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The Potential Well-Depth $U_{\Sigma}^{(N)}$ Constraints on the Surface Gravitational Red-shift of a Proto Neutron Star

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Abstract. The influence of the potential well depth $U_{\Sigma}^{(N)}$ of Σ in nuclear matter on the surface gravitational red-shift of a proto neutron star is examined within the framework of the relativistic mean field theory for the baryon octet system. It is found that as $U_{\Sigma}^{(N)}$ increases from -35 MeV to $+35$ MeV, the surface gravitational red-shift increases and the influence of the negative $U_{\Sigma}^{(N)}$ on the surface gravitational red-shift is larger than that of the positive ones. Furthermore, the $M_{\max}/R$ and the surface gravitational red-shift corresponding to the maximum mass all increase as the $U_{\Sigma}^{(N)}$ increases, $M_{\max}$ and R being the maximum mass of the proto neutron star and the corresponding radius respectively.

Key words. Surface gravitational red-shift—the relativistic mean field thoery—proto neutron star.

1. Introduction

Proto neutron star (PNS) is formed just after the supernova collapses and its temperature becomes greater than 10 MeV. In the next evolutional stage, it evolves either as a black hole or as a stable neutron star (NS) (Burrows & Lattimer 1986). So the calculation of its maximum mass is very important.

The surface gravitational red-shift of a NS is connected with a mass-to-radius ratio (M/R), Therefore, the maximum mass of the NS can be obtained by defining the surface gravitational red-shift of the NS (Glendenning 1997). In fact, the mass of a NS or PNS is so large that it should be studied by general relativity (Oppenheimer & Volkoff 1939).

The relativistic mean field (RMF) theory is a fairly good method to describe the nuclear matter and the properties of a compact star. However, several sets of hyperon coupling constants need to be defined to study a NS or PNS using the RMF theory (Glendenning 1985). The coupling constants of mesons ω and ρ can be got by the constituent quark model [SU(6) symmetry] (Glendenning 1997; Schaffner & Mishustin 1996). Mesons σ, can be determined by fitting the Λ, Σ, Ξ well-depth in nuclear matter (Schaffner *et al.* 1992). The experimental data of the depth $U_{\Lambda}^{(N)}$ of

Λ in nuclear matter is $U_{\Lambda}^{(N)} = -30$ MeV (Tan Yu-hong *et al.* 2004). For the depth $U_{\Xi}^{(N)}$ of Ξ in nuclear matter, the experimental values indicate a nonrelativistic potential of about -16 MeV (Fukuda *et al.* 1998) and -14 MeV (Khaustov *et al.* 2000) or less, respectively. But Dover & Gal (1983), based on their analysis of emulsion data, found the Ξ-nucleus potential well-depth to be 21−24 MeV. In actual calculation, a more relativistic potential $U_{\Xi}^{(N)} = -28$ MeV is adaptive (Schaffner-Bielich & Gal 2000).

For the potential well depth $U_{\Sigma}^{(N)}$, their values have a considerably uncertain region. In 1989, Dover *et al.* obtained an attractive Σ-nucleus potential of about -27 MeV. By the analysis of Σ nucleus optical potential constructed, Mares *et al.* (1995) got potentials with a repulsive real part in the nuclear interior. A more sophisticated theoretical analysis of these KEK (π^-, K^+) spectra have also confirmed that the Σ-nuclear potential is repulsive within the nuclear volume and they yield a weaker repulsion in the range of 10–40 MeV (Kohno *et al.* 2004, 2006; Harada & Hirabayashi 2005, 2006; Friedman & Gal 2007). However, because the Σ-nuclear potentials are not finally defined by the Σ-atom data in the nuclear interior, they have considerable uncertainty.

In this paper, the effect of $U_{\Sigma}^{(N)}$ on the surface gravitational red-shift of a PNS is examined within the RMF approach considering the baryon octet.

