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Relativistic Beaming and Orientation Effects in BL Lacertae Objects

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Abstract. We use the correlation between the core-to-lobe radio luminosity ratio (R) and the linear size (D) of a sample of BL Lacertae objects to investigate the relativistic beaming and radio source orientation paradigm for high peaked and low-peaked BL Lacs (X-ray and radio selected BL Lacs respectively) and to constrain relativistic beaming model for this extreme class of active galactic nuclei. We show that the $R - D$ distributions of the BL Lac populations contradict blazar orientation sequence, with the X-ray selected BL Lacs (XBLs) being more consistent with the beaming and orientation model. On the premise that Fanaroff-Riley Type I radio galaxies are the unbeamed parent population of these objects, we derive the bulk Lorentz factor of the jets, $\gamma \sim 7-20$ corresponding to a critical cone angle for optimum boosting, ϕ_c of $\sim 1°-4°$, while on average, these objects are inclined at $5°-12°$ to the line-of-sight. The implications of these results for the blazar unification sequence are discussed.

Key words. Galaxies: BL Lacertae objects: general.

1. Introduction

Active galactic nuclei (AGN) are believed to be the central regions of certain galaxies from which enormous quantity of energy is released by processes different from what we obtain in normal stars. They generally show continuum variability at all wave bands at which they are observed. BL Lacertae objects (BL Lacs) are among the most violent AGNs. BL Lacs and core-dominated optically violently variable (OVV) quasars form the extreme subclass of AGNs called blazars. Blazars are generally characterized by compact, core-dominant radio morphology, flat radio spectra ($\alpha < 0.5$; $S_\nu \sim \nu^{-\alpha}$), high degree of variability and polarization, non-thermal radio-to-X-ray continuum and evidence of frequent superluminal motion (Kollgaard 1994; Urry 1998). These extreme properties of blazars can be accommodated in the relativistic jet model (e.g. Blandford and Königl 1979), which requires that the radiations from these objects be highly anisotropic. Unified schemes have been proposed in which matter accretion onto a supermassive blackhole at the galactic centre produces an accretion disk that accelerates jets to relativistic speeds. In this context, the relative orientation of the radio jets with respect to our line-of-sight produces the apparent diversity in the observed properties of the various classes of AGNs via Doppler boosting and/or obscuration. BL Lacs, however, differ from OVV quasars in many respects, namely: absence of strong (if any) optical lines in the spectra of BL Lacs (e.g. Stocke *et al.* 1991), lower redshift and luminosity distribution than OVV quasars (e.g. Murphy *et al.* 1993).

Traditionally, BL Lacs have been discovered in either radio or X-ray band, which led to their classification as radio and X-ray selected BL Lacs (RBLs and XBLs respectively). This has recently been supplanted by a new classification into high energy peaked BL Lacs (HBLs) and low energy peaked BL Lacs (LBLs) based on synchrotron peak luminosity, with XBLs and RBLs being HBLs and LBLs, respectively (Laurent-Muehleisen *et al.* 1999). These two subclasses of BL Lacs show different observational properties. For instance, RBLs are known to be more variable, more luminous at radio and optical bands and have more prominent cores than XBLs (Ghisellini *et al.* 1993; Perlman & Stocke 1993). Due to these differences, the two subclasses of BL Lacs are perceived as separate classes of AGN (e.g. Stocke *et al.* 1985; Ledden & O'Dell 1985; Sambruna *et al.* 1996). While, some authors have argued that the BL Lac population is somewhat continuous with RBLs and XBLs representing the extremes of the continuum (Laurent-Muehleisen *et al.* 1999; Caccianiga *et al.* 1999; Landt *et al.* 2002; Nieppola *et al.* 2006, 2007), it is not yet clear how a continuous BL Lac sequence fits in with the bimodality of spectral energy distribution in their parent FR I radio galaxy population (Bai & Lee 2001).

Within the framework of the long-standing unification scheme for radio-loud AGNs, the apparent differences between blazar subclasses suggest that BL Lacs and OVV quasars are, respectively, the aligned counterparts of low- and high-luminosity/redshift radio galaxies which, in turn, form their respective parent populations (Kollgaard 1994; Urry & Padovani 1995). However, there is increasing evidence suggesting that BL Lacs and OVV quasars can be unified in a continuous blazar sequence (Fossati *et al.* 1997; Meyer *et al.* 2011). In a simple relativistic beaming and radio source orientation unification scheme, both BL Lacs and Fanaroff-Riley (1974) Type-I radio galaxies (FR I) are believed to be intrinsically the same type of objects. The apparent differences observed between the two classes of objects have been attributed to the orientations of their radio axes with respect to the observer's line-of-sight (Urry & Padovani 1995; Bai & Lee 2001; Odo *et al.* 2011). In this scheme, the extreme properties of BL Lacs could be attributed to beamed emissions from the jets near the nucleus of the parent galaxy when viewed end-on. However, recent results (e.g. Kharb *et al.* 2010; Landt *et al.* 2006), in which powerful FR II-like BL Lacs and low power flat spectrum radio quasars appear to be inconsistent with the orientation-based unification scheme for radio loud AGNs.

