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Self-Similar Solutions for Viscous and Resistive Advection Dominated Accretion Flows

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Abstract. In this paper, self-similar solutions of resistive advection dominated accretion flows (ADAF) in the presence of a pure azimuthal magnetic field are investigated. The mechanism of energy dissipation is assumed to be the viscosity and the magnetic diffusivity due to turbulence in the accretion flow. It is assumed that the magnetic diffusivity and the kinematic viscosity are not constant and vary by position and α-prescription is used for them. In order to solve the integrated equations that govern the behavior of the accretion flow, a self-similar method is used. The solutions show that the structure of accretion flow depends on the magnetic field and the magnetic diffusivity. As the radial infall velocity and the temperature of the flow increase by magnetic diffusivity, the rotational velocity decreases. Also, the rotational velocity for all selected values of magnetic diffusivity and magnetic field is sub-Keplerian. The solutions show that there is a certain amount of magnetic field for which rotational velocity of the flow becomes zero. This amount of the magnetic field depends upon the gas properties of the disc, such as adiabatic index and viscosity, magnetic diffusivity, and advection parameters. The mass accretion rate increases by adding the magnetic diffusivity and the solutions show that in high magnetic pressure, the ratio of the mass accretion rate to the Bondi accretion rate is reduced with an increase in magnetic pressure. Also, the study of Lundquist and magnetic Reynolds numbers based on resistivity indicates that the linear growth of magnetorotational instability (MRI) of the flow reduces by resistivity. This property is qualitatively consistent with resistive magnetohydrodynamics (MHD) simulations.

Key words. Accretion—accretion disks—magnetohydrodynamics: MHD.

1. Introduction

The standard geometrically thin, optically thick accretion disc model can successfully explain most of the observational features in active galactic nuclei (AGN) and X-ray binaries (Shakura & Sunyaev 1973). In the standard thin model, the motion of the matter in the accretion disc is nearly Keplerian, and the gravitational energy released in the disc is radiated away locally. During recent years, another type of accretion flow has been studied in which it is assumed that energy released through dissipation processes in the disc may be trapped within the accreting gas, and only a small fraction of the energy released in the accretion flow is radiated away due to inefficient cooling, and most of the energy is stored in the accretion flow and advected to the central object. This kind of accretion flow is known as advection-dominated accretion flow (ADAF). The basic ideas of ADAF models have been developed by a number of researchers (e.g. Ichimaru 1977; Rees *et al*. 1982; Narayan & Yi 1994, 1995; Abramowicz *et al*. 1995; Ogilvie 1999; Blandford & Begelman 1999).

It seems that accretion discs, whether in star-forming regions, in X-ray binaries, in cataclysmic variables, or in the centers of active galactic nuclei are likely to be threaded by magnetic fields. Consequently, the role of magnetic fields on ADAFs has been analyzed in detail by a number of investigators (Bisnovatyi-kogan & Lovelace 2001; Akizuki & Fukue 2006; Shadmehri 2004; Ghanbari *et al.* 2007; Bu *et al*. 2009; Khesali & Faghei 2009). The existence of the toroidal magnetic field has been proven in the outer regions of the discs of young stellar objects (YSOs) (Aitken *et al*. 1993; Wright *et al*. 1993) and in the Galactic center (Novak *et al*. 2003; Chuss *et al*. 2003). Thus, considering the accretion discs with a toroidal magnetic field have been studied by several authors (Akizuki & Fukue 2006; Begelman & Pringle 2007; Khesali & Faghei 2008; Bu *et al*. 2009; Khesali & Faghei 2009).

The resistive diffusion of magnetic field is important in some systems, such as the protostellar discs (Stone *et al*. 2000; Fleming & Stone 2003), discs in dwarf nova systems (Gammie & Menou 1998), the discs around black holes (Kudoh & Kaburaki 1996), and Galactic center (Kaburaki *et al*. 2010). Also, two- and three-dimensional simulations of the local shearing box have shown that resistive dissipation is one of the crucial processes that determines the saturation amplitude of the magnetorotational instability (MRI). Also, linear growth rate of MRI can be reduced significantly due to the suppression by ohmic dissipation (Fleming *et al*. 2000; Masada & Sano 2008).

Akizuki & Fukue (2006) proposed a self-similar advection-dominated accretion flow that the disc plasma is highly ionized, so they assumed that resistivity of the plasma is zero, and only viscosity is due to turbulence and dissipation in the disc. However, recent works represent importance of magnetic diffusivity in accretion discs (e.g. Kuwabara *et al*. 2000; Kaburaki 2000; Kaburaki *et al*. 2002; Shadmehri 2004; Ghanbari *et al*. 2007; Krasnopolsky *et al*. 2010; Kaburaki *et al*. 2010). Shadmehri (2004) studied a quasi-spherical accretion flow and that dominant mechanism of energy dissipation was assumed to be the magnetic diffusivity due to turbulence in the accretion flow and the viscosity of the fluid was completely neglected. The main focus of Shadmehri (2004) was nonrotating accretion flow that was ignored from toroidal magnetic field. Also, he studied the induction equation of magnetic field in a steady state that is not according to anti-dynamo theorem (e.g. Cowling 1981) and is useful only in particular systems where the magnetic dissipation time is very long, much longer than the age of the system. Ghanbari *et al*. (2007) considered an axisymmetric, rotating, isothermal steady accretion flow, which contains a poloidal magnetic field of the central star and was neglected from toroidal magnetic field of the flow. They assumed that the mechanisms of energy dissipation are the turbulence viscosity and magnetic diffusivity due to the magnetic field of the central star. They explored the effect of viscosity on a rotating disc in the presence of constant magnetic diffusivity. Like Shadmehri (2004), they too considered the flow in balance between escape and creation of the magnetic field, that was ignored from the toroidal component of magnetic field.

As mentioned the observational evidences and the MHD simulations express that the toroidal component of the magnetic field and the magnetic diffusivity are important in accretion discs. Thus in this paper, by using the Akizuki & Fukue (2006) technique, we will study the influence of the presence of the toroidal component of the magnetic fields in a viscous and resistive accreting gas, and investigate the role of non-constant magnetic diffusivity in systems that escaping and creating of magnetic fields are unbalanced. We will show that the present model from some aspects is qualitatively consistent with the observational evidences and hence resistive MHD simulation results. The paper is organized as follows. In section 2, the basic equations of constructing a model for quasi-spherical magnetized advection dominated accretion flow will be defined. In section 3, self-similar method for solving equations which govern the behavior of the accreting gas was utilized. The summary of the model is given in section 4.

