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The Work Function Associated with Ultra-relativistic Electron Emission from Strongly Magnetized Neutron Star Surface

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Abstract. Following an extremely interesting idea (Schieber 1984), published long ago, the work function associated with the emission of ultra-relativistic electrons from magnetically deformed metallic crystal (mainly iron) at the outer crust of a magnetar is obtained using relativistic version of Thomas–Fermi type model for electron distribution around the nuclei in this region. In the present scenario, surprisingly, the work function becomes anisotropic; the longitudinal part is an increasing function of magnetic field strength, whereas the transverse part diverges.

Key words. Work function—magnetar—Landau levels—cold emission—pulsar emission.

1. Introduction

In condensed matter physics the work function is the minimum energy needed to remove an electron from a solid to a point immediately outside the solid surface or equivalently, the energy needed to move an electron from the Fermi level into vacuum. Here the word ‘immediately’ means that the final electron position is far from the surface on the atomic scale but still close to the solid on the macroscopic scale. It is also known that the work function is a characteristic property for any solid face of a substance with a conduction band, which may be empty or partly filled. For a metal, the Fermi level is inside the conduction band, indicating that the band is partly filled. For an insulator, however, the Fermi level lies within the band gap, indicating an empty conduction band; in this case, the minimum energy to remove an electron is about the sum of half the band gap and the work function.

In the free electron model the valence electrons move freely inside the metal but find a confining potential step, say C at the boundary of the metal. In the system’s ground state, energy levels below the Fermi energy are occupied, and those above the Fermi level are empty. The energy required to liberate an electron in the Fermi level is the work function and is given by $W_f = C - \mu_e$, where μ_e is the Fermi energy. Therefore, the work function of a metal is usually defined as the smallest energy needed to extract an electron at zero temperature. Formally, this definition is made for an infinitely large crystal plane in which one takes an electron from infinitely deep inside the crystal and brings it through the surface, infinitely far away into the vacuum. The exact definition of work function, which is valid in the atomic scale is the work done in bringing an electron from far below the surface, compared to atomic dimensions but not far compared to crystal dimensions.

The work function of metals varies from one crystal plane to another and also varies slightly with temperature. For a metal, the work function has a simple interpretation. At absolute zero, the energy of the most energetic electrons in a metal is referred to as the Fermi energy; the work function of a metal is then equal to the energy required to raise an electron with the Fermi energy to the energy level corresponding to an electron at rest in vacuum. The work function of a semiconductor or an insulator, however, has the same interpretation, but in these materials the Fermi level is in general not occupied by electrons and thus has a more abstract meaning (Sommerfeld 1927, 1928; Frenkel 1928; Tamm & Blochinzev 1932; Slater & Krutter 1935; Wigner & Bardeen 1935; Bardeen 1936 are some of the fundamental papers on work function).

The work function is associated with three types of electron emission processes: photo-emission, thermionic emission and field emission. All of them have a large number of applications in various branches of science and technology; starting from electronic valves – a very old type of electronic device, to modern opto-electronic devices used to convert optical signals to electrical signals etc. The other important applications are in photo-multiplier tube (PMT), CCD, etc. In recent years, it is found that the field emission process has a lot of important applications in modern nanotechnology.

Similar to the down-to-earth application of work function in condensed matter physics and its various scientific and technological applications, the work function also plays a vital role in many astrophysical processes, e.g., in the formation of magneto-sphere by the emission of electrons from the polar region induced by strong electrostatic field in strongly magnetized neutron stars or magnetars. The present investigation is mainly associated with the emission of high energy electrons from the dense metallic iron crystals present at the crustal region of strongly magnetized neutron stars. The study of the formation of plasma in a pulsar magnetosphere is quite old but is still an unresolved astrophysical issue. In the formation of magnetospheric plasma, it is generally assumed that there must be an initial high energy electron flux from the magnetized neutron stars. At the poles of a neutron star the emitted charged particles flow only along the magnetic field lines. Further, a rotating magnetized neutron star generates extremely high electro-static potential difference at the poles. The flow of high energy electrons along the direction of magnetic field lines and their penetration through the light cylinder is pictured as the current carrying conductors. Naturally, if the conductor is broken near the pulsar surface the entire potential difference will be developed across a thin gap, called the polar gap. This is of course based on the assumption that above a critical height from the polar gap, because of high electrical conductivity of the plasma, the electric field $E_{||}$, parallel to the magnetic field near the poles is quenched. Further, a steady acceleration of electrons originating at the polar region of neutron stars, travelling along the field lines, will produce magnetically convertible curvature γ-rays. If these curvature γ-ray photons have energies $> 2m_ec^2$ (with m_e the electron rest mass and c the velocity of light), then pairs of $e^- - e^+$ will be produced in enormous amount with very high efficiency near the polar gap. These produced $e^- - e^+$ pairs form what is known as the magneto-spheric plasma (Jessner *et al.* 2001; Ruderman & Sutherland 1975; Diver *et al.* 2009; Shapiro & Teukolsky 1983; Michel 1982, 2004; Harding & Lai 2006; Ruderman 1971). The emission of electrons from the polar region of neutron stars is mainly dominated by the cold emission or the field emission (Fowler & Nordheim 1928), driven by electro-static force at the poles, produced by the strong magnetic field of rotating neutron stars. The work function plays a major role in the emission processes. Moreover, it is interesting to note that electrons are only emitted from the polar region of magnetized neutron stars, but never from some region far from the poles. A large number of theoretical and numerical techniques were developed to obtain analytical expression and also the numerical values of work functions of various materials for which the experimental values are also known. Self-consistent jellium-background model, embedding atom-jellium model, density functional theory (DFT) etc., were used to obtain numerical values for work functions (Ekardt 1984; Wang & Cheng 2007; Mamonova & Prudnikov 1998). In a recent article the work function has been identified with the exchange energy of electrons within the material (Mastwijk *et al.* 2007; Liu *et al.* 2008; Smoluchowski 1941).