Relativistic beaming and source orientation paradigm consistently finds plausible connections between radio galaxies and their beamed (aligned) counterparts (Kollgard 1994), which appears difficult using optical spectra (e.g. Ghisellini *et al.* 2010). Quantitatively, a knowledge of the values of the bulk Lorentz factor of any AGN class is required to unify the aligned and progressively misaligned counterparts through beaming and orientation hypothesis. The projected linear size (D) and the core-dominance parameter, defined as the ratio (R) of core-to-lobe radio luminosity are two crucial orientation parameters which can be used not only to provide consistency tests of the orientation based unification schemes for radio-loud AGN, but to provide quantitative constraints on the relativistic beaming model (e.g. Kapahi & Saikia 1982; Ubachukwu 1998). In particular, the correlation between R and D can be used to derive the value of the bulk Lorentz factor which characterizes the Doppler boosting of the radio emission and the radio morphology of AGNs. In previous papers (Ubachukwu 1998; Ubachukwu & Chukwude 2002), statistical consequences of relativistic beaming and geometric projection were studied in high-luminosity radio-loud AGN using the observed correlation in the $R - D$ data. In this paper, we wish to extend the $R - D$ analyses to their low-luminosity counterparts.

2. The relativistic beaming model

The relativistic beaming and radio source orientation model posits that radio sources observed at close angles to the line-of-sight should be characterized by relativistically boosted core emissions relative to the lobes (R) and foreshortened linear sizes (D), which are apparently, due to the beaming and projection effects respectively (e.g. Orr & Browne 1982). In the context of this scheme, both D and R should, respectively, depend on the viewing angle according to the following equations:

$$D = D_0 \sin\phi \tag{1}$$

and

$$R = \frac{R_T}{2}[(1 - \beta\cos\phi)^{-n+\alpha} + (1 + \beta\cos\phi)^{-n+\alpha}], \tag{2}$$

where D_0 is the intrinsic linear size of the source, ϕ is the viewing angle with respect to the radio axes, $R_{\rm T}$ is the value of R at $\phi = 90°$, β is the velocity of the radiating plasma (in units of the speed of light), α is the spectral index of the core emissions and n is a parameter which depends on the assumed flow model. For continuous beam model and a radiating plasma consisting of blob (e.g. Orr & Browne 1982; Ubachukwu & Chukwude 2002), $n = 2$ and 3, respectively.

An obvious implication of equations (1) and (2) is that, since BL Lacs are believed to form the beamed counterparts of FRI sources, an $R - D$ anti-correlation should be envisaged. At small angles to the line-of-sight, relativistic beaming in AGNs is fundamentally characterised by a beaming enhancement/Doppler boosting factor defined as

$$\delta = \gamma^{-1}(1 - \beta\cos\phi)^{-1}, \tag{3}$$

where $\gamma = (1 - \beta^2)^{-1/2}$ is the bulk Lorentz factor of the jet (e.g. Bai & Lee 2001; Ubachukwu & Chukwude 2002). By this definition of the bulk Lorentz factor, R can be conveniently written as

$$R = \frac{\delta^{n+\alpha} S_{\rm oc}}{S_{\rm e}}, \tag{4}$$

where $S_{\rm oc}$ is the intrinsic core flux density, $\delta^{n+\alpha}$ is the core enhancement parameter and $S_{\rm e}$ is the isotropic lobe flux density.

Equations (2) and (4) suggest that the distribution of R should, in principle, provide an indication of the range of values of ϕ and γ which can be used to test specific predictions of the beaming model, provided that $R_{\rm T}$ is known for any assumed flow model. If we assume $\alpha = 0$ (for synchrotron self-absorbed sources) and $\beta \sim 1$, it can be shown, from equation (2), that the mean angle to the line-of-sight (ϕ_{m}) for beamed sources whose radio axes lie closer to the line-of-sight is (to a first approximation) given by

$$\cos\phi_{\mathrm{m}} \sim 1 - \left(\frac{2R_{\mathrm{m}}}{R_{\mathrm{T}}}\right)^{-\frac{1}{n}}, \tag{5}$$

where R_{m} is the mean value of the R distribution for a particular sample of sources. Similarly, equation (2) implies that maximum boosting occurs at $\phi = 0°$, which combines with equation (4) to yield a maximum core-dominance parameter (R_{max}) as

$$R_{\mathrm{max}} = R_{\mathrm{T}}\gamma^{n}(2\gamma^{n} - 1). \tag{6}$$

In principle, this corresponds to $D = 0$ in equation (1). The implication is that if we assume a flow model and a functional form of the $R - D$ relation, the beaming index γ can be determined once R_{T} is known for a given sample of sources.