2. Basic equations

We have used spherical coordinates (r, θ, φ) centered on the accreting object and have made the following standard assumptions:

(i) The flow is assumed to be steady and axisymmetric, $\partial_t = \partial_\varphi = 0$, so all flow variables are a function of only r;
(ii) The magnetic field has only an azimuthal component;
(iii) The gravitational force on a fluid element is characterized by the Newtonian potential of a point mass, $\Psi = -GM_*/r$, with G representing the gravitational constant and M_* stands for the mass of the central star;
(iv) The equations written in spherical coordinates are considered in the equatorial plane $\theta = \pi/2$ and terms with any θ and φ dependence are neglected (Ogilvie 1999; Khesali & Faghei 2009).
(v) For the sake of simplicity, the self-gravity and general relativistic effects have been neglected;

The behavior of such system can be analyzed by magnetohydrodynamics (MHD) equations. The general MHD equations are written as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] = -\nabla p_{\rm gas} - \rho \nabla \Psi + \frac{1}{4\pi} \mathbf{J} \times \mathbf{B} + \mathbf{F}_{\rm vis}, \tag{2}$$

$$\mathbf{J} = \nabla \times \mathbf{B} \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J}), \tag{4}$$

$$\rho\left[\frac{1}{\gamma-1}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_{\mathrm{gas}}}{\rho}\right)+\left(\frac{p_{\mathrm{gas}}}{\rho}\right)(\nabla\cdot\mathbf{v})\right]=Q_{\mathrm{diss}}-Q_{\mathrm{cool}}\equiv fQ_{\mathrm{diss}}, \tag{5}$$

$$\nabla\cdot\mathbf{B}=0, \tag{6}$$

where ρ, $\mathbf{v}$ and p_{gas} are the density, the velocity field and the gas pressure, respectively, $\mathbf{F}_{\mathrm{vis}}$ is the viscous force per unit volume, $\mathbf{B}$ is the magnetic field, $\mathbf{J}$ is the current density, η represents the magnetic diffusivity and γ is the adiabatic index. The term on the right-hand side of the energy equation, Q_{diss}, is the rate of heating of the gas by the dissipation and Q_{cool} represents the energy loss through radiative cooling. The advection factor, f ($0\le f\le 1$), describes the fraction of the dissipation energy which is stored in the accretion flow and advected into the central object rather than radiating away. The advection factor of f in general depends on the details of the heating and cooling mechanism and will vary with position (e.g. Watari 2006, 2007; Sinha *et al.* 2009). However, we assume a constant f for simplicity. Clearly, the case $f=1$ corresponds to the extreme limit of no radiative cooling and in the limit of efficient cooling, we have $f=0$.

Under the assumptions (i)–(v), the approximation of quasi-spherical symmetry, the equations (1)–(6) become

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2\rho v_{\mathrm{r}})=\dot{\rho}, \tag{7}$$

$$v_r\frac{\mathrm{d}v_r}{\mathrm{d}r}+\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}r}+\frac{GM_*}{r^2}=r\Omega^2-\frac{B_\varphi}{4\pi r\rho}\frac{\mathrm{d}}{\mathrm{d}r}(rB_\varphi), \tag{8}$$

$$\rho v_{\mathrm{r}}\frac{\mathrm{d}}{\mathrm{d}r}(r^2\Omega)=\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[\nu\rho r^4\frac{\mathrm{d}\Omega}{\partial r}\right], \tag{9}$$

$$\frac{1}{\gamma-1}\left[v_{\mathrm{r}}\frac{\mathrm{d}p}{\mathrm{d}r}+\frac{\gamma p}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2v_{\mathrm{r}}\right)\right]=fQ_{\mathrm{diss}}, \tag{10}$$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[rv_{\mathrm{r}}B_\varphi-\eta\frac{\mathrm{d}}{\mathrm{d}r}(rB_\varphi)\right]=\dot{B}_\varphi. \tag{11}$$

Here v_{r} is the radial velocity, $\dot{\rho}$ the mass-loss rate per unit volume, Ω the angular velocity, B_φ the toroidal component of magnetic field, $\dot{B}_\varphi$ the field escaping/creating rate due to the magnetic instability or dynamo effect and ν the kinematic viscosity coefficient. We assume that since both are due to the kinematic coefficient of viscosity and the magnetic diffusivity due to turbulence in the accretion flow, it is reasonable to use these parameters in analogy to the α-prescription of Shakura & Sunyaev (1973) for the turbulence,

$$\nu=\alpha\frac{p_{\mathrm{gas}}}{\rho\Omega_{\mathrm{K}}}(1+\beta)^{1-\mu}, \tag{12}$$

$$\eta=\eta_0\frac{p_{\mathrm{gas}}}{\rho\Omega_{\mathrm{K}}}(1+\beta)^{1-\mu}, \tag{13}$$

where $\Omega_{\mathrm{K}}=(GM_*/r^3)^{1/2}$ is the Keplerian angular velocity. Narayan & Yi (1995) used a similar form for kinematic coefficient of viscosity, i.e. $\nu=\alpha(p_{\mathrm{gas}}/\rho\Omega_{\mathrm{K}})$, also Shadmehri (2004) applied a similar form for magnetic diffusivity, i.e. $\eta=$

$\eta_0(p_{\rm gas}/\rho\Omega_{\rm K})$. Thus in comparison to Narayan & Yi (1995) and Shadmehri (2004) prescriptions, we are using the above equations for the kinematic coefficient of viscosity ν and the magnetic diffusivity η. The parameters of α and η_0 are assumed to be positive constants, and we assume that $\alpha,\ \eta_0\ \leq\ 1$ (Campbell 1999; Kuwabara *et al.* 2000; Shadmehri 2004; King *et al.* 2007). The parameter of $\beta[= p_{\rm mag}/p_{\rm gas}]$ is the degree of magnetic pressure, $p_{\rm mag} = B_\varphi^2/8\pi$, to the gas pressure. Since, we apply steady self-similar method for solving system equations, this parameter will be constant throughout the disc, but really this parameter varies by position (see Khesali & Faghei 2008, 2009). The parameter of μ is a constant and states importance of the degree of total pressure in the kinematic viscosity and the magnetic diffusivity. Clearly, the case of $\mu = 1$ corresponds to the traditional α-model and in case of $\mu = 0$, total pressure is used. Note that the kinematic coefficient of viscosity and the magnetic diffusivity are not constant and depend on the physical quantities of the flow. We will show the quantities of ν and η in our self-similar solution which vary with radius $r^{1/2}$.

The ratio of the kinematic coefficient of viscosity to the magnetic diffusivity is defined by the magnetic Prandtl number, $P_{\rm m} = \nu/\eta$. By using equations (12) and (13) the magnetic Prandtl number in the present model is $P_{\rm m} = \alpha/\eta_0$. We will consider conditions that $P_{\rm m} = \infty$ (in case that magnetic diffusivity is zero), $P_{\rm m} \geq 1$ and $P_{\rm m} < 1$.

For the heating term in equation (10), $Q_{\rm diss}$, we use two sources of dissipation: the viscous and resistive dissipations. Thus, for $Q_{\rm diss}$ we can write

$$Q_{\rm diss} = \nu\rho r^2\left(\frac{{\rm d}\Omega}{{\rm d}r}\right)^2 + \frac{\eta}{4\pi}J^2, \tag{14}$$

where the first term is related to viscous dissipation and the second term is related to resistive dissipation. By converting the gas pressure and the magnetic pressure in terms of sound speed ($c_{\rm s}^2 = p_{\rm gas}/\rho$) and Alfvén speed ($c_{\rm A}^2 = B_\varphi^2/4\pi\rho$), and by using equations (12)–(14) for equations (7)–(10), we can write

$$\frac{1}{r^2}\frac{\rm d}{{\rm d}r}(r^2\rho v_{\rm r}) = \dot{\rho}, \tag{15}$$

$$v_{\rm r}\frac{{\rm d}v_r}{{\rm d}r} + \frac{1}{\rho}\frac{\rm d}{{\rm d}r}(\rho c_{\rm s}^2) + \frac{GM_*}{r^2} = r\Omega^2 - \frac{c_{\rm A}^2}{r} - \frac{1}{2\rho}\frac{\rm d}{{\rm d}r}(\rho c_{\rm A}^2), \tag{16}$$

$$\rho v_{\rm r}\frac{\rm d}{{\rm d}r}(r^2\Omega) = \frac{\alpha}{r^2}\frac{\rm d}{{\rm d}r}\left[\frac{\rho c_{\rm s}^2}{\Omega_{\rm K}}(1+\beta)^{1-\mu}r^4\left(\frac{{\rm d}\Omega}{{\rm d}r}\right)\right], \tag{17}$$

$$\frac{1}{\gamma-1}\left[v_{\rm r}\frac{\rm d}{{\rm d}r}(\rho c_{\rm s}^2) + \frac{\gamma\rho c_{\rm s}^2}{r^2}\frac{\rm d}{{\rm d}r}\left(r^2 v_{\rm r}\right)\right] = fQ_{\rm diss}, \tag{18}$$

$$\frac{1}{r}\frac{\rm d}{{\rm d}r}\left[\sqrt{4\pi\rho c_{\rm A}^2}\left(rv_{\rm r} - \frac{\eta_0(1+\beta)^{1-\mu}}{4\rho\beta\Omega_{\rm K}}\frac{\rm d}{{\rm d}r}(r^2\rho c_{\rm A}^2)\right)\right] = \dot{B}_\varphi, \tag{19}$$

where

$$Q_{\rm diss} = \frac{(1+\beta)^{1-\mu}}{\Omega_{\rm K}}\left[\alpha r^2\rho c_{\rm s}^2\left(\frac{{\rm d}\Omega}{{\rm d}r}\right)^2 + \frac{\eta_0}{8r^4\rho\beta}\left(\frac{\rm d}{{\rm d}r}(r^2\rho c_{\rm A}^2)\right)^2\right]. \tag{20}$$

3. Self-similar solutions

We seek self-similar solutions in the following forms:

$$v_r = -c_1\alpha\sqrt{\frac{GM_*}{r}}, \tag{21}$$

$$\Omega = c_2\sqrt{\frac{GM_*}{r^3}}, \tag{22}$$

$$c_{\rm s}^2 = c_3\frac{GM_*}{r}, \tag{23}$$

$$c_{\rm A}^2 = \frac{B_\varphi^2}{4\pi\rho} = 2\beta c_3\frac{GM_*}{r}, \tag{24}$$

where coefficients c_1, c_2 and c_3 are determined later. We assume a power-law relation for density

$$\rho(r) = \rho_0 r^s, \tag{25}$$

where ρ_0 and s are constants. By using the above self-similar quantities, the mass-loss rate and the field escaping/creating rate must have the following form:

$$\dot{\rho} = \dot{\rho}_0 r^{s-3/2}, \tag{26}$$

$$\dot{B}_\varphi = \dot{B}_0 r^{\frac{s-4}{2}}, \tag{27}$$

where $\dot{\rho}_0$ and $\dot{B}_0$ are constants.

Substituting the above solutions in the continuity, momentum, angular momentum, energy and induction equations (15)–(20), we obtain the following relations:

$$\dot{\rho}_0 = -\left(s+\frac{3}{2}\right)\alpha\rho_0 c_1\sqrt{GM_*}, \tag{28}$$

$$-\frac{1}{2}c_1^2\alpha^2 + 1 - c_2^2 + c_3\left(s-1+\beta(1+s)\right) = 0, \tag{29}$$

$$c_1 = 3(s+2)(1+\beta)^{1-\mu}c_3, \tag{30}$$

$$-\frac{1}{\gamma-1}\alpha c_1(2s-2+3\gamma) = \frac{1}{2}f(1+\beta)^{1-\mu}(9\alpha c_2^2 + 2\eta_0\beta c_3(1+s)^2), \tag{31}$$

$$\dot{B}_0 = -\frac{s}{2}GM_*\sqrt{2\pi\rho_0\beta c_3}(2c_1\alpha + \eta_0 c_3(1+s)(1+\beta)^{1-\mu}). \tag{32}$$

The above equations indicate that, for $s = -3/2$, there is no mass loss, while for $s > -3/2$ mass loss (wind) exists. The escape and creation of magnetic fields are balanced in $s = -2 + (3\eta_0)/(6\alpha + \eta_0)$ such that $\dot{B}_0$ becomes zero, solving it in $s = -3/2$ (no wind) implies that $\eta_0 = (6/5)\alpha$. This amount of η_0 corresponds to the magnetic Prandtl number $5/6$. Thus, when the flow has $s = -3/2$ and $\eta_0 = (6/5)\alpha$, we expect the escape and creation of magnetic field to be balanced and hence there is no mass loss. In $\eta_0 = 0$, for balance of escape and creation of magnetic field, we have $s = -2$. In this paper, only the case of no wind ($s = -3/2$) is considered such that $\dot{\rho}_0 = 0$ and $\dot{B}_\varphi \propto r^{-11/4}$. We also note that $\dot{B}_\varphi$ in $P_\mathrm{m} = 5/6$ will be zero.

Equations (29)–(31) imply that for a given α, η_0, β, s and f, a closed set of equations of c_1, c_2 and c_3 are formed which will determine the behavior of the accretion flow.

3.1 *Analysis*

By using equations (29)–(31), the coefficients of c_i have the following forms:

$$c_1 = \frac{1}{2\alpha^2}\left(D_4 + \sqrt{D_4^2 + 8\alpha^2}\right), \tag{33}$$

$$c_2^2 = -\frac{2}{9} c_1 D_1 D_2, \tag{34}$$

$$c_3 = \frac{1}{3(s+2)} c_1 D_1, \tag{35}$$

where

$$D_1 = \frac{1}{(1+\beta)^{1-\mu}}, \tag{36}$$

$$D_2 = \frac{(2s - 2 + 3\gamma)}{(\gamma - 1) f} + \frac{\eta_0 \beta}{3\alpha} \frac{(1+s)^2}{(s+2)}, \tag{37}$$

$$D_3 = \frac{s - 1 + \beta(1+s)}{s+2}, \tag{38}$$

$$D_4 = \frac{2}{3} D_1 \left(\frac{2}{3} D_2 + D_3\right). \tag{39}$$

The obtained results imply that the model is parametrized by the ratio of specific heat, γ, the standard viscous parameter, α, the magnetic diffusivity parameter, η_0, the degree of magnetic pressure to gas pressure, β, the advection parameter, f and the mass-loss rate parameter, s.

