

The Dynamics of Radio Galaxies and Double–Double Radio Galaxies

C. Konar^{1,*}, M. Jamrozy², M. J. Hardcastle³, J. H. Croston⁴, S. Nandi⁵,
D. J. Saikia⁶ & J. Machalski²

¹*Institute of Astronomy and Astrophysics, AS, Taipei 10617, Taiwan.*

²*Astronomical Observatory, Jagiellonian University, ul. Orla 171, 30244 Kraków, Poland.*

³*University of Hertfordshire, College Lane, Hatfield, UK.*

⁴*University of Southampton, Southampton SO17 1BJ, UK.*

⁵*ARIES, Manora Peak, Nainital 263 129, India.*

⁶*National Centre for Radio Astrophysics, TIFR, Pune University Campus, Pune 411 007, India.*

**e-mail: chiranjib.konar@gmail.com*

Abstract. Relativistic and magnetized plasma ejected by radio loud AGNs through jets form the diffuse lobes of radio galaxies. The radiating particles (electron/electron–positron) in lobes emit in radio via the synchrotron process and X-ray via inverse-Compton scattering of cosmic microwave background photons. The thermal environment around radio galaxies emits X-rays via the thermal bremsstrahlung process. By combining information from these processes we can measure physical conditions in and around the radio lobes and thus study the dynamics of radio galaxies, including double–double radio galaxies.

Key words. Galaxies: active—galaxies: jets—galaxies: nuclei—radio continuum: galaxies—galaxies: individual: 3C457, J0041+3224.

1. Introduction

Radio galaxies are created by Active Galactic Nuclei (AGNs) which eject jets in two opposite directions and inflate what are called lobes. This jet-forming activity of an AGN may be episodic in nature. A striking example of episodic jet forming activity in radio galaxies is seen when a new pair of radio lobes form closer to the nucleus before the ‘old’ and more distant radio lobes have faded (e.g. Subrahmanyan *et al.* 1996; Lara *et al.* 1999). Such radio galaxies have been christened ‘Double–Double Radio Galaxies’ (DDRGs) by Schoenmakers *et al.* (2000). Hereafter, we will call the double lobed sources ‘radio galaxies’ and sources with two pairs of lobes with a common centre (core) DDRGs (see Saikia & Jamrozy 2009 for review).

Lobes consist of charged particles which can be either electron–positron pairs or electron–proton pairs. The exact composition of lobes is not known. Whatever may be the composition of lobes, it is well established that the lobes contain electrons (negatively charged particles) which radiate via the synchrotron and inverse-Compton (IC) processes. The synchrotron and IC power emitted by a single electron can be written as

$$P_{\text{syn}} = \frac{4}{3} c \sigma_T \gamma^2 \beta^2 U_B \quad (1)$$

and

$$P_{\text{IC}} = \frac{4}{3}c\sigma_{\text{T}}\gamma^2\beta^2U_{\text{ph}}, \quad (2)$$

(see Rybicki & Lightman 1979) where c is velocity of light, β is the velocity of the electron in units of c and γ is the corresponding Lorentz factor. U_B and U_{ph} are the magnetic field energy density and photon field energy density respectively, in which the electron is moving. σ_{T} is the Thomson cross-section and is given by $\sigma_{\text{T}} = \frac{8\pi}{3} \frac{e^4}{c^4 m_e^2}$, where e is the charge of an electron and m_e is the mass of an electron.

Since there cannot be charge separation, there must be positively charged particles in lobes. If there are positrons (as the positive component) then they will radiate (via synchrotron and IC processes) at the same rate as the electrons, as they have the same mass and magnitude of charge ($P_{\text{syn}} \propto e^4$ and $P_{\text{IC}} \propto e^4$, where P_{syn} is synchrotron power and P_{IC} is IC power emitted by an electron). However, protons will radiate at a very low rate compared to electrons/positrons ($P_{\text{syn/IC}} \propto \frac{1}{m_p^2}$). So, we can say that the radio lobes contain radiating particles and non-radiating particles in general. Since, ‘whether the radio galaxy lobes are in the minimum energy condition’ has always been an issue in the field of radio astronomy, astronomers test the minimum energy hypothesis whenever possible. In that case, we need to estimate the ratio (κ) of kinetic energy in non-radiating particles to that of radiating particles. Therefore, particle content in radio lobes or at least the knowledge of κ is a very important issue. From recent studies of large radio galaxies ($l =$ a few 100 kpc to Mpc), it has been found that the lobes are close to the minimum energy condition (Croston *et al.* 2004, 2005; Konar *et al.* 2009) with the assumption that the non-radiating particles are not energetically dominant. Since, the outer lobes of DDRGs have been created in the same way as the lobes of radio galaxies, those outer lobes are likely to have similar physical properties. But the density in relativistic plasma in the old cocoons into which the inner lobes are expanding, if equipartition holds, is orders of magnitude below that needed to ram-pressure confine the inner lobes even if we take protons (as non-radiating particles) into consideration. In other words, in such a tenuous medium the inner jets should have moved ballistically, had there been only relativistic matter fed by the jets into outer lobes in the previous cycle. But we clearly see well-defined inner lobes embedded in the outer lobes of the DDRGs. This compels us to think that besides electron–positron/proton fed by the jets, thermal matter from the ambient medium must have been ingested into the outer lobes. Therefore, we can say that we would not expect to observe the inner lobes unless some additional, higher density material is present in the outer cocoons. Kaiser *et al.* (2000) argued that entrainment of material from surrounding hot gaseous environment is not a viable source of material to confine the inner lobes, as it is a very slow process. Instead, they proposed a two fluid scenario in which dense clumps of warm (10^4 K) gas embedded in the intragroup/intracluster medium are ingested inside the expanding lobes and mixed in with the cocoon material after the clumps get disrupted. It is possible that those warm clumps get heated up by the interaction with the relativistic cocoon material. For the cases where thermal material might provide the missing pressure (e.g., in the case of FRI lobes), it has to be heated in order to make its density lower so that cavities are observed in the positions of the lobes (e.g. Croston *et al.* 2003; Hardcastle *et al.* 2005). So, there must be some heating mechanism in order to provide the missing pressure in the FRI lobes and this must be provided by the microphysics, which

seem unlikely to be different for FRIs and FRIIs. We do not know what temperature it is heated to, *but* it must be considerably larger than the ambient temperature. Once the entrained material has been heated, it is possible that some processes, like shocks, will drive some protons into a non-thermal tail. The work of Safouris *et al.* (2008) has shown that there is strong evidence for the ambient thermal matter being ingested into the outer lobes of DDRGs. Therefore, the inner jets are propagating through a medium which is in general a mixture of relativistic gas and ingested thermal gas from the ambient medium. However, Brocksopp *et al.* (2007, 2011) put forward an alternative model according to which the inner lobes arise from the emission of relativistic electrons within the outer lobes, which are compressed and re-accelerated by the bow shock in front of the restarted jets and within the outer lobes.

As per discussion in the previous paragraph, the issues that are crucial for an understanding of the radio galaxy dynamics are: (1) the proton-to-electron energy density ratio κ , (2) the magnetic field strength, (3) internal and external pressures of lobes, (4) the ingested thermal matter density inside the lobes and the kinetic energy density in the ingested thermal matter, and (5) the actual picture of the formation of inner lobes. To obtain these pieces of information, it is necessary to study radio galaxies as well as DDRGs both in X-ray and radio wavelengths.

2. How we can study the dynamics of radio galaxies by radio and X-ray observations?

As described above, leptons (e^-/e^+) in the lobes radiate via the synchrotron process at radio wavelengths. We know that Cosmic Microwave Background (CMB) photons are ubiquitous in the Universe. So the same charged particles, which radiate via synchrotron, also up-scatter CMB photons from millimeter wavelengths to X-ray wavelengths. We will hereafter refer to this IC scattering of CMB photons as the IC-CMB process. In addition, there is an ambient thermal medium in which a radio galaxy is always embedded. Such a medium radiates via the thermal bremsstrahlung in the X-ray. If the radio lobes are strong enough and the ambient thermal medium is relatively weak, we may be able to measure the X-ray from the lobes. The detection of lobe related X-ray depends on how well the background subtraction can be done. Once we detect such lobe-related X-ray, we can try to fit the data with IC-CMB and thermal bremsstrahlung models. The lobe-related X-rays detected so far in all radio galaxies are consistent with IC-CMB process, though the thermal origin of lobe related X-ray cannot be ruled out in some cases. In some cases, e.g. Pictor A (Hardcastle & Croston 2005) it is clearly ruled out. As an example, we describe below radio and X-ray observational results of a giant radio galaxy, 3C457, in brief in § 2.1 as a case study.