Also, following Vermeulen and Cohen (1994), it is evident that relativistic beaming in extragalactic radio sources is optimized within certain cone angles between $\phi = 0°$ and some critical angle (ϕ_{c}), given by

$$\sin\phi_{\mathrm{c}} = \frac{1}{\gamma}. \tag{7}$$

All sources within this cone angle are expected to move at superluminal speeds with an apparent speed given by (e.g. Kollgaard 1994)

$$\beta_{\mathrm{app}} = \frac{\beta\sin\phi}{1 - \beta\cos\phi}. \tag{8}$$

Consequently, we can derive the values of γ and hence ϕ_{c} and β_{app} from $R - D$. Note that in this scenario, we are using R as the beaming indicator and D as an orientation indicator. Actually, ϕ_{c} corresponds to D_{min} ($\neq 0$) which is the minimum linear size for optimum beaming. In practice, therefore, R_{max} should correspond to D_{min} at $\phi \simeq \phi_{\mathrm{c}}$.

3. Data analysis and results

Although there are over a thousand BL Lacs known from various surveys in literature, those with linear size (D) information are still very few. In the current study, we employ a heterogeneous sample of BL Lacs (containing both XBLs and RBLs), taken from Murphy *et al.* (1993) compilation of VLA core-dominated sources, HEAO 1 LASS complete sample of X-ray selected BL Lacs (Kollgaard *et al.* 1996) and the largest pure radio-selected BL Lac sample (Rector & Stocke 2001). This sample was updated with the compilation by Fan and Zhang (2003). On the whole, there are 56 sources with complete 5-GHz data on R-parameter and linear size (D). Out of the 56 sources, 21 sources are XBLs while 35 are RBLs.

The R-distribution of the two sub-samples on a log scale is shown in Figure 1. Apparently, the sample taken together shows a normal distribution of the core-to-lobe luminosity ratio, with a mean value $R_{\mathrm{m}} = 34\pm1$. However, it is evident from Fig. 1 that the sample shows a bimodal distribution in R, with RBLs having larger

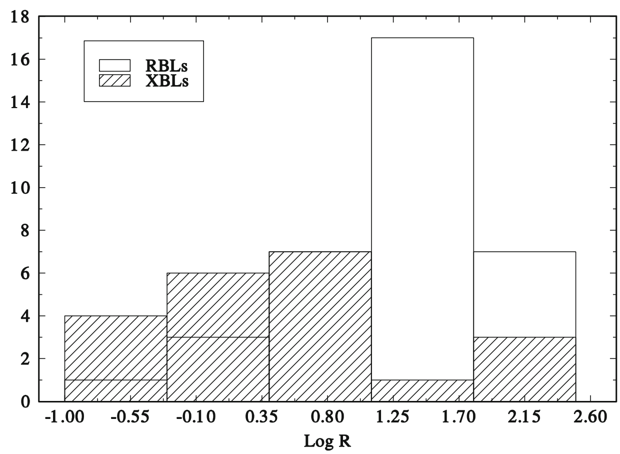

Figure 1. The distribution of core-dominance parameter (R) for the 21 X-ray selected BL Lacertae objects (XBLs) and the 35 radio BL Lacertae objects (RBLs) on a logarithmic scale.

values than the XBLs. Separating the sources into RBL and XBL yields $R_{\rm m} = 41\pm2$ for RBLs and 19 ± 3 for XBLs. Several authors (e.g. Stickel *et al.* 1991; Bai & Lee 2001) have demonstrated that BL Lacs and FRI sources are the same objects seen at different orientation angles. This implies that the intrinsic values of both D and R should be the same for the two sub-samples of BL Lacs and FRI once they are drawn from the same parent sample. In fact, Zirbel & Baum (1995) and Capetti *et al.* (2000) have shown that for FRI-BL Lac unification, the R-distribution is consistent with intrinsic value $R_{\rm T}$ of 0.0004. Using the distribution above and $R_{\rm T} = 0.0004$ in equation (5), we derive mean viewing angles $\phi_{\rm m} \sim 4°$ and $11°$ for $n = 2$ and 3, respectively, for the complete BL Lac sample. When the subsamples are considered separately, we find $\phi_{\rm m} \sim 11°$ and $12°$, respectively, for RBLs and XBLs for $n = 3$ and $\phi_{\rm m} \sim 4°$ and $5°$, respectively for $n = 2$.

Furthermore, the distribution of the linear sizes of the sample is shown in Fig. 2. The figure does not apparently show a normal distribution in the linear sizes. In addition, the XBLs and RBLs are uniformly distributed in a unimodal configuration. This, however, is not expected in a unified blazar orientation-based sequence in which XBLs and RBLs represent progressively aligned blazars, with decreasing projected linear sizes. The distribution seems to suggest that any difference between the two BL Lac populations may not be attributed solely to orientation effect. The observation is quite consistent with some previous results (Padovani & Giommi 1995; Landt *et al.* 2002), which propose that XBLs and RBLs essentially share the same range in orientation, but have different spectral energy distributions (SED). However, the data give a mean value $D_{\rm m} = 118 \pm 2$ Kpc, when taken together, and $D_{\rm m} = 122\pm3$ and 108 ± 6 Kpc for RBL and XBL subsamples, respectively. Laurent-Muehleisen *et al.* (1999) have shown that the distributions of source parameters reveal RBL-XBL-FRI sequence for progessively misaligned sources. For consistency test using FRI sample by Zirbel & Baum (1995), we obtain $R_{\rm m} = 0.20 \pm 0.04$ and $D_{\rm m} = 149 \pm 5$ Kpc, corresponding to $\phi_{\rm m} \sim 14°$ and $26°$ for $n = 2$ and 3 respectively. The distributions are quite consistent with BL Lac-FR I unification, but with XBLs and RBLs showing overlapping distributions in linear sizes.