The process of extracting electrons from the outer crust region of strongly magnetized neutron stars, including the most exotic stellar objects, the magnetars, requires a more or less exact description of the structure of matter in that region. From the knowledge of structural deformation of atoms in strong magnetic field, we expect that the departure from spherical nature to a cigar shape allows us to assume that because of high density the electron distribution around the iron nuclei at the outer crust region may be replaced by Wigner-Seitz type cells of approximately cylindrical in structure (Lieb *et al.* 1992; Canuto & Ventura 1977; Nag *et al.* 2009; Nag & Chakrabarty 2010). We further assume that the electron gas inside the cells are strongly degenerate and at zero temperature (in the presence of strong quantizing magnetic field the atoms get deformed from their usual spherical shape and also get contracted, then it is quite natural that the electron density becomes so high inside the cells that their chemical potential $\mu_e \gg T$, the temperature of the degenerate electron gas. Then the temperature of the system may be assumed to be very close to zero). It is well-known that the presence of extraordinarily large magnetic field not only distorts the crystalline structure of the dense metallic iron, but also significantly modifies the electrical properties of such matter. As for example, the electrical conductivity, which is otherwise isotropic, becomes highly anisotropic in the presence of strong quantizing magnetic field (Potekhin *et al.* 1999; Potekhin 1996; Potekhin & Yakovlev 1997). In the presence of strong magnetic field, iron crystal is highly conducting in the direction parallel to the magnetic field, whereas flow of current in the perpendicular direction is severely inhibited.

The aim of this article (i) is to show that the work function associated with the emission of electrons along the direction of magnetic field at the polar region of strongly magnetized neutron stars increases with the strength of the magnetic field and (ii) it will be shown that the same quantity associated with the emission of electrons in the direction transverse to the magnetic field direction is infinitely large. Therefore the main result of this article is to show that in the presence of a strong quantizing magnetic field, the work function becomes anisotropic (see also Liu *et al.* 2008; Smoluchowski 1941; Medin & Lai 2006, 2007). We have noticed that the result does not depend on the dimension of the crustal matter. The scenario is very much analogous to the charge transport mechanism within the metal in the presence of strong magnetic field. The electron emission process may be assumed to be a kind of charge transport process from inside the metal to outside. We shall show in this article that analogous to the internal charge transport, this is also anisotropic in the presence of strong magnetic field and if the magnetic field strength is high enough, the emission process in the transverse direction is totally forbidden. To the best of our knowledge, using this simple idea for cold cathode emission (Schieber 1984), the study of anisotropic nature of work function in the presence of strong quantizing magnetic field, relevant for strongly magnetized neutron star crustal region, has not been studied earlier. Of course, a completely different approach, popularly known as density functional theory (DFT) has been used by Medin & Lai (2006, 2007) and Harding & Lai (2006) to study properties of dense neutron star crustal matter. They have also obtained the work function as a function of the magnetic field (Medin & Lai 2006, 2007; Harding & Lai 2006; Lang & Kohn 1970, 1971). Surprisingly, our simple model gives almost the same kind of magnetic field dependence for the work function as obtained by Medin & Lai (2006, 2007) and Harding & Lai (2006).

We have organized this article as follows: In section 2, we obtain an approximate analytical expression for work function associated with the emission of electrons along the field direction. In section 3, we obtain the same quantity corresponding to the emission transverse to the direction of the magnetic field, and show that this particular component is infinitely large. Whereas in the last section we give conclusions and future prospects of this work.

2. Longitudinal emissions

In this section, for the sake of completeness, we shall repeat some of the derivations presented in Schieber (1984). Based on this particular work we shall develop a formalism to obtain the work function associated with the emission of electrons along the direction of the magnetic field from dense crystalline structure of mainly metallic iron in the outer crust region of neutron stars. It should be noted that the physical picture of our model is completely different from Schieber (1984). In our model calculation the arrangement of cylindrically deformed WS cells in the crustal region are such that the field lines and the axes of the cylinders are parallel to each other. To obtain the work function for both axial and transverse emission processes, let us consider Fig. 1, where we have considered a cylindrically deformed WS cell of the crustal matter. Here the magnetic field is along the z-axis and the r-axis is orthogonal to the direction of magnetic field lines, and we assