

(1) $x_\sigma = x_\omega = x_\rho = \sqrt{\frac{2}{3}}$;
(2) $x_\sigma = x_\omega = x_\rho = 1$;
(3) $x_\sigma = x_\omega = x_\rho = 0.65$;
(4) $x_\sigma = x_\omega = 0.62$, $x_\rho = 0.7$;
(5) $x_\sigma = 0.6$, $x_\omega = 0.66$, $x_\rho = 0.7$.

In addition, because the properties of the neutron stars including neutrons and protons are different from those including octet (Glendenning 1985), it is meaningful to consider the case not including hyperons, i.e.

(6) no hyperons.

At last, as the x_ω and x_ρ are chosen by SU(6) symmetry,

$$g_{\omega N}/3 = g_{\omega\Sigma}/2 = g_{\omega\Lambda}/2 = g_{\omega\Xi}, \tag{14}$$

$$g_{\rho\Lambda} = 0,\ g_{\rho\Sigma} = 2g_\rho,\ g_{\rho\Xi} = g_\rho. \tag{15}$$

Then x_σ can be given by fitting to the hyperon–nucleus potential depths $U_\Lambda^{(N)} = U_\Sigma^{(N)}$=−30 MeV, $U_\Xi^{(N)}$=−28 MeV as follows:

$$U_h^{(N)} = m_{\mathrm{B}} \left(\frac{m_n^*}{m_n} - 1 \right) \left(\frac{g_{\sigma h}}{g_{\sigma N}} \right) + \left(\frac{g_{\omega N}}{m_\omega} \right)^2 \rho_0 \left(\frac{g_{\omega h}}{g_{\omega N}} \right). \tag{16}$$

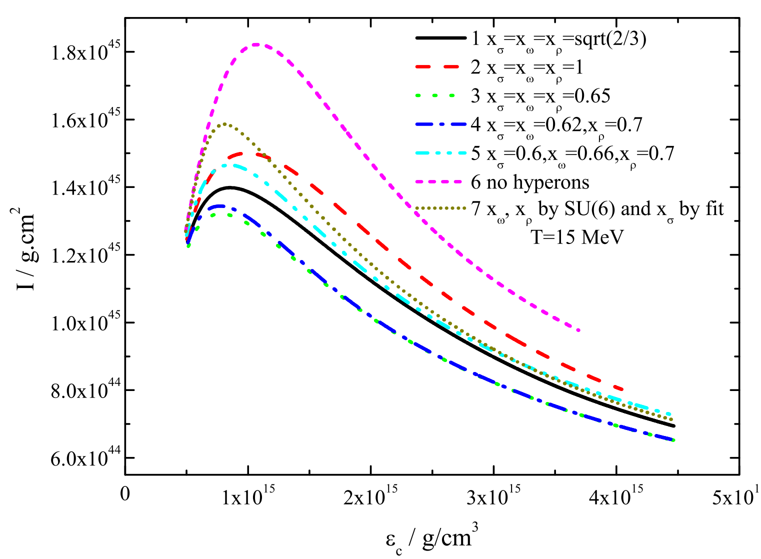

Figure 1. The moment of inertia as a function of central energy density.

Table 2. The maximum values of moment of inertia and the corresponding mass and radius.

Hyperon coupling constants	ϵ_c (10^{15} g/cm^3)	r (km)	M ($M_\odot$)	$I_{\max}$ (10^{45} g.cm^2)
(1) $x_\sigma = x_\omega = x_\rho = \sqrt{\frac{2}{3}}$	0.8489	15.096	1.3263	1.3984
(2) $x_\sigma = x_\omega = x_\rho = 1$	0.9855	14.420	1.4793	1.5001
(3) $x_\sigma = x_\omega = x_\rho = 0.65$	0.7687	15.555	1.2262	1.3211
(4) $x_\sigma = x_\omega = 0.62$, $x_\rho = 0.7$	0.7727	15.530	1.2431	1.3443
(5) $x_\sigma = 0.6$, $x_\omega = 0.66$, $x_\rho = 0.7$	0.8440	15.082	1.3677	1.4654
(6) No hyperons	1.0999	13.830	1.7707	1.8210
(7) x_ω, x_ρ by SU(6) and x_σ by fit	0.8061	15.107	1.4256	1.5858

This set of parameter is

(7) x_ω, x_ρ by SU(6) and x_σ by fit.

Our previous work shows that such hyperon coupling constants can give the neutron star matter a better description (Zhao 2010) and therefore in this work we also select this parameter.