Equation (37) implies that for a value of the magnetic pressure fraction, β, the value of D_2 will be zero, and from equations (22) and (34) we know that $\Omega \propto c_2^2 \propto D_2$. So we conclude that for a value of the magnetic pressure fraction we define it as β_b (braking β) and the angular velocity will be zero. Above that value, c_2^2 becomes negative that it is not physical. By solving $D_2 = 0$ for the β parameter, we can write

$$\beta_\mathrm{b} = -\frac{3\alpha}{f\eta_0} \frac{(s+2)}{(s+1)^2} \frac{(2s - 2 + 3\gamma)}{(\gamma - 1)}. \tag{40}$$

In the case of no mass-loss, $s = -3/2$, we can write

$$\beta_\mathrm{b} = \frac{18\alpha}{f\eta_0}\left(\frac{5/3 - \gamma}{\gamma - 1}\right). \tag{41}$$

The above equation in terms of the magnetic Prandtl number becomes

$$\beta_{\rm b} = 18 \left(\frac{P_{\rm m}}{f}\right) \left(\frac{5/3 - \gamma}{\gamma - 1}\right). \tag{42}$$

Equations (40)–(42) express that the amount of the $\beta_{\rm b}$ parameter depends on f, $P_{\rm m}$, and γ. $\beta_{\rm b}$ decreases by adding the advection degree, and it increases by adding the magnetic Prandtl number. Due to high conductivity $P_{\rm m}$ becomes very large and so $\beta_{\rm b}$ also becomes very large. Due to high resistivity and low viscosity, $\beta_{\rm b}$ becomes small. Since the magnetic pressure fraction is always positive or equal to zero ($\beta \geq 0$), $\gamma \leq 5/3$.

Examples of the coefficients c_i in the two cases of $\mu = 0$ and 1.0 are shown in Figures 1 and 2 as a function of the degree of magnetic pressure to the gas pressure, (β), for different values of the magnetic diffusivity, η_0. In Figures 1 and 2, c_4 is the viscous torque that is obtained from the right-hand side of equation (30), c_5 is the total energy dissipation by the viscosity and the resistivity, and is calculated by the right-hand side of equation (31), and c_6 is the ratio of the resistive dissipation to the viscous dissipation.

3.1.1 *First case:* $\mu = 0$. The parameter of μ has appeared in the equations of the kinematic coefficient of the viscosity and the magnetic diffusivity to indicate important degree of total pressure in them. In the case of $\mu = 0$, the kinematic coefficient of viscosity and the magnetic diffusivity become

$$\begin{aligned} \nu &= \alpha \frac{c_{\rm s}^2}{\Omega_{\rm K}} (1 + \beta) \\ &= \alpha c_3 (1 + \beta) \sqrt{G M_*}\, r^{1/2}, \end{aligned} \tag{43}$$

$$\begin{aligned} \eta &= \eta_0 \frac{c_{\rm s}^2}{\Omega_{\rm K}} (1 + \beta) \\ &= \eta_0 c_3 (1 + \beta) \sqrt{G M_*}\, r^{1/2}. \end{aligned} \tag{44}$$

In this case the function forms of the kinematic coefficient of viscosity and the magnetic diffusivity by a factor $(1 + \beta)$ deviate from the function forms that Narayan & Yi (1995) and Shadmehri (2004) have used. Also we add profiles of non-resistive and non-magnetic case to our profiles to compare the present model with canonical ADAF solutions (e.g. Narayan & Yi 1995). The existence of $(1 + \beta)$ in ν and η causes the value to increase by adding the toroidal magnetic field, and because ν and η have direct effects on the viscous torque (c_4) and the energy dissipation (c_5), we expect c_4 and c_5 to increase by adding the toroidal magnetic field and the profiles of c_4 and c_5 confirm it. Also, by adding the magnetic diffusivity (η_0) of the flow, c_4 and c_5 increase, because adding η_0 parameter causes the resistivity of the flow to increase and so the energy dissipation due to ohmic dissipation increases. The rise in the energy dissipation causes the temperature of the flow to increase (the profiles