2. The RMF theory and the surface gravitational red-shift of a PNS

The Lagrangian density of hadron matter reads as follows (Glendenning 1987):

$$\begin{aligned}\mathcal{L} = & \sum_B \overline{\Psi}_B \Big[i\gamma_\mu \left(\partial^\mu + i g_{\omega B}\gamma_\mu \omega^\mu + \frac{i}{2} g_{\rho B} \tau \cdot \rho^\mu \right) \\ & - (m_B - g_{\sigma B}\sigma) \Big] \Psi_B + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) \\ & - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\rho_{\mu\nu} \cdot \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu \\ & - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 + \sum_{\lambda = e,\mu} \overline{\Psi}_\lambda (i\gamma_\mu \partial^\mu - m_\lambda)\Psi_\lambda. \end{aligned} \tag{1}$$

The usual RMF approximation is used to solve the field equations. The equations of baryon field are obtained as follows:

$$\left(i\gamma_\mu \kappa^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_0 \omega_0 - \frac{1}{2} g_{\rho B}\gamma_0 \tau_3 \rho_{03} \right) \Psi_B = 0 \tag{2}$$

and their eigenvalues are

$$e_B(\kappa) = g_{\omega B}\omega_0 + g_{\rho B}\rho_{03} I_{3B} + \sqrt{\kappa^2 + m_B^{*2}}. \tag{3}$$

Here,

$$m^* = m_B - g_{\sigma B}\sigma, \tag{4}$$

where m^* is the effective mass of baryons.

The mesons field equations are given by

$$\begin{aligned} m_\sigma^2\sigma &= -g_2\sigma^2 - g_3\sigma^3 + \sum_B \frac{2J_B+1}{2\pi^2} g_{\sigma B} \\ &\quad \times \int_0^\infty \frac{m^*}{\sqrt{k^2+m^{*2}}} (\exp[(\varepsilon_B(k)-\mu_B)/T]+1)^{-1} k^2 \mathrm{d}k, \end{aligned} \tag{5}$$

$$\begin{aligned} m_\omega^2\omega_0 &= \sum_B \frac{2J_B+1}{2\pi^2} g_{\omega B} b_B \\ &\quad \times \int_0^\infty (\exp[(\varepsilon_B(k)-\mu_B)/T]+1)^{-1} k^2 \mathrm{d}k, \end{aligned} \tag{6}$$

$$\begin{aligned} m_\rho^2\rho_{03} &= \sum_B \frac{2J_B+1}{2\pi^2} g_{\rho B} I_{3B} b_B \\ &\quad \times \int_0^\infty (\exp[(\varepsilon_B(k)-\mu_B)/T]+1)^{-1} k^2 \mathrm{d}k. \end{aligned} \tag{7}$$

The energy density and pressure of a PNS are given by

$$\begin{aligned} \varepsilon &= \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ &\quad + \sum_B \frac{2J_B+1}{2\pi^2} \int_0^\infty \kappa^2 \mathrm{d}\kappa \sqrt{\kappa^2+m^{*2}} (\exp[(\varepsilon_B(k)-\mu_B)/T]+1)^{-1} \\ &\quad + \sum_{\lambda=e,\mu} \frac{2J_\lambda+1}{2\pi^2} \int_0^\infty \kappa^2 \mathrm{d}\kappa \sqrt{\kappa^2+m_\lambda^2} (\exp[(\varepsilon_\lambda(k)-\mu_\lambda)/T]+1)^{-1}, \end{aligned} \tag{8}$$

$$\begin{aligned} p &= -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ &\quad + \frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2} \int_0^\infty \frac{\kappa^4}{\sqrt{\kappa^2+m^{*2}}} \mathrm{d}\kappa (\exp[(\varepsilon_B(k)-\mu_B)/T]+1)^{-1} \\ &\quad + \frac{1}{3}\sum_{\lambda=e,\mu} \frac{2J_\lambda+1}{2\pi^2} \int_0^\infty \frac{\kappa^4}{\sqrt{\kappa^2+m_\lambda^2}} \mathrm{d}\kappa (\exp[(\varepsilon_\lambda(k)-\mu_\lambda)/T]+1)^{-1}. \end{aligned} \tag{9}$$