Since we are interested in the dynamics of the radio lobes, age is an important parameter of the source. We use spectral ageing analysis for estimating the ages. Caveats related to the evolution of the local magnetic field in the lobes need to be borne in mind (e.g. Rudnick *et al.* 1994; Jones *et al.* 1999; Blundell & Rawlings 2000). Furthermore, while Kaiser (2000) have suggested that spectral and dynamical ages are comparable if bulk back-flow and both radiative and adiabatic losses are taken into account in a self-consistent manner, Blundell & Rawlings (2000) suggest that this may be so only in the young sources with ages much less than 10 Myr. In

the study of the FR II-type giant radio galaxy, J1343 + 3758, Jamrozy *et al.* (2005) find the dynamical age to be approximately four times the maximum synchrotron age of the emitting particles. In spite of the caveats of the spectral ageing analysis, we get a reasonably good idea about the dynamics of the sources by using spectral ages. Spectral ageing analysis not only infers the ages, but also tell us *whether* the particles are re-accelerated in certain portion of the lobes (after being accelerated at the hotspot during injection). This hints us some physical processes (possibly lobe-environment interactions) which are not clearly known.

2.1 The giant radio galaxy 3C457: A case study

This particular source (as shown in Fig. 1) is a giant radio galaxy of size ~ 1 Mpc at a red-shift of 0.428. Our detailed work on this source has been published by Konar *et al.* (2009). Figure 1 shows, both in colour and contour, the GMRT images depicting diffuse relativistic plasma filling the huge lobes of this source. At the ends of the lobes we can see the clear double hotspots, which might have been created due to jittering of the jet direction. These hotspots are the characteristic features of FR II (Fanaroff & Riley 1974) radio galaxies and the sites of particle acceleration. IC-CMB modelling of the lobes yields that the magnetic fields, B , are $0.68^{+0.06}_{-0.09}$ and $0.40^{+0.04}_{-0.02}$ nT for the NE and SW lobes respectively (the internal pressure of the lobes are $0.224^{+0.034}_{-0.019} \times 10^{-12}$ and $0.378^{+0.043}_{-0.042} \times 10^{-12}$ Pa for the NE and SW lobe respectively). The corresponding minimum energy fields, B_{\min} , are 0.86 and 0.82 nT, which are within a factor of two of what we measure. This is in good agreement with the results of Croston *et al.* (2004, 2005). Minimum energy fields are calculated with the assumptions that the filling factor of lobes is unity, the energetically dominant particles are the radiating particles only (the contribution of protons has been neglected, i.e., $\kappa = 0$) and the electron energy spectra extend from $\gamma = 10$ to 10^5 . Croston *et al.* (2005) argue that the matching of the values between B (constrained from IC-CMB modelling) and B_{\min} (estimated with such a choice of parameters) has to be a coincidence for all radio lobes, unless these parameters have values close to what have been assumed. However, such a coincidence in all the lobes studied by Croston *et al.* (2004, 2005) and Konar *et al.* (2009) is unlikely. So in that sense, the IC-CMB modelling of radio lobes (including the lobes of 3C457) says that the non-radiating particles are not energetically dominant, i.e., $\kappa \sim 0$. Our spectral ageing analysis of the same source has yielded an age of ~ 30 Myr.

The right panel of Fig. 1 shows that the lobes as well as the environment are detected in X-ray. The bottom panel of Fig. 1 shows the X-ray surface brightness profile of the environment. We fitted the spectrum of the environment with the mekal model (thermal bremsstrahlung model) at a red-shift of 0.428 (see Konar *et al.* 2009 for details of the fit). This yields a reasonable fit with the gas temperature equal to $2.62^{+1.15}_{-0.69}$ keV. Our estimated luminosity of the hot gaseous environment is $\sim 1.51^{+1.33}_{-0.97} \times 10^{44}$ erg s $^{-1}$. This is a poor cluster scale environment as we find from the luminosity–temperature correlation (Osmond & Ponman 2004).