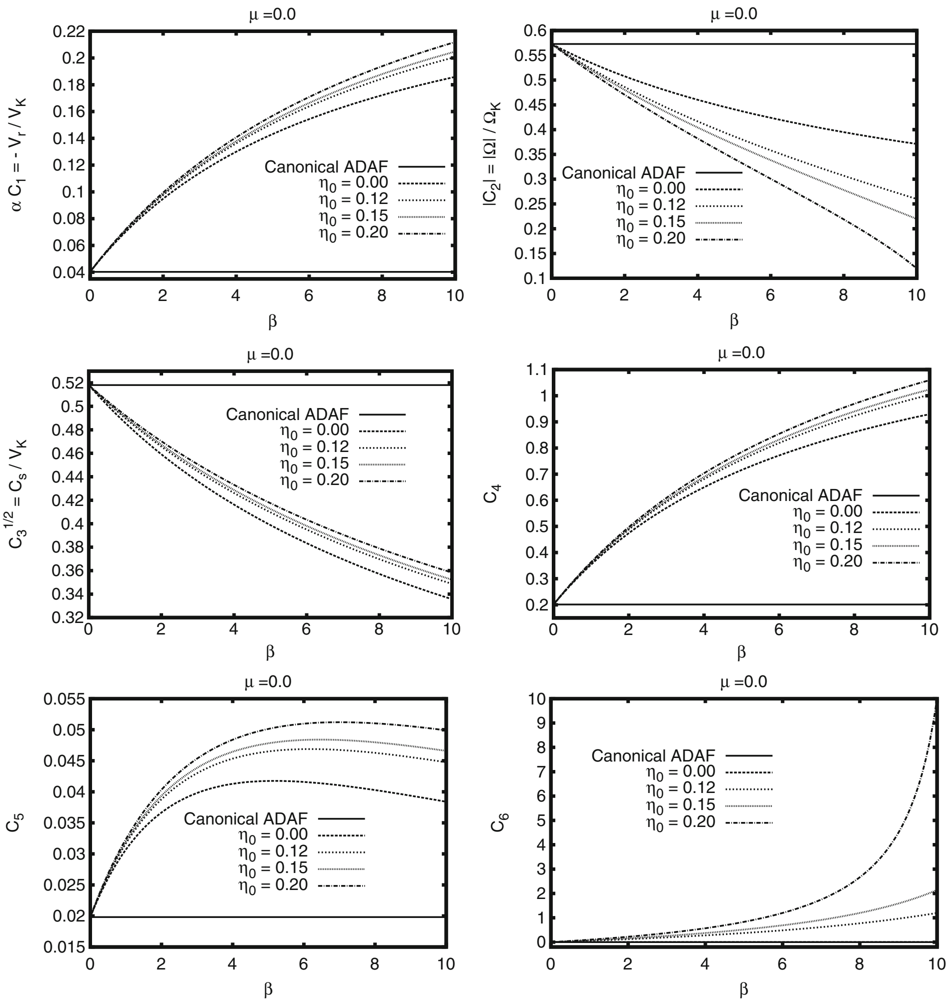

Figure 1. Physical quantities of the flow as a function of the degree of magnetic pressure to the gas pressure, for several values of $\eta_0 = 0, 0.12, 0.15$ and 0.2 that corresponds to $P_{\mathrm{m}} = \infty$, 5/6, 2/3 and 5/10. The disc density profile is set to be $s = -3/2$ (no wind), the ratio of the specific heat is set to be $\gamma = 1.3$, the viscous parameter is $\alpha = 0.1$ and the advection parameter is $f = 1.0$.

of c_3 confirm it), and because of the viscous torque proportional with temperature (sound speed), the viscous torque, c_4, increases on adding the magnetic diffusivity. Also, the profiles of c_4 and c_5 imply that for all values of β and η_0, the total energy dissipation and the viscous torque are larger than the canonical ADAF solutions. The profiles of c_3 show that the temperature of the flow decreases by adding the toroidal component of magnetic field. This property is qualitatively consistent with the results of Bu *et al*. (2009) and Khesaki & Faghei (2009). The profiles of c_6 show that the ratio of the resistive dissipation to the viscous dissipation increases by adding the toroidal component of magnetic field (β) and the magnetic diffusivity (η_0). In case

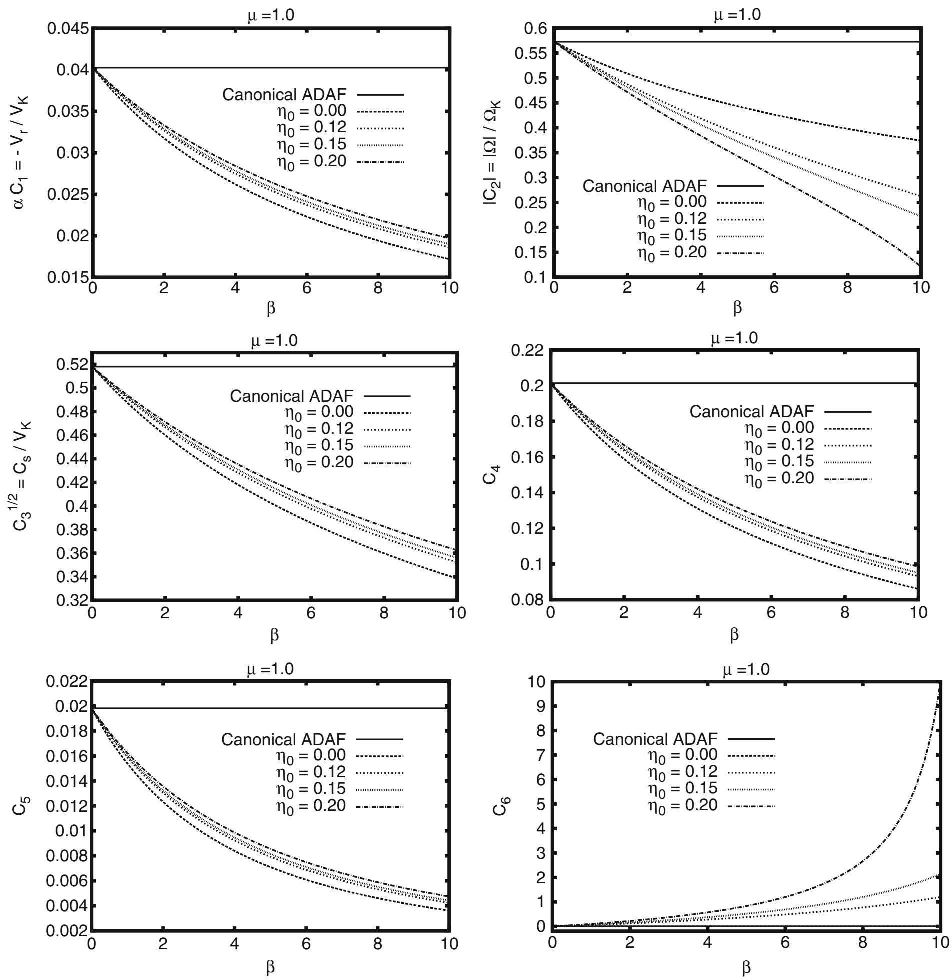

Figure 2. Physical quantities of the flow as a function of the degree of magnetic pressure to the gas pressure, for several values of $\eta_0 = 0, 0.12, 0.15$ and 0.2 that corresponds to $P_{\rm m} = \infty$, $5/6$, $2/3$ and $5/10$. The disc density profile is set to be $s = -3/2$ (no wind), the ratio of the specific heat is set to be $\gamma = 1.3$, the viscous parameter is $\alpha = 0.1$ and the advection parameter is $f = 1.0$.

of small amounts of magnetic field and the magnetic diffusivity, the dominant heat generator is the viscous dissipation, while in large values of magnetic field and the magnetic diffusivity, the dominant heat generator will be the resistive dissipation. Also, Fig. 1 shows that by adding the β and η_0 parameters, the radial infall velocity, c_1, increases, and the angular velocity, c_2, decreases. This can be due to the behavior of the viscous torque (c_4) in terms of parameters of β and η_0, because rise in viscous torque by adding η_0 and β parameters generate a larger negative torque in the angular momentum equation and causes the angular velocity of the flow to decrease, and the matter accretes with larger speed. In the present model, the flow rotates slower than the canonical ADAF and accretes faster than it. The increase in the radial infall velocity by adding the parameter of β is qualitatively consistent with the results by Bu *et al*. (2009) and Khesali & Faghei (2009).