The condition of β-equilibrium is given by

$$\mu_i = b_i\mu_b + q_i\mu_q. \tag{10}$$

The conditions of the conservation of baryon number and the charge neutrality respectively are given by

$$\rho = \sum_B \frac{2J_B+1}{2\pi^2} b_B \int_0^\infty (\exp[(\varepsilon_B(k)-\mu_B)/T]+1)^{-1} k^2 \mathrm{d}k, \tag{11}$$

$$0 = \sum_B \frac{2J_B+1}{2\pi^2} b_B \int_0^\infty (\exp[(\varepsilon_B(k)-\mu_B)/T]+1)^{-1} k^2 \mathrm{d}k + \sum_{\lambda=e,\mu} \frac{2J_\lambda+1}{2\pi^2} q_\lambda \int_0^\infty (\exp[(\varepsilon_\lambda(k)-\mu_\lambda)/T]+1)^{-1} k^2 \mathrm{d}k. \tag{12}$$

We use the Oppenheimer–Volkoff equation (O–V equation) to obtain the mass and the radius of a PNS (Oppenheimer & Volkoff 1939)

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{(p+\varepsilon)(M+4\pi r^3 p)}{r(r-2M)}, \tag{13}$$

$$M = 4\pi \int_0^r \varepsilon r^2 \mathrm{d}r. \tag{14}$$

The gravitational red-shift of a PNS is given by (Glendenning 1997)

$$z = \left(1 - \frac{2M}{R}\right)^{-1/2} - 1. \tag{15}$$

3. Parameters

The coupling constants of the nucleons are chosen as GL97 sets (Glendenning 1997) as seen in Table 1 and the temperature of the PNS is $T = 15$ MeV in our calculations.

We define the ratios: $x_{\sigma h} = \frac{g_{\sigma h}}{g_{\sigma N}}$, $x_{\omega h} = \frac{g_{\omega h}}{g_{\omega N}}$, $x_{\rho h} = \frac{g_{\rho h}}{g_{\rho N}}$, where h denotes hyperons. For the coupling constants of mesons ω and ρ, we obtain ratios

$$g_{\omega N}/3 = g_{\omega \Sigma}/2 = g_{\omega \Lambda}/2 = g_{\omega \Xi}, \tag{16}$$

$$g_{\rho N} = g_{\rho \Sigma}, g_{\rho N} = g_{\rho \Xi}, g_{\rho \Lambda} = 0, \tag{17}$$

Table 1. The coupling constants of the nucleons.

m	939	g_3	−9.835
m_σ	500	C_3	0
m_ω	782	ρ_0	0.153
m_ρ	770	B/A	16.3
g_σ	7.9835	K	240
g_ω	8.7	a_{sym}	32.5
g_ρ	8.5411	m^*/m	0.78
g_2	20.966		

by the constituent quark model [SU(6)] symmetry (Glendenning 1997; Schaffner & Mishustin 1996). The couplings constants of mesons σ are then determined by fitting the Λ, Σ and Ξ well-depth in the nuclear matter as follows (Glendenning 1997):

$$U_h^{(N)} = m_B \left(\frac{m_n^*}{m_n} - 1\right)\left(\frac{g_{\sigma h}}{g_{\sigma N}}\right) + \left(\frac{g_{\omega N}}{m_\omega}\right)^2 \rho_0 \left(\frac{g_{\omega h}}{g_{\omega N}}\right). \tag{18}$$

In our calculation, we choose $U_\Lambda^{(N)} = -30$ MeV, $U_\Xi^{(N)} = -28$ MeV and $U_\Sigma^{(N)} = -25, -15, -5, 5, 15, 25$ MeV, respectively.