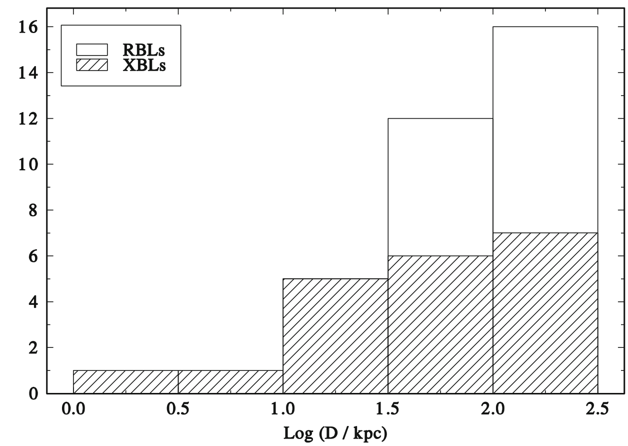

Figure 2. The histogram showing the distributions of the linear size (D) for the 21 X-ray selected BL Lacertae objects (XBLs) and the 35 radio selected BL Lacertae objects (RBLs) on a logarithmic scale.

To test for beaming effect, as can be inferred from equation (6), the scatter plot of the $R - D$ data on log scales is shown in Fig. 3 for the current BL Lac sample. Perhaps, a more striking feature of the plot is the apparent difference in the $R - D$ relation between the two BL Lac subsamples.

Simple regression analyses of the data yield the following results:

$$\log R = 0.63 \log D + 0.03, \tag{9}$$

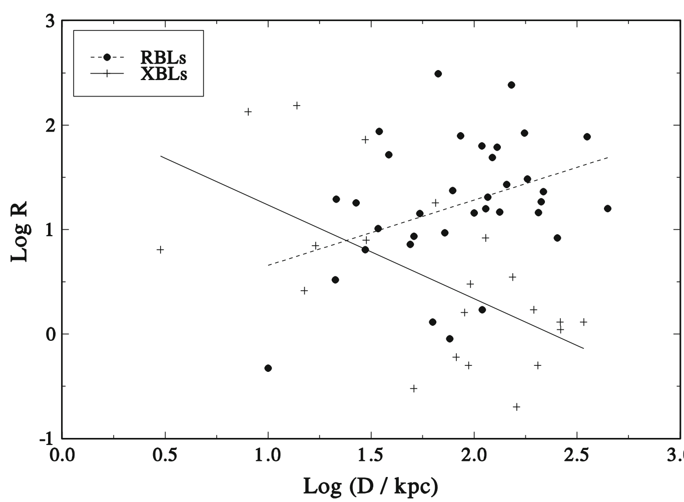

Figure 3. The log–log scatter plot of the 35 radio selected BL Lac objects (•) and the 21 X-ray selected BL Lac objects (+).

and

$$\log R = 2.13 - 0.89 \log D, \tag{10}$$

with the correponding correlation coefficients $r \sim +0.37$ and -0.61, respectively, for RBL and XBL subsamples at confidence level of $\sim 95.0\%$. It could be observed that the RBLs apparently suggest a positive $R - D$ correlation, which is contrary to the beaming prediction, while the XBLs indicate a significant $R - D$ anti-correlation, which is consistent with the beaming hypothesis. This again suggests the existence of some intrinsic differences between the XBLs and RBLs. However, both the linear size and luminosity of radio sources have been known to undergo some form of cosmological evolution (see Okoye & Onuora 1982). The observed $R - D$ correlation could therefore be an artifact if there is any significant cosmological evolution or luminosity selection effects in the sample (in flux density limited samples, luminosity and redshift are strongly correlated due to Malmquist bias). To check for these effects in our sample, we carried out a simple statistical investigation of a possible dependence of D and R on redshift (z). Basically, our analyses involve a linear regression of the $D - z$ and $R - z$ data. The results reveal no significant $D - z$ or $R - z$ correlation (with correlation coefficient $r \sim 0.06$ and 0.29 respectively). Although the statistics are seemingly poor, the results nonetheless suggest that the difference between the two populations may not be only due to different orientation angles with respect to the line-of-sight, but could as well be intrinsic (see Stocke *et al.* 1985; Perlman & Stocke 1993; Sambruna *et al.* 1996). However, the classical RBL-XBL classification is believed by many authors (e.g. Laurent-Muehleisen *et al.* 1999) to be a result of selection effect which sampled the BL Lac sources at, respectively, low X-ray and low radio luminosities, so that their sharp dichotomic SEDs become apparent (Padovani and Giommi 1995; Brinkmann *et al.* 1996). We show the distributions of the current BL Lac samples in the X-ray luminosity (L_x) at 1 KeV, 5-GHz radio luminosity (L_r) and X-ray-to-radio luminosity ratio (L_x/L_r) on logarithmic scales in Figures 4(a), 4(b) and 4(c) respectively. In Figures 4(a) and 4(b), the tendency for XBLs to have similar X-ray, but lower radio luminosities compared to RBLs, which have high radio luminosities, is apparent. Correspondingly, it appears, from Fig. 4c, that the X-ray-to-radio luminosity ratio increases from RBLs to XBLs, with mean values of -6.8 ± 0.5 and -4.8 ± 0.2 for RBLs and XBLs, respectively, but does not show a clear evidence for sharp division between the two BL Lac samples. We note that for all of the distributions, there is continuity in properties between the XBLs and RBLs. This is in close agreement with Fossati *et al.* (1997) for the 1 Jy (RBL) and Slew (XBL) BL Lac samples and suggests that X-ray luminosity does not seem to have any significant effect on the detection of the radio properties of the BL Lac samples. Therefore, in subsequent analysis, we will assume that the difference in the $R - D$ data for the two populations is intrinsic. Hence, we focus attention on the XBL subsample, whose $R - D$ relation appears more consistent with the prediction of the beaming model.