3.1.2 *Second case: $\mu = 1$.* In the case of $\mu = 1$, for the kinematic coefficient of viscosity and the magnetic diffusivity we can write

$$\nu = \alpha \frac{c_s^2}{\Omega_K} = \alpha c_3 \sqrt{GM_*}\, r^{1/2}, \tag{45}$$

$$\eta = \eta_0 \frac{c_s^2}{\Omega_K} = \eta_0 c_3 \sqrt{GM_*}\, r^{1/2}. \tag{46}$$

In this case the function forms of the kinematic coefficient of viscosity and the magnetic diffusivity are the same as the function forms that Shadmehri (2004) and Narayan & Yi (1995) have used. The behavior of the physical quantities of the flow in this case are shown in Fig. 2. The absence of $(1 + \beta)$ in this case for ν and η relate to the previous case and cause the value to decrease by a factor of $(1 + \beta)$, and the absence of this factor by adding a parameter of β increases. The profiles of the angular momentum transport (c_4) and the total energy dissipation (c_5) imply that the amount decreases by a factor of $(1 + \beta)$, related to the previous case. As their behavior in terms of the toroidal component of magnetic field have changed, they decrease by adding the β parameter. However, the magnetic diffusivity has previous effects and causes these two quantities to increase by adding η_0. Also, the solutions show that the viscous torque and the total dissipation in this case are smaller than the canonical ADAF. The weakening of c_4 and c_5 by adding the β parameter causes the radial infall velocity of the flow to decrease, and due to rise in c_4 and c_5 by adding η_0, the radial infall velocity increases by adding the η_0 parameter. The behavior of the other physical quantities in terms of β and η_0 parameters do not change, and represent small variations. Comparison of the radial infall velocity profiles with canonical ADAF solutions implies that the flow accretes slower than the canonical ADAF that is different from the previous case. The decrease in temperature and the viscous torque by adding the parameter of β is qualitatively consistent with the results by Bu *et al*. (2009).

3.2 *Mass accretion rate*

According to assumptions in section 2 the mass accretion rate is defined as

$$\dot{M} = -4\pi r^2 \rho v_r. \tag{47}$$

The mass accretion rate under self-similar transformations of equations (21) and (25) becomes

$$\dot{M} = 4\pi \alpha \rho_0 c_1 \sqrt{GM_*}\, r^{s+3/2}. \tag{48}$$

In our case, $s = -3/2$ (no wind). For the mass accretion rate, we can write

$$\dot{M} = 4\pi\alpha\rho_0 c_1 \sqrt{GM_*} \tag{49}$$

which implies that the mass accretion rate does not vary by position. This result is qualitatively consistent with the results by Shadmehri (2004), Ghanbari *et al*. (2007) and Akizuki & Fukue (2006). Although the present model of accretion flow is different from the accretion flow of Bondi (1952) in various aspects, we can define Bondi accretion rate as

$$\dot{M}_{\text{Bondi}} = \pi G^2 M_*^2 \left(\frac{\rho(\infty)}{c_{\text{s}}^3(\infty)}\right)\left[\frac{2}{5-3\gamma}\right]^{(5-3\gamma)/2(\gamma-1)} \tag{50}$$

where $\rho(\infty)$ and $c_{\text{s}}(\infty)$ are the density and the sound speed in the gas far away from the star (Frank *et al*. 2002). Bondi accretion rate in terms of our self-similar transformations becomes

$$\dot{M}_{\text{Bondi}} = \pi\sqrt{GM_*}\left(\frac{\rho_0}{c_3^{3/2}}\right)\left[\frac{2}{5-3\gamma}\right]^{(5-3\gamma)/2(\gamma-1)}. \tag{51}$$

Thus, we can write the mass accretion rate in terms of Bondi accretion rate as follows:

$$\dot{M}/\dot{M}_{\text{Bondi}} = 4\alpha c_1 c_3^{3/2}\left[\frac{2}{5-3\gamma}\right]^{(3\gamma-5)/2(\gamma-1)} r^{s+3/2}. \tag{52}$$

In our case, $s = -3/2$ (no wind), we can write

$$c_7 = \dot{M}/\dot{M}_{\text{Bondi}} = 4\alpha c_1 c_3^{3/2}\left[\frac{2}{5-3\gamma}\right]^{(3\gamma-5)/2(\gamma-1)}. \tag{53}$$

Here, we defined a new parameter of c_7 that indicates the ratio of the mass accretion rate to Bondi accretion rate. Examples of the coefficient of c_7 in two cases of $\mu = 0$ and 1.0 are shown in Fig. 3 as a function of the degree of magnetic pressure to the gas pressure, (β), for different values of the magnetic diffusivity, η_0. The profiles of c_7 show that in the case $\mu = 0$, the mass accretion rate increases by adding the toroidal magnetic field and magnetic diffusivity. The solutions for $\mu = 1$ imply that the mass accretion rate decreases by adding the toroidal magnetic field and increases by adding the magnetic diffusivity. On the other hand, the magnetic diffusivity in two cases causes the mass accretion rate to increase. In high magnetic pressure, the c_7 profiles for both cases show that the ratio of the mass accretion rate to the Bondi accretion rate is reduced with an increase in the magnetic pressure. This property is qualitatively consistent with the results of Kaburaki (2007). Comparison of the present model with the canonical ADAF solutions implies that in the case of $\mu = 0$ the flow accretes quicker than the canonical ADAF, however in the case of $\mu = 1$ the flow accretes slower than the canonical ADAF. Also, the profiles of c_7 in both cases show that the mass accretion rate in our model is smaller than the Bondi accretion rate that is, in adapting with observational evidence from Sgr A*, M87 and NGC 4261 (Kaburaki 2007).

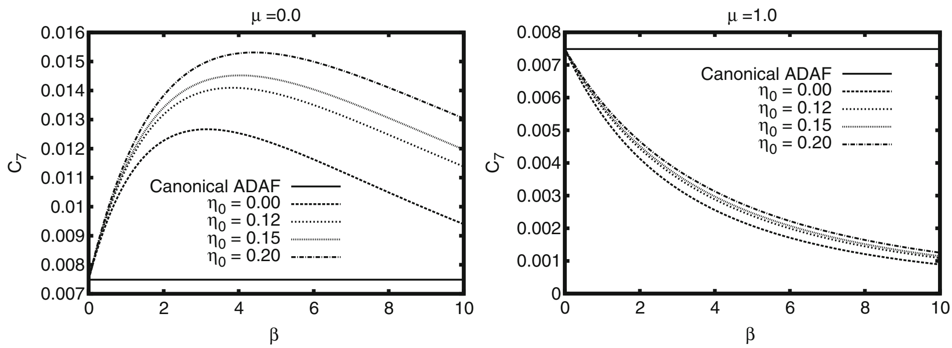

Figure 3. The ratio of mass accretion rate to Bondi accretion rate ($c_7 = \dot{M}/\dot{M}_{\rm Bondi}$) as a function of the degree of magnetic pressure to the gas pressure, for several values of $\eta_0 = 0$, 0.12, 0.15 and 0.2 that corresponds to $P_{\rm m} = \infty$, 5/6, 2/3 and 5/10. The disc density profile is set to be $s = -3/2$ (no wind), the ratio of the specific heat is set to be $\gamma = 1.3$, the viscous parameter is $\alpha = 0.1$ and the advection parameter is $f = 1.0$.