The radial surface brightness in units of counts arcsec $^{-2}$ was extracted from concentric annuli with point sources, chip gaps and lobes masked out (see Fig. 1). We fitted a model consisting of a point source situated at the position of the core of 3C457 and a single β model. The Bayesian estimates of β and core radius (r_c)

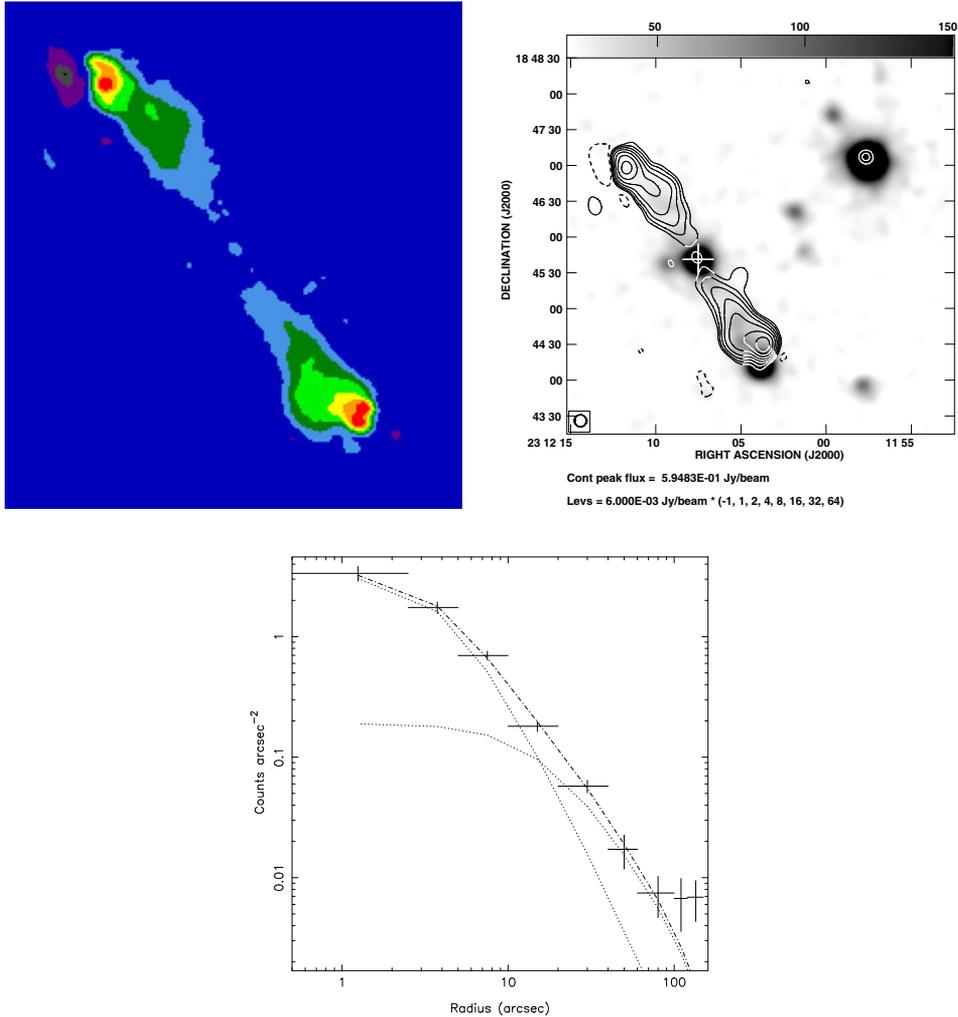


Figure 1. *Left panel:* A false colour GMRT map of the giant radio galaxy 3C457 at 610 MHz at full resolution ($\sim 6''$). *Right panel:* X-ray image, convolved with $10''$ Gaussian, of the source is in grey scale which is overlaid in contour with a 610-MHz GMRT image of $10''$ resolution. A '+' sign indicates the position of the optical host galaxy. *Bottom panel:* The surface brightness profile of the environment of 3C 457. The dot-dashed line is the total model (a point source plus a β -model) fitted to the data from all three cameras and dotted lines are two components of the total model. For details, see the work done by Konar *et al.* (2009), from which the right and the bottom panels have been reproduced.