4. Theoretical results and analysis

The surface gravitational red-shift as a function of central energy density is shown in Fig. 1, where the central energy density is in units of the density of ordinary nuclear matter. Figure 1 shows that the surface gravitational red-shift increases as the central energy density increases. The mass of a PNS as a function of the central energy density is given in Fig. 2. Part AB of these curves correspond to stable stars because of $\mathrm{d}M/\mathrm{d}\varepsilon_c > 0$, while part BC corresponds to unstable stars because of $\mathrm{d}M/\mathrm{d}\varepsilon_c < 0$. As $U_\Sigma^{(N)}$ increases from -25 MeV to -5 MeV, the surface gravitational red-shift increases. But as the $U_\Sigma^{(N)}$ increases from $+5$ MeV to $+25$ MeV, the effect of $U_\Sigma^{(N)}$ on the surface gravitational red-shift is not very obvious.

Figure 3 shows the surface gravitational red-shift as a function of mass. For the section AB corresponding to the stable PNS, the surface gravitational red-shift

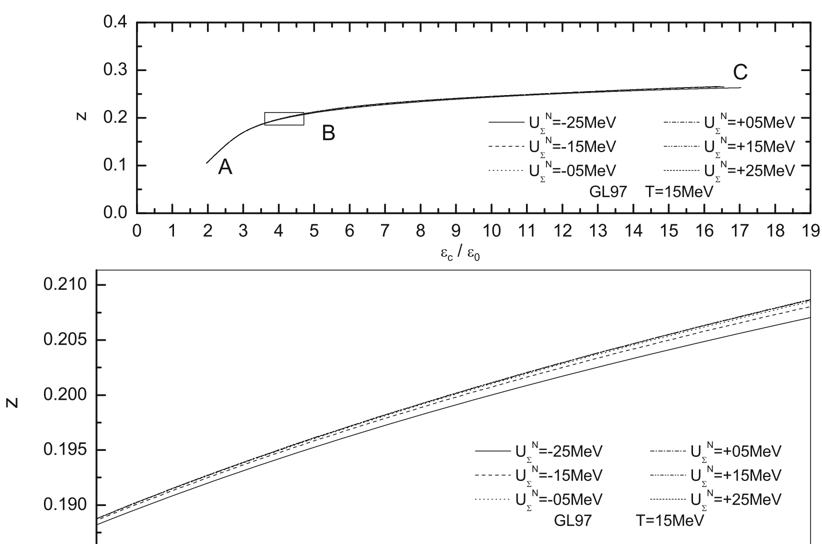

Figure 1. The surface gravitational red-shift as a function of central energy density.

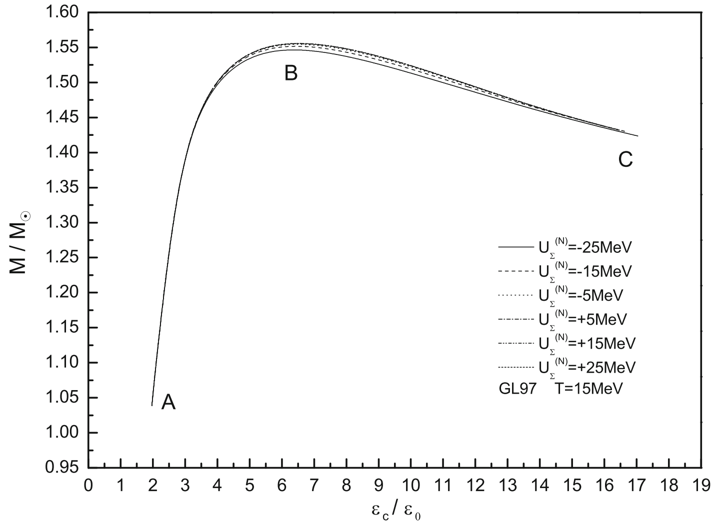

Figure 2. The mass of a PNS as a function of the central energy density.