An analysis of the functional $R - D$ relation obtained for XBL subsample, (see equation (10)), gives $R_{\max} = 134.9$. Putting this result and $R_T = 0.0004$ in Eq. (6) yields $\gamma = 7.4$ and 20.2 for $n = 3$ and 2, respectively. Furthermore, it can be deduced from Eq. (8) that $\beta_{\rm app}$ is a maximum at ϕ_c and this corresponds to $\beta_{\rm app}^{\max} = \gamma\beta = \sqrt{\gamma^2 - 1} \simeq 7.3$ and 20.1 for $n = 2$ and $n = 3$, respectively.

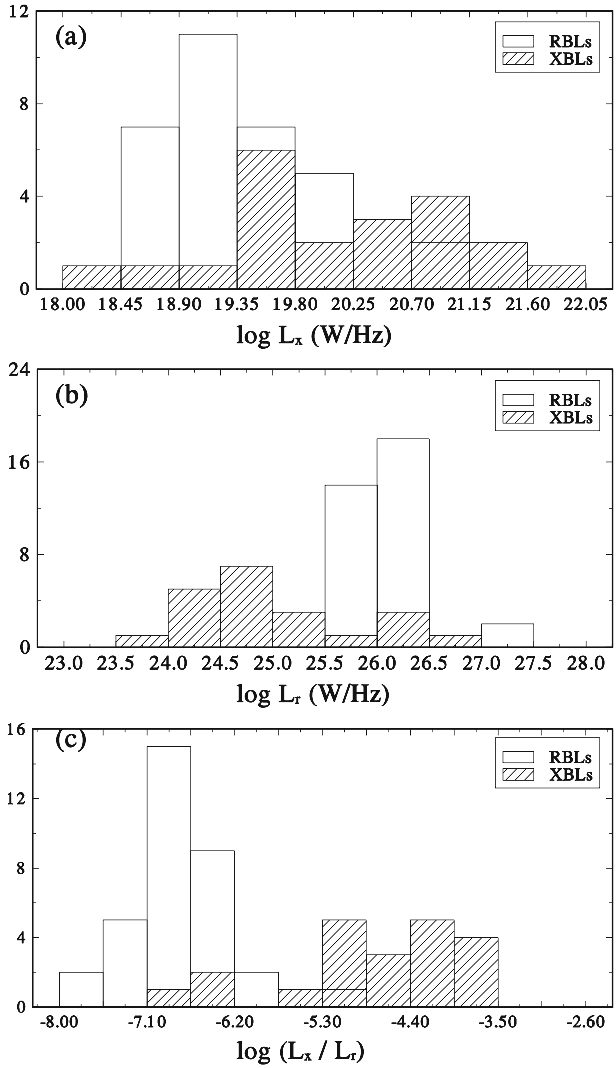

Figure 4. The histogram showing the distributions of the X-ray luminosity (**a**), radio luminosity (**b**), and the X-ray-to-radio luminosity ratio (**c**) for the 21 X-ray selected BL Lacertae objects (XBLs) and the 35 radio selected BL Lacertae objects (RBLs) on logarithmic scales.