are $\beta = 0.51^{+0.02}_{-0.17}$ and $r_c = 11.2^{+22.7}_{-6.4}$ arcsec. Errors are the $1-\sigma$ credible interval. The detailed analysis of the spatially extended emission around the core of 3C 457 and the β -model fitting procedure were carried out in the same way as described in Croston *et al.* (2008). Knowing the temperature and the surface brightness profile of the ambient medium, we have estimated the pressure profile. Fig. 2 shows the pressure profile of the hot gaseous ambient medium of 3C 457. Now it is evident from Fig. 2 that the lobe heads are close to pressure balance with the ambient medium.

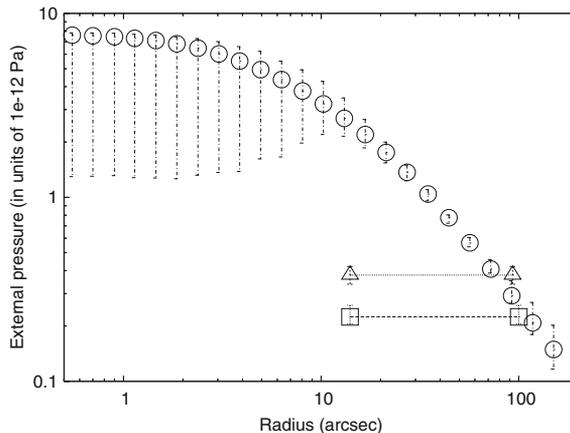


Figure 2. Pressure vs. deprojected radius plot of the environment of 3C 457 at the best-fitting temperature of 2.62 keV. The deprojected radius (on the X-axis) is measured from the position of the core of 3C 457. Open circles: Pressure of the environment vs. radial distance from the core. Open squares: Pressure of the NE lobe vs. radial distance from the core. Open triangles: Pressure of the SW lobe vs. radial distance from the core. The open square on the right-hand side indicates the positions of the hotspots of the two respective lobes. The lobes are assumed to be in the plane of the sky in this plot. The error in pressure shown in this plot is only due to the error in measurement of the emission measure only. Errors on temperature are not shown for clarity. This figure has been reproduced from the work of Konar *et al.* (2009).

The middle and the tail-part of the lobes are little underpressured, which is in contradiction with what has been assumed in self-similar models of radio galaxy (Falle 1991; Kaiser & Alexander 1997; Kaiser *et al.* 1997). We believe that the lateral expansion is subsonic, as otherwise; we would not have been able to model the lobe-related X-ray as due to pure IC-CMB process. So, the results are consistent with our paradigm of radio galaxy dynamics with radiating particles as energetically dominant.

Another quantity which is important for understanding the dynamics of radio galaxies is the lobe head velocity. Since the lobe heads of radio galaxies propagate through the dense thermal ambient medium, lobe head velocity is non-relativistic. So the ram pressure balance equation at the lobe head gives us the following equation:

$$n_p \mu m_p v_h^2 = \frac{L_{\text{jet}}}{v_{\text{jet}} A_h}, \quad (3)$$

where n_p is the particle density of the environment, μ is the mean molecular weight of particles in the environment and can be assumed to be 1.4, m_p is the proton mass, v_h is the jet head velocity, L_{jet} is the jet power and can be expressed as $\frac{U_{\text{tot}}}{t}$ with $t =$ spectral age, v_{jet} is the jet bulk speed which is usually assumed to be close to the speed of light and A_h is the lobe-head area over which the jet is impinging the environment. All the parameters except A_h in Equation (3) have unique values for the source 3C457. However, for A_h , we have estimated ~ 2 kpc and ~ 25 kpc as lower and upper limits of the diameters of the hotspots for 3C457. Our estimation yields that v_h is equal to $\sim 0.04c$ (for both lobes) and $0.003c$ (for both lobes) for lower and

higher limits of A_h respectively. So, the lobe heads can be supersonic, though the lateral expansion is subsonic, which is consistent with our IC-CMB modelling of the lobe related X-rays. Hence the entire analysis of the dynamics of FR II radio galaxies is self-consistent.