4. Theoretical results and analysis

The moment of inertia of a PNS as a function of central energy density is shown in Figure 1 and its maximum value and the corresponding mass and radius in Table 2. From Fig. 1, it can be seen that for each case the moment of inertia has a maximum value, which is sensitive to the hyperon coupling constants and is distinct for different constants. For the case of no hyperons, the moment of inertia is the

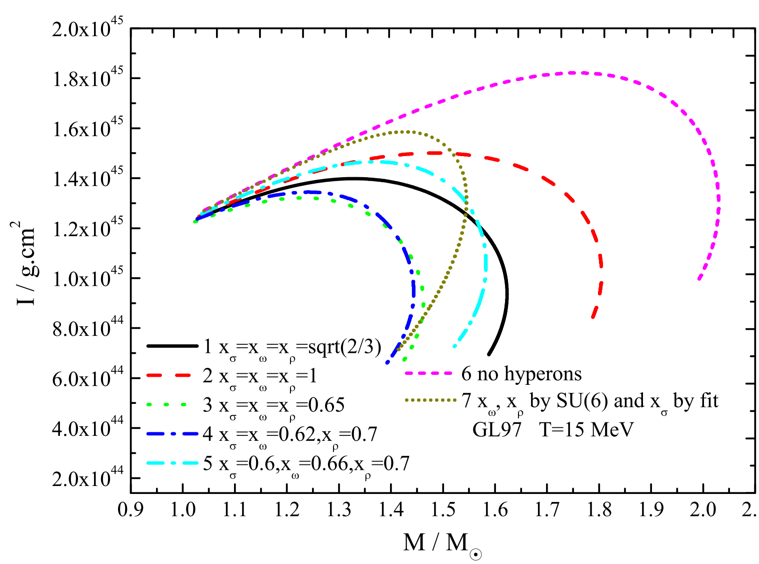

Figure 2. The moment of inertia of a PNS as a function of mass.

Table 3. The maximum values of mass and the corresponding radius and the moment of inertia.

Hyperon coupling constants	ϵ_c (10^{15} g/cm^3)	R (km)	$M_{\max}$ ($M_\odot$)	I (10^{45} g.cm^2)
(1) $x_\sigma = x_\omega = x_\rho = \sqrt{\frac{2}{3}}$	2.7989	11.277	1.6232	0.9370
(2) $x_\sigma = x_\omega = x_\rho = 1$	2.8586	10.966	1.8048	1.0197
(3) $x_\sigma = x_\omega = x_\rho = 0.65$	2.5221	11.909	1.4622	0.9037
(4) $x_\sigma = x_\omega = 0.62$, $x_\rho = 0.7$	2.3228	12.290	1.4437	0.9448
(5) $x_\sigma = 0.6$, $x_\omega = 0.66$, $x_\rho = 0.7$	2.3154	12.199	1.5827	1.0556
(6) no hyperons	2.4319	11.564	2.0298	1.2997
(7) x_ω, x_ρ by SU(6) and x_σ by fit	1.5872	13.802	1.5450	1.3170

largest. Table 2 shows the maximum values of the moment of inertia $I_{\max}$ and the corresponding mass and radius for differential hyperon coupling constants.

Figure 2 shows the moment of inertia of a PNS as a function of the mass. From Fig. 2, we can see that the moment of inertia is also very sensitive to the hyperon coupling constants. For the case of no hyperons, the maximum mass is the largest. The observational data of the mass of a neutron star are in the range of ∼1.25 to 1.8 $M_\odot$ (Lnminet 1997) or 1.97 $M_\odot$ (Demorest *et al.* 2010) and our results are in the range of ∼1.4437 to 2.2098 $M_\odot$. From our previous work we know that the mass of a PNS is larger than that of a neutron star (Zhao 2011). So our results about the mass of PNS are reasonable. And then we can derive that our conclusions about the moment of inertia of a PNS are reasonable too. Table 3 shows the values of mass, radius and the corresponding moment of inertia for differential hyperon coupling constants.

The moments of inertia of a PNS as a function of M/R are given in Figure 3. For different hyperon coupling constants, the relationships between the moment of

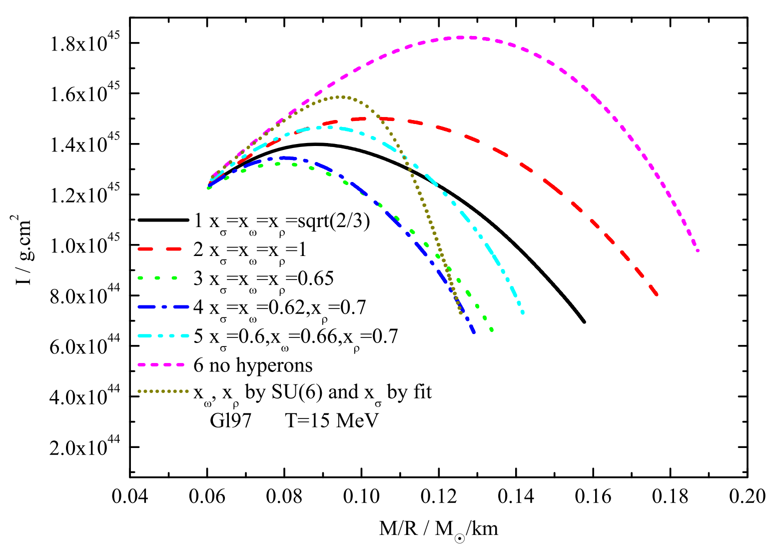

Figure 3. The moment of inertia as a function of M/R.