4. Discussion

Relativistic beaming model has been proposed to explain certain observed properties of different classes of active galactic nuclei and applied extensively in the unification of beamed sources and their presumably misaligned counterparts. A simple consequence of this model is that relativistic Doppler boosting and geometric projection effects are expected to be more pronounced in sources whose radio axes are inclined at smaller angles to the line-of-sight than in those sources that are, more or less, observed at the plane of the sky. Hence, the relativistic beam from the core of an AGN is expected to be narrow and characterized, for a particular class of the AGN, by a single Lorentz factor (γ) and a projection angle (ϕ), for any assumed flow model. However, each of the assumed flow models ($n = 2$ or 3) leads to different values of γ and ϕ. In particular, smaller value of n results in larger value of γ and a smaller ϕ. It has therefore been suggested in Ubachukwu & Chukwude (2002) that the exact values of these beaming parameters (γ and ϕ), which characterize the radio morphology as well as the Doppler boosting of the core of any class of AGN, should lie within a range bounded by the two flow models. Furthermore, the distributions of the beaming and orientation parameters of the radio sources are expected to show smooth transition from the beamed sources to their misaligned counterparts (Laurent-Muehlesien *et al.* 1999).

The distributions of the observed R parameter of our sample seem to suggest a smooth transition from RBLs to FR I radio galaxies, through XBLs, which is consistent with the relativistic beaming and orientation-based unification scheme for low luminosity sources (e.g. Laurent-Muehleisen *et al.* 1999), with XBLs and RBLs being the high- and low-peaked blazars respectively. This result is apparently not in agreement with the work by Meyer *et al.* (2011), in which the radio core-dominance decreases with decreasing synchrotron peak for progressively misaligned blazars. Nevertheless, the authors did predict bi-modality in the distribution of high-peaked and low-peaked blazars on the core dominance − frequency peak ($R - \nu_{\rm peak}$) plane, consistent with the current R-distribution results for XBLs and RBLs. Furthermore, the R-distributions for individual subsamples are found to be consistent with mean viewing angles $\phi_{\rm m} \sim 4^\circ - 11^\circ$ for RBLs and $5^\circ - 12^\circ$ for XBLs. This does not evidently show that RBLs and XBLs have different orientations with respect to the line-of-sight as suggested by the blazar sequence. Perhaps, the difference in R-distributions of these objects could be interpreted in line with Beckmann *et al.* (2003), in which RBLs and XBLs have different intrinsic jet power. Similarly, there is evidence in the $R - D$ data of the XBLs and RBLs suggestive of some forms of intrinsic, rather than mere orientation, differences between the two sub-samples. In the blazar sequence scenario (Fossati *et al.* 1997), in which RBLs (LBLs) and XBLs (HBLs) represent progressively misaligned blazars, one would expect the distribution to be continuous, with RBLs concentrated on the upper left and XBLs on the lower right envelopes of the $R - D$ plot. Apparently, this is not observed in Figure 3, rather the two sub-samples cover similar ranges in D. More specifically, the $R - D$ relation of XBL subsample appears to be more consistent with beaming model than that of the RBL subsample. The results presented here are in close agreement with those obtained with newer samples (e.g. Caccianiga and Marcha 2004; Landt & Bignall 2008), and appear to challenge the blazar orientation sequence, suggesting some intrinsic differences between XBLs and RBLs.

Although orientation scenario provides a natural explanation for the more extreme properties of RBLs compared with XBLs, it has been suggested that the two subsamples have intrinsically different spectral energy distributions (e.g. Sambruna *et al.* 1996), which implies that there are some intrinsic differences between XBLs and RBLs. Our result has shown that, on logarithmic scales, the X-ray-to-radio luminosity ratio is higher for XBLs, with a mean value of −4.8, than for RBLs, with a mean value of −6.8. This is consistent with a dividing line of −5.5 for HBL-LBL dichotomy (e.g. Padovani & Giommi 1995; Beckmann *et al.* 2003). Furthermore, Bai & Lee (2001) have used the distribution of radio-to-X-ray broadband spectral index

(α_{rx}) to show that FR I sources also have a double peaked structure in their spectral energy distribution (SED) and that there exist two intrinsically different subclasses of FRI sources, with a dividing line at $\alpha_{rx} = 0.75$. The authors found that the RBL-like and XBL-like are, respectively, characterized by $\alpha_{rx} \geq 0.75$ and $\alpha_{rx} < 0.75$. Apparently, these subclasses of FRI sources form the parent populations of the RBLs and XBLs respectively. Moreover, Brinkmann *et al.* (1996) have proposed that the two types of BL Lacs are intrinsically different and either originate from different parent populations, or have different emission conditions. Perhaps, the difference in the $R - D$ relations of the two BL Lac subsamples may be a manifestation of the difference in their spectral energy distributions. If this is actually the case, then the difference in the SED of XBLs and RBLs is consistent with relativistic beaming and source orientation in BL Lacs. Similarly, the bi-modality in the broadband SED of FR I radio galaxies therefore agrees with FR I-BL Lac unification. Our result also suggests that the effects of cosmological evolution and luminosity selection are negligibly small in the sample since neither R nor D shows significant correlation with redshift (z). In fact, for BL Lacs, it is expected that evolutionary effects would not be important since they are mostly located at low redshifts.