All our calculations presented here are by the assumption that the lobes are on the plane of the sky. Since 3C457 is a radio galaxy, it can be aligned at an angle between 45° and 90° with the line-of-sight. If we repeat our calculation with the assumption that our sources are aligned at an angle of 45° with the line-of-sight, then the values of magnetic fields and internal pressure of lobes do not change much. Hence the qualitative results related to the pressure balance of lobes remain more or less the same.

3. Double–double radio galaxies

In the case of DDRGs, the inner lobes are often confined within the cocoon material of the outer lobes. Illustrative examples of such sources are J0041 + 3224, J0116–4722 (Saikia *et al.* 2006; Sefouris *et al.* 2008; Saripalli *et al.* 2002, 2003).

We have made multifrequency radio observations of a small sample of DDRGs to study their dynamics. Preliminary results for the source J0041 + 3224 (see Fig. 3) are presented here. From our spectral ageing analysis of J0041 + 3224, we found, in the spectral age vs. distance plot as presented in Fig. 3, that the age is increasing monotonically with the distance of the strip from the hotspot for the eastern lobe. However, for the western lobe, the spectral age increases with the distance from the hotspot up to 200 kpc from the hotspot. After that the spectral age does not vary with the distance from the hotspot. This we interpret as due to particle re-acceleration in that portion of the western lobe. So, this result hints at some unknown physical processes causing particle re-acceleration in the lobes. We have found that the average injection spectral index, α_{inj} ($S_\nu \propto \nu^{-\alpha_{inj}}$) is similar (~ 0.76) for both the outer as well as the inner double for J0041 + 3224. We also found similar α_{inj} values, in both episodes of jet activity, of ~ 0.57 and 0.80 for J1453 + 3308 (see Fig. 4, Konar *et al.* 2006) and 4C29.30 (Jamrozy *et al.* 2007) respectively. Readers can notice the similarity of the slopes of two power-law spectra (spectrum of inner double and low

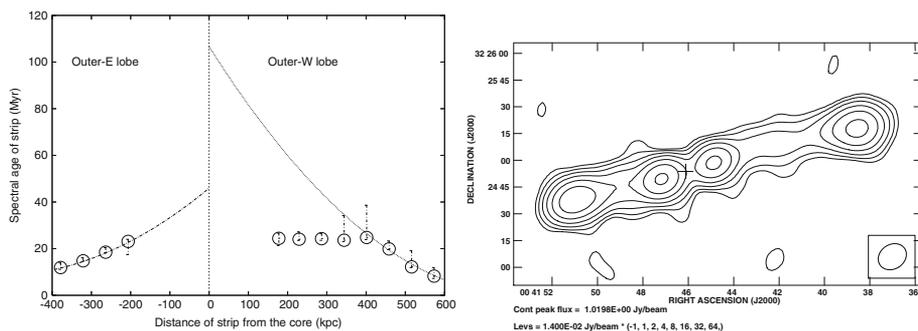


Figure 3. *Left panel:* A plot of spectral age vs. strip distance from the core. *Right panel:* 240-MHz GMRT image of DDRG J0041 + 3224. A ‘+’ sign indicates the position of the optical host galaxy.