In section 3.1, we obtained the upper limit for the magnetic field ($\beta_{\rm b}$); using $\beta_{\rm b}$, equations (33)–(39), and assuming no wind case ($s = -3/2$), the mass accretion rate to the Bondi accretion rate (c_7) approximately is

$$c_7 = \dot{M}/\dot{M}_{\rm Bondi} \approx 24\sqrt{2}\,\alpha\, g_1 \frac{(1+\beta_{\rm b})^{1-\mu}}{(5+\beta_{\rm b})^{5/2}} = 24\sqrt{2}\,\alpha\, g_1 \frac{\left(1+\frac{18\, g_2 P_{\rm m}}{f}\right)^{1-\mu}}{\left(5+\frac{18\, g_2 P_{\rm m}}{f}\right)^{5/2}}, \tag{54}$$

where

$$g_1 = \left[\frac{2}{5-3\gamma}\right]^{\frac{(3\gamma-5)}{2(\gamma-1)}},$$

$$g_2 = \left[\frac{5/3-\gamma}{\gamma-1}\right].$$

To obtain the root derivative of c_7 in terms of f, we can write

$$\frac{{\rm d}c_7}{{\rm d}f} = 0 \;\Rightarrow\; f_{\rm max} = \left(\frac{18}{5}\right)\left(\frac{3+2\mu}{1-2\mu}\right) g_2 P_{\rm m}.$$

The above equation states that the mass accretion rate in $f_{\rm max}$ becomes maximum and for $f > f_{\rm max}$ the mass accretion rate reduces by advection degree. The $f_{\rm max}$ for $\mu > 1/2$ becomes negative, while $0 \le f \le 1$. Thus, the relation of $f_{\rm max}$ is valid only for the case $\mu = 0$. In case of $\mu = 0$, $f_{\rm max} = (54/5)\, g_2 P_{\rm m}$ and $\beta_b = 5/3$. Thus, we expect that in dominant magnetic case ($\beta > 1$), the accretion efficiency decreases by advection degree parameter. In the case of $\mu = 1$, due to the lack of any extremum, the mass accretion rate only increases by advection degree and does not show any decrease.

In the case of high magnetic pressure ($\beta \gg 1$), equation (54) becomes

$$c_7 = \dot{M}/\dot{M}_{\rm Bondi} \approx 24\sqrt{2}\,\alpha\; g_1 \left(\frac{f}{18\; g_2 P_{\rm m}}\right)^{3/2+\mu}.$$

The above equation implies that the mass accretion rate to Bondi accretion is strongly dependant on viscosity parameter, Prandtl number and the advection degree. The mass accretion rate increases by f and decreases by $P_{\rm m}$. Thus, accretion efficiency increases by the advection degree parameter in high magnetic pressure.

3.3 *Time-scales*

To estimate the effect of viscosity and resistivity on the accretion discs, we compare the viscous and resistive time-scales with accretion time-scale. The accretion time-scale, $t_{\rm acc}$, and the viscous time-scale, $t_{\rm visc}$, are given by

$$t_{\rm acc} = \frac{r}{-v_{\rm r}}, \tag{55}$$

$$t_{\rm visc} = \frac{r^2}{\nu}. \tag{56}$$

We are using a similar functional form of $t_{\rm visc}$ for the resistive time-scale, $t_{\rm resis}$, that is given by

$$t_{\rm resis} = \frac{r^2}{\eta}. \tag{57}$$

By using self-similar forms of physical quantities, we can write

$$\begin{aligned}\frac{t_{\rm resis}}{t_{\rm acc}} &= \frac{\alpha}{\eta_0}\frac{c_1}{c_3}(1+\beta)^{\mu-1} \\ &= 3(\frac{\alpha}{\eta_0})(s+2).\end{aligned} \tag{58}$$

Equation (30) is used for fraction of c_1/c_3. As we said in the previous section, the present model α/η_0 is the magnetic Prandtl number, $P_{\rm m}$, so the above equation becomes

$$\frac{t_{\rm resis}}{t_{\rm acc}} = 3P_{\rm m}(s+2). \tag{59}$$

In our case, $s = -3/2$ (no wind), we can write

$$\frac{t_{\rm resis}}{t_{\rm acc}} = \frac{3}{2}P_{\rm m}. \tag{60}$$

The above equation implies that for $P_{\rm m} \leq 2/3$, the magnetic diffusivity time-scale is shorter than or equal to accretion time-scale, while for $P_{\rm m} > 2/3$ the accretion time-scale is shorter.

Similar calculations for the viscous time-scale is expressed as

$$\frac{t_{\rm visc}}{t_{\rm acc}} = 3(s+2), \tag{61}$$

where in no wind case ($s = -3/2$) becomes

$$\frac{t_{\rm visc}}{t_{\rm acc}} = (3/2). \tag{62}$$

Thus, the viscosity time-scale will be longer than the accretion time-scale. By comparing the magnetic diffusivity time-scale and the viscous time-scale, we can write

$$\begin{aligned} \frac{t_{\rm resis}}{t_{\rm visc}} &= \frac{r^2/\eta}{r^2/\nu} \\ &= \frac{\nu}{\eta} \\ &= P_{\rm m}. \end{aligned} \tag{63}$$

Thus, the magnetic Prandtl number specifies which one is shorter. For example in flow with high conductivity (e.g. Akizuki & Fukue 2006; Bu *et al*. 2009), $\eta \to 0$, the magnetic Prandtl number limits to infinity, and so the magnetic diffusivity time-scale will be much longer than the viscous time-scale. On the other hand, for a flow with finite resistivity and tiny viscosity (e.g. Shadmehri 2004), the magnetic Prandtl number limits to zero, and so the magnetic diffusivity time-scale is much shorter than the viscous time-scale. When the resistivity and the viscosity are approximately equal, $P_{\rm m} \sim 1$, we expect $t_{\rm resis} \sim t_{\rm visc}$. Also, in the special case of $P_{\rm m} = 5/6$ and $s = -3/2$ that escape and creation of magnetic field are balanced, there is no mass-loss, $t_{\rm resis} = (5/6)t_{\rm visc}$.

4. Summary and discussion

In this paper, the influences of the resistivity on the structure of the advection-dominated accretion flow is investigated. It is used only as an azimuthal component of the magnetic field that is consistent with the observational evidence of Galactic center (Novak *et al*. 2003; Chuss *et al*. 2003; Yuan 2006). The α-prescription is used for the kinematic coefficient of viscosity and the magnetic diffusivity. The equations of the model are solved by a semi-analytical self-similar method in comparison with the self-similar solution by Akizuki & Fukue (2006).

The physical quantities of the disc are sensitive to the amounts of the magnetic pressure fraction (β) and the magnetic diffusivity (η_0) parameters. As the angular velocity of the flow decreases by adding the β and η_0 parameters, the solutions also show that for a value of the magnetic pressure fraction, the angular velocity of disc becomes zero. This amount of the magnetic pressure fraction strongly depends upon the properties of the accreting gas, such as the viscosity, resistivity, adiabatic index and advection degree. The solutions represent the radial infall velocity as the magnetic diffusivity increases. Also the solutions show that the temperature of the flow decreases by adding the toroidal component of magnetic field, so this result is qualitatively consistent with the results of Bu *et al*. (2009) and Khesali & Faghei (2009). Also, the profiles of the temperature of the flow show that the temperature of the flow increases by adding the magnetic diffusivity that is due to rise in resistive dissipation. The comparison of the present model with the Bondi accretion implies that for all values of the β and η_0 parameters, the Bondi accretion rate is larger than the mass accretion rate that is in accordance with the observational evidence from Sgr A*, M87 and NGC 4261 (Kaburaki 2007). Also, the mass accretion rate profiles at high magnetic pressure state that the magnetic pressure reduces mass accretion rate that is similar to results of Kaburaki (2007). Moreover, the results represent that the viscous dissipation will be dominant for small amounts of magnetic field, while the resistive dissipation will be dominant for large amount of magnetic field and magnetic diffusivity.