increases as the central energy density increase. Considering the influence of the $U_{\Sigma}^{(N)}$, it will decrease as $U_{\Sigma}^{(N)}$ increases from -25 MeV to -5 MeV. For part BC corresponding to the unstable PNS, the surface gravitational red-shift decreases as the central energy density increase. When $U_{\Sigma}^{(N)}$ increases from -25 MeV to -5 MeV, it will increase. For the above two cases, the effect of the positive $U_{\Sigma}^{(N)}$ on the surface gravitational red-shift is not very obvious.

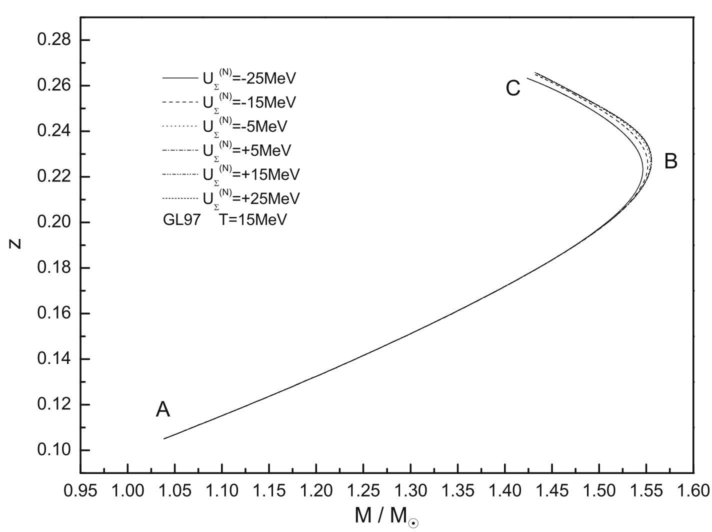

Figure 3. The surface gravitational red-shift as a function of the mass.

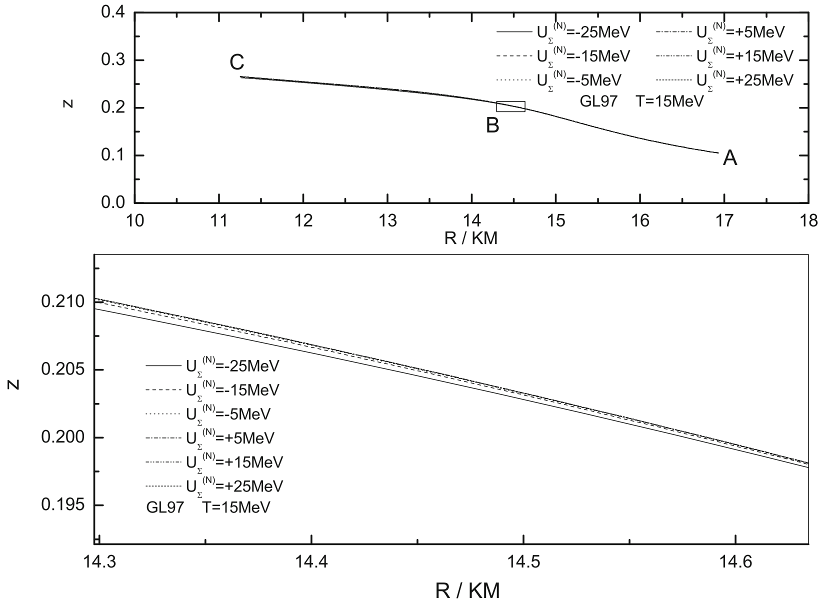

Figure 4. The surface gravitational red-shift as a function of the radius.