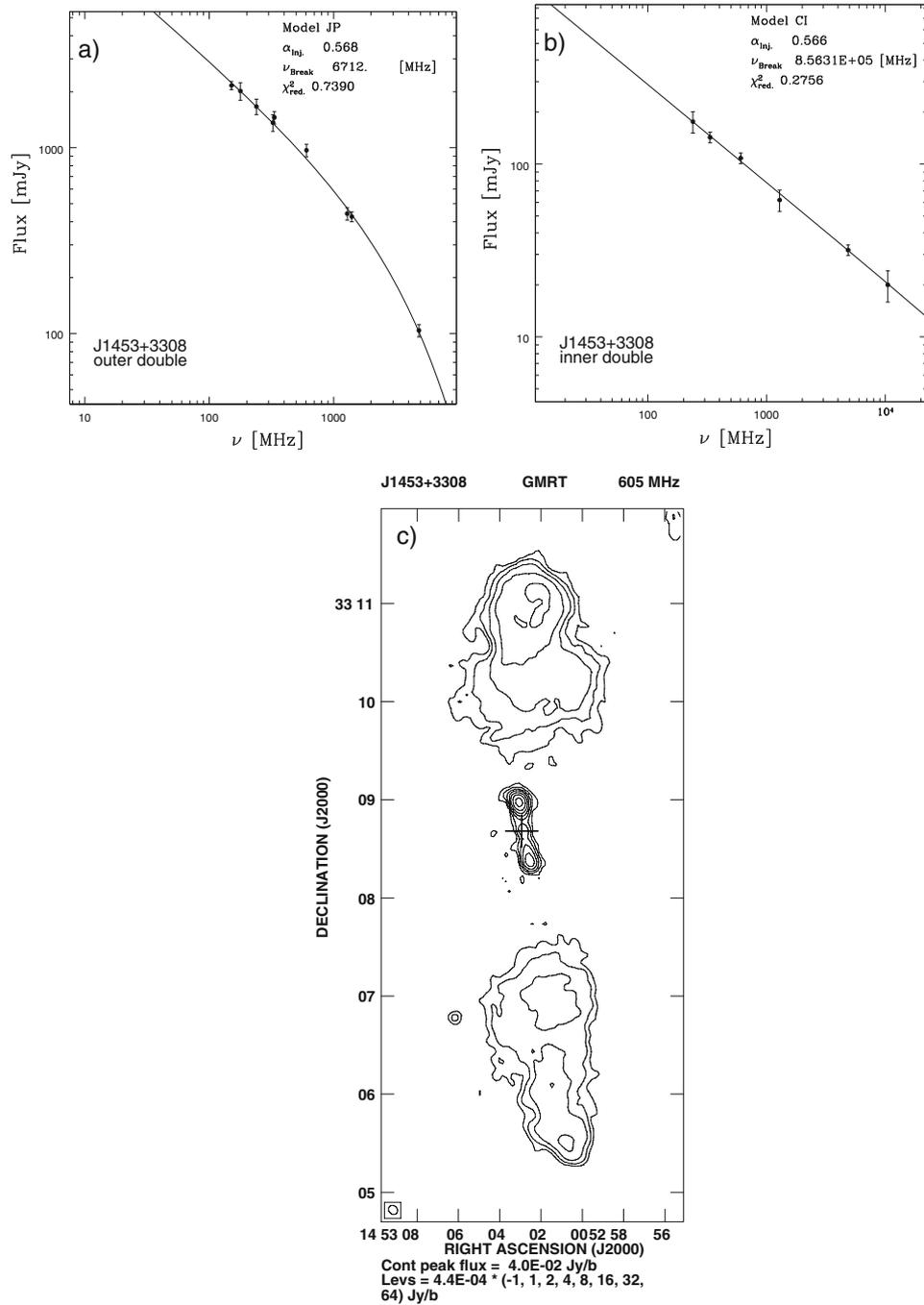


Figure 4. *Left panel:* The total spectrum of the outer lobes of J1453 + 3308. *Right panel:* The total spectrum of the inner double (excluding the core) of the same source. *Bottom panel:* 610-MHz GMRT image of the same source. These are reproduced from the work of Konar et al. (2006).

frequency part of the outer double spectrum) of J1453 + 3308 (Fig. 4). The outer double spectrum (left panel of Fig. 4) of J1453 + 3308, has been fitted with Jaffe-Perola model (Jaffe & Perola 1973). All these results will be discussed in more detail by Konar *et al.* (2012).

4. Summary of the results from the study of the dynamics of radio galaxy and double-double radio galaxies

The results that we have obtained from our study of the dynamics of 3C457 are as follows:

- (1) The magnetic fields of radio lobes are close to the minimum energy values.
- (2) Non-radiating particles in lobes do not seem to be energetically dominant.
- (3) The lobes appear to be approximately at pressure balance towards the edge and underpressured towards the core.
- (4) In our paradigm, the lobe heads can be supersonic, though the lateral expansion is subsonic.
- (5) There can be re-acceleration of particles in the lobes (other than hotspot regions), which hints at some physical process (possibly lobe-environment interaction) inside the lobes.
- (6) The injection spectral index seem to be similar in two episodes of jet activity, which hints that intrinsic source properties should determine the values of α_{inj} .

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