As noted in the Introduction, the MHD simulations show that the linear growth of MRI reduces significantly by ohmic dissipation. Linear growth of the MRI in the resistive fluid can be characterized by the Lundquist number ($S_{\rm MRI} = c_{\rm A}^2/\eta\Omega$) and magnetic Reynolds number ($\mathrm{Re_M} = c_{\rm s}^2/\eta\Omega$), where $c_{\rm A}$, $c_{\rm s}$, η and Ω have the usual meaning. In terms of our self-similar transformations, the Lundquist number and magnetic Reynolds number become $S_{\rm MRI} = 2\beta/\eta_0 c_2(1+\beta)^{1-\mu}$ and $\mathrm{Re_M} = 1/\eta_0 c_2(1+\beta)^{1-\mu}$. The solutions of the present model show that $S_{\rm MRI}$ and $\mathrm{Re_M}$ decrease by resistivity. This property is qualitatively consistent with MHD simulation results (Fleming *et al*. 2000; Masada & Sano 2008).

Here, latitudinal dependence of physical quantities is ignored, while some authors showed that latitudinal dependence is important in structure consideration of a disc (Narayan & Yi 1995; Shadmehri 2004; Ghanbari *et al*. 2007). One can investigate latitudinal behavior of such discs. Furthermore, in a realistic model the advection parameter f is a function of position, if one can consider such discs.

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References

Abramowicz, M., Chen, X., Kato, S., Lasota, J. P., Regev, O. 1995, *Astrophys. J.*, **438**, L37.
Aitken, D. K., Wright, C. M., Smith, C. H., Roche, P. F. 1993, *Mon. Not. R. Astron. Soc.*, **262**, 456.
Akizuki, C., Fukue, J. 2006, *Publ. Astron. Soc. Japan*, **58**, 469.
Begelman, M. C., Pringle, J.E. 2007, *Mon. Not. R. Astron. Soc.*, **375**, 1070.
Bisnovatyi-Kogan, G. S., Lovelace, R. V. E. 2001, *New Astron. Rev.* **45**, 663.
Blandford, R. D., Begelman, M. C. 1999, *Mon. Not. R. Astron. Soc.*, **303**, L1.
Bondi, H. 1952. *Mon. Not. R. Astron. Soc.*, **112**, 195.
Bu, D., Yuan, F., Xie, F. 2009, *Mon. Not. R. Astron. Soc.*, **392**, 325.
Campbell, C. G. 1999, *Geophys. Astrophys. Fluid Dynamics*, **90**, 113.
Chuss, D. T., Davidson, J. A., Dotson, J. L., Dowell, C. D., Hildebrand, R. H., Novak, G., Vaillancourt, J. E. 2003, *Astrophys. J.*, **599**, 1116.

Cowling, T. G. 1981, *Ann. Rev. Astron. Astrophys.*, **19**, 115.
Fleming, T. P., Stone, J. M., Hawley, J. F. 2000, *Astrophys. J.*, **530**, 464.
Fleming, T. P., Stone, J. M. 2003, *Astrophys. J.*, **585**, 908.
Frank, J., King, A. R., Raine, D. 2002, *Accretion Power in Astrophysics*, Cambridge Univ. Press, Cambridge.
Gammie, C. F., Menou, K. 1998, *Astrophys. J.*, **332**, 659.
Ghanbari, J., Salehi, F., Abbassi, S. 2007, *Mon. Not. R. Astron. Soc.*, **381**, 159.
Ichimaru, S. 1977, *Astrophys. J.*, **214**, 840.
Kaburaki, O. 2000, *Astrophys. J.*, **531**, 210.
Kaburaki, O. 2007, *Astrophys. J.*, **662**, 102.
Kaburaki, O., Yamazaki, N., Okuyama, Y. 2002, *New A*, **7**, 283.
Kaburaki, O., Nankou, T., Tamura, N., Wajima, K. 2010, *Publ. Astron. Soc. Japan*, **62**, 1177.
Khesali, A., Faghei, K. 2008, *Mon. Not. R. Astron. Soc.*, **389**, 1218.
Khesali, A., Faghei, K. 2009, *Mon. Not. R. Astron. Soc.*, **398**, 1361.
King, A. R., Pringle, J. E., Livio, M. 2007, *Mon. Not. R. Astron. Soc.*, **376**, 1790.
Krasnopolsky, R., Li, Z. Y., Shang, H. 2010, *Astrophys. J.*, **716**, 1541.
Kudoh, T., Kaburaki, O. 1996, *Astrophys. J.*, **460**, 199.
Kuwabara, T., Shibata, K., Kudoh, T., Matsumoto, R. 2000, *Publ. Astron. Soc. Japan*, **52**, 1109.
Masada, Y., Sano, T., 2008, *Astrophys. J.*, **689**, 1234.
Narayan, R., Yi, I. 1994, *Astrophys. J.*, **428**, L13.
Narayan, R., Yi, I. 1995, *Astrophys. J.*, **453**, 710.
Novak, G., Chuss, D. T., Renbarger, T. *et al.* 2003, *Astrophys. J. Lett.*, **583**, L83.
Ogilvie, G. I. 1999, *Mon. Not. R. Astron. Soc.*, **306**, L9O.
Rees, M. J., Begelman, M. C., Blandford, R. D., Phinney, E. S. 1982, *Nature*, **295**, 17.
Shadmehri, M. 2004, *Astron. Astrophys.*, **424**, 379.
Shakura, N. I., Sunyaev, R. A. 1973, *Astron. Astrophys.*, **24**, 337.
Sinha, M., Rajesh, S. R., Mukhopadhyay, B. 2009, *Res. Astron. Astrophys.*, **9**, 1331.
Stone, J. M., Gammie, C. F., Balbus, S. B., Hawley, J. F. 2000, in: *Protostars and Planets IV*, (eds) Manning, V., Boss, A., Russell, S., Uni. Arizona Press, Tuncsun.
Watarai, K. 2006, *Astrophys. J.*, **648**, 523.
Watarai, K. Y. 2007, *Publ. Astron. Soc. Japan*, **59**, 443.
Wright, C. M., Aitken, D. K., Smith, C. H., Roche, P. F. 1993, *Publ. Astron. Soc. Australia*, **10**, 247.
Yuan, F. 2006, *J. Phys. Conf. Ser.*, **54**, 427.