The surface gravitational red-shift as a function of the radius is given in Fig. 4, from which it can be seen that the surface gravitational red-shift decreases as the radius increases. Considering the effect of the variety of the $U_{\Sigma}^{(N)}$ on the surface gravitational red-shift, it is found that the surface gravitational red-shift increases as

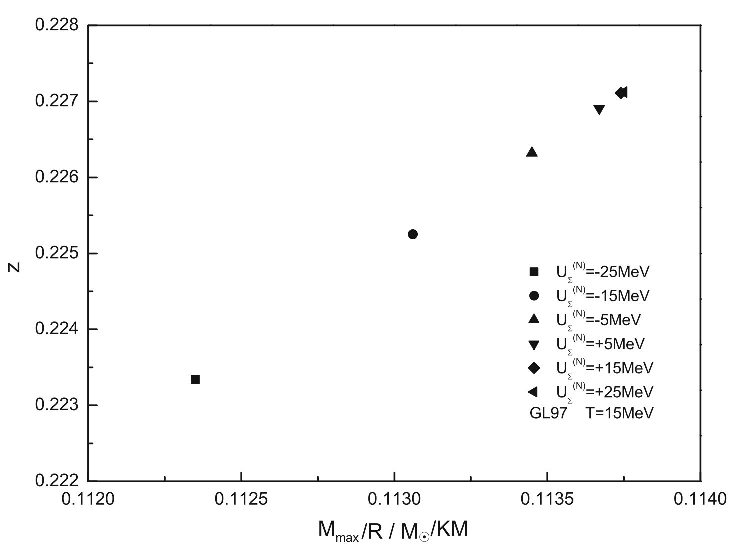

Figure 5. The surface gravitational red-shift as a function of $M_{\max}/R$.

Table 2. The surface gravitational red-shift corresponding to the maximum mass of a PNS.

$U_{\Sigma}^{(N)}$ (MeV)	$M_{\max}$ ($M_{\odot}$)	R (km)	z	Increasing rate (per cent)
−25	1.54643	13.76400	0.22334	
−15	1.55163	13.72400	0.22525	0.855
−5	1.55430	13.70000	0.22632	0.475
+5	1.55534	13.68300	0.22691	0.261
+15	1.55563	13.67700	0.22711	0.088
+25	1.55569	13.67700	0.22712	0.004

$U_{\Sigma}^{(N)}$ increases from −25 MeV to −5 MeV but the influence is not obvious as $U_{\Sigma}^{(N)}$ increases from +5 MeV to +25 MeV.

Figure 5 and Table 2 show the surface gravitational red-shift as a function of $M_{\max}/R$, with $M_{\max}$ being the maximum mass of the PNS and R the corresponding radius. It is found that, as $U_{\Sigma}^{(N)}$ are −25,−15,−5,+5 ,+15 and +25 MeV, the $M_{\max}/R$ increases and the surface gravitational red-shift corresponding to the maximum mass respectively are 0.22334, 0.22525, 0.22632, 0.22691, 0.22711 and 0.22712, i.e. the increasing rate respectively being 0.855, 0.475, 0.261, 0.088 and 0.004 per cent. It is quite evident that the influence of the negative $U_{\Sigma}^{(N)}$ on the surface gravitational red-shift is more than that of the positive ones.

5. Summary

In conclusion, in this paper the influence of the depth $U_{\Sigma}^{(N)}$ of Σ in nuclear matter on the surface gravitational red-shift of a proto neutron star is examined within the framework of the relativistic mean field theory for the baryon octet system. It is found that as $U_{\Sigma}^{(N)}$ increases from −35 MeV to +35 MeV, the surface gravitational red-shift increases and the influence of the negative $U_{\Sigma}^{(N)}$ on the surface gravitational red-shift, being close to 1 per cent, is larger than that of the positive ones. Furthermore, as $U_{\Sigma}^{(N)}$ increases the $M_{\max}/R$ and the surface gravitational red-shift corresponding to the maximum mass also increase.

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