Another important result of the current analysis comes from the significant $R - D$ anti-correlation ($r \simeq -0.6$) seen in the XBL subsample, which corresponds to bulk Lorentz factor $\gamma \sim 7 - 20$. This appears to suggest that the XBL subsample is, more or less, consistent with beaming and source orientation model. Equation (6) actually predicts the maximum value of R, which perhaps, corresponds to the extreme values of other beaming parameters for any given sample. Hence, the extreme values of these beaming parameters for any sample are obtainable at the upper envelope $R - D$ function where relativistic beaming is expected to dominate other effects (see Ubachukwu 1998, 2002) and appears apparent in Fig. 3. Regression analysis of the upper envelope $R - D$ data in four uniform bins of D: $D \leq 75$ Kpc, $75 < D \leq 150$ kpc, $150 < D \leq 225$ kpc and $D > 225$ kpc gives $R_{max} = 7943.3$, with a correlation coefficient $r \sim -0.99$. Using $R_T = 0.0004$ and $R_{max} = 7943.3$ in equation (6) shows that for $2 \leq n \leq 3$, the upper envelope $R - D$ data are consistent with $\gamma \sim 15 - 56$, which correspond to critical cone angles ϕ_c in the range of 1° to 4°. The extreme values of bulk Lorentz factor ($\gamma > 50$) obtained from the upper envelope $R - D$ data of the XBL subsample suggest that jets in XBLs must be highly relativistic. This scenario has been suggested (Ghisellini *et al.* 2009) for high energy (TeV) blazars as a condition necessary to avoid absorption of the synchrotron radiations by co-spatial infrared photons. These results are largely consistent with relativistic beaming and orientation-based unification scenario for low luminosity AGNs, in which relativistic flows from the jets of BL Lacs observed at close angles to the line-of-sight are expected to be amplified by Doppler boosting (Kollgaard 1994). This, however, creates the illusion that the apparent transverse speed of the jet exceeds the speed of light, as frequently observed among blazars (e.g. Gabuzda 1994; Cohen *et al.* 2007). For consistency check, we have shown in the results that the $R - D$ data of XBLs, on average, are consistent with apparent superluminal motion in the jets with the apparent speed in the range $7c \leq \beta_{app} \leq 20c$. These results agree well with those reported in Ubachukwu & Chukwude (2002) for core-dominated quasars and suggest that radiations from the blazars are highly anisotropic owing to their relatively smaller viewing angles compared with their unbeamed parent population of radio galaxies.

Kollgaard *et al.* (1996) have used the distributions of core and extended radio luminosity of FR I radio galaxies and BL Lacs to show that there is high core-enhancement in BL Lacs which generally requires their γ to exceed 4.5, with XBLs (including 16 sources from our sample) and RBLs, on average, inclined at 20° and 10°, respectively, to the line-of-sight. Also, Landt *et al.* (2002) obtained $2 \leq \gamma \leq 4$ for BL Lacs, which include five sources from current sample, and low power galaxies using Ca H&K break value as an orientation indicator. Similarly, from the jet-to-counter jet flux density distribution, Giroletti *et al.* (2006) used synchrotron self-Compton model to derive $2 \leq \gamma \leq 9$ and $\phi_{\rm m} \sim 13° - 23°$ for nearby ($z < 0.2$) BL Lacs. Recent results (e.g. Ghisellini *et al.* 2009) have suggested $\gamma \geq 50$ for very high energy peaked (TeV) blazars, which are in good agreement with the results presented in this paper.

5. Conclusions

We have used the ratio (R) of core-to-lobe radio luminosity and linear size (D) of extragalactic radio sources as orientation indices to quantitatively test a simple statistical consequence of relativistic beaming and radio source orientation hypothesis in low luminosity radio loud active galactic nuclei. We show that the two BL Lac sub classes (XBLs and RBLs: representing high- and low-peaked BL Lacs, respectively) show somewhat contradicting $R - D$ relations. Using the R-distribution and $R - D$ anti-correlation, it is shown that the difference in radio core-dominance between FR I radio galaxies and X-ray selected BL Lacertae objects can be accounted for by a bulk Lorentz factor $\gamma \sim 7$–20 and viewing angle $\phi \sim 5°$–12°. Our results are consistent with the relativistic beaming and radio source orientation-based unification paradigm for low luminosity radio sources in which FRI radio galaxies are believed to form the unbeamed parent population of BL Lacertae objects (particularly with XBLs). However, the sample size is small and more data would be required to put these results on a firmer footing.

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References

Bai, J. M., Lee, M. G. 2001, *Astrophys. J.*, **548**, 244.
Beckmann, V., Engles, D., Bade, N., Wucknitz, O. 2003, *Astron. Astrophys.*, **401**, 927.
Blandford, R. D., Königl, A. 1979, *Astrophys. J.*, **232**, 34.
Brinkmann, W., Siebert, J., Kollgaard, R. I., Thomas, H. C. 1996, *Astron. Astrophys.*, **313**, 356.
Capetti, A., de Ruitar, H. R., Fanti, R. *et al.* 2000, *Astron. Astrophys.*, **362**, 871.
Caccianiga, A., Maccacaro, T., Wolter, O., della Ceca, R., Gioia, I. M. 1999, *Astrophys. J.*, **513**, 51.
Caccianiga, A., Marcha, M. J. 2004, *Mon. Not. R. Astron. Soc.*, **348**, 937.
Cohen, M. H., Lister, M. L., Homan, D. *et al.* 2007, *Astrophys. J.*, **658**, 232.
Fan, J. H., Zhang, J. S., 2003, *Astron. Astrophys.*, **407**, 899.