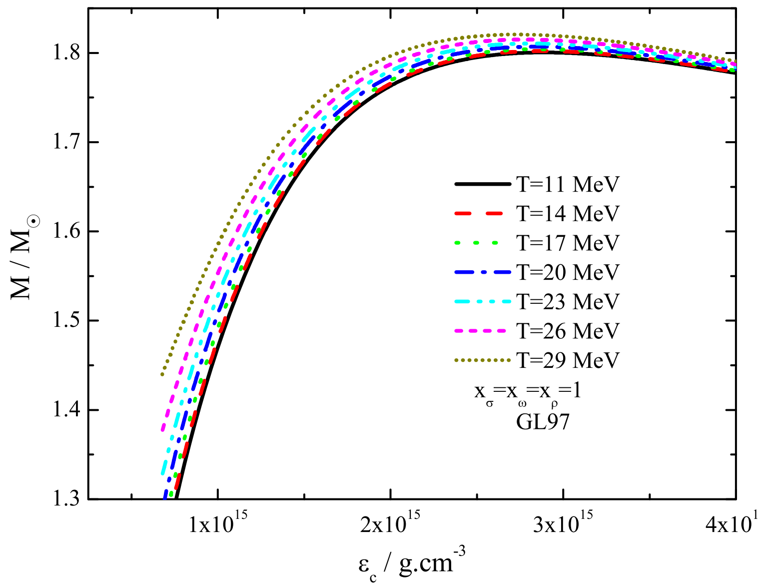

Figure 4. The effect of temperature on the mass of neutron stars.

inertia and M/R are more distinct. So this also shows that the moment of inertia is sensitive to the hyperon coupling constants. From Figure 3 we can see that, if the moment of inertia is defined then M/R can also be derived. Furthermore, if M/R is known and if either M or R is known from other measurements then the other quantity can also be derived.

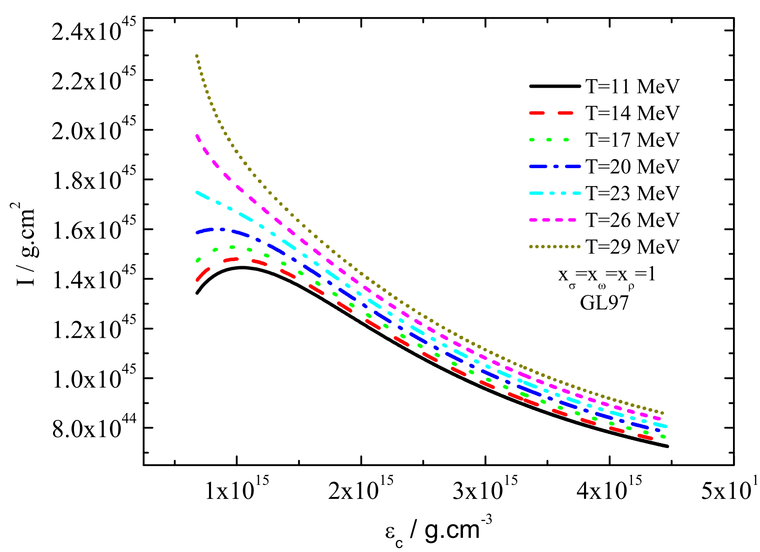

Figure 5. The effect of temperature on the moment of inertia of neutron stars.

How about the results if other values of T are used? To elucidate it, we calculate the mass and the moment of inertia of the PNS by selecting temperatures $T = 11$, 14, 17, 20, 23, 26 and 29 MeV, respectively. The results are shown in Figures 4 and 5, from which it can be seen that the mass and the moment of inertia of the PNS all increase as the temperature increases.

5. Summary

In conclusion, in this paper the influence of the hyperon coupling constants on the moment of inertia of a proto neutron star has been investigated within the framework of the relativistic mean field theory for the baryon octet $\{n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0\}$ system. It is found that the moment of inertia is more sensitive to the hyperon coupling constants. For different hyperon coupling constants, the moment of inertia and its maximum value are distinct from each other. Meanwhile, the moment of inertia as a function of the mass and that as a function of M/R are also sensitive to the hyperon coupling constants. For the case of no hyperons, the moment of inertia is the largest.

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