Fanaroff, B. L., Riley, J. M. 1974, *Mon. Not. R. Astron. Soc.*, **167**, 13.
Fossati, G., Celloti, A., Ghisellini, G., Maraschi, L. 1997, *Mon. Not. R. Astron. Soc.*, **289**, 136.
Gabuzda, D. C., Mullan, C. M., Cawthorne, T. V., Wardle, J. F., Roberts, D. H. 1994, *Astrophys. J.*, **435**, 140.
Ghisellini, G., Padovani, P., Celotti, A., Maraschi, L. 1993, *Astrophys. J.*, **407**, 67.
Ghisellini, G., Tavecchio, F., Bodo, G., Celotti, A. 2009, *Mon. Not. R. Astron. Soc.*, **393**, L16.
Ghisellini, G., Tavecchio, F., Foschini, L., Ghirlanda, G. 2010, *Mon. Not. R. Astron. Soc.*, **414**, 2674.
Giroletti, M., Giovannini, G., Taylor, G. B., Falomo, R. 2006, *Astrophys. J.*, **646**, 801.
Kapahi, V. K., Saikia, D. J. 1982, *J. Astrophys. Astron.*, **3**, 465.
Kharb, P., Lister, M. L., Cooper, N. J. 2010, *Astrophys. J.*, **710**, 764.
Kollgaard, R. I. 1994, *Vistas in Astronomy*, **38**, 29.
Kollgaard, R. I., Palma, C., Laurent-Muchleisen, S., Feigelson, E. D. 1996, *Astrophys. J.*, **46**, 115.
Landt, H., Bignall, H. E. 2008, *Mon. Not. R. Astron. Soc.*, **391**, 967.
Landt, H., Padovani, P., Perlman, E. S. 2006, *Astrophys. J.*, **637**, 183.
Landt, H., Padovani, P., Perlman, E. S., Giommi, P. 2002, *Mon. Not. R. Astron. Soc.*, **336**, 945.
Laurent-Muehleisen, S. A., Kollgaard, R. I., Feigelson, E. D., Brickmann, W., Siebert, J. 1999, *Astrophys. J.*, **525**, 127.
Ledden, J. E., O'Dell, S. L. 1985, *Astrophys. J.*, **258**, 630.
Meyer, E. T., Fossati, G., Georganopoulos, M., Lister, M. L. 2011, *Astrophys. J.*, **740**, 98.
Murphy D. W., Browne, I. W. A., Perley, R. A. 1993, *Mon. Not. R. Astron. Soc.*, **264**, 298.
Nieppola, E., Tornikoski, M., Valtaoja, E., Hakala, T. *et al.* 2007, *Astron. J.*, **133**, 1947.
Nieppola, E., Tornikoski, M., Valtaoja, E. 2006, *Astron. Astrophys.*, **445**, 441.
Odo, F. C., Ubachukwu, A. A., Chukwude, A. E. 2011, *African Skies*, **15**, 15.
Okoye, S. E., Onuora, L. I. 1982, *Astrophys. J.*, **260**, 37.
Orr, M. J., Browne, I. W. A. 1982, *Mon. Not. R. Astron. Soc.*, **200**, 1067.
Padovani, P., Giommi, P. 1995, *Astrophys. J.*, **444**, 567.
Perlman, E. S., Stocke, J. T. 1993, *Astrophys. J.*, **406**, 430.
Rector, T. A., Stocke, J. T. 2001, *Astron. J.*, **122**, 565.
Sambruna, R. M., Maraschi, L., Urry, C. M. 1996, *Astrophys. J.*, **463**, 444.
Stickel, M., Padovani, P., Urry, C. M., Fried, H. 1991, *Astrophys. J.*, **374**, 431.
Stocke, J. T., Liebert, J., Schmidt, G. *et al.* 1985, *Astrophys. J.*, **298**, 619.
Stocke, J. T., Morris, S. L., Giola, I. M. *et al.* 1991, *Astrophys. J. Suppl.*, **76**, 813.
Ubachukwu, A. A. 1998, *Astrophys. Space Sci.*, **257**, 23.
Ubachukwu, A. A. 2002, *Astrophys. Space Sci.*, **279**, 251.
Ubachukwu, A. A., Chukwude, A. E. 2002, *J. Astrophys. Astron.*, **23**, 253.
Urry, C. M. 1998, *Publ. Adv. Sp. Res.*, **21**, 189.
Urry, C. M., Padovani, P. 1995, *Publ. Astron. Soc. Pacific*, **803**, 107.
Vermeulen, R. C., Cohen, M. H. 1994, *Astrophys. J.*, **430**, 467.
Zirbel, E., Baum, S. A. 1995, *Astrophys. J.*, **448**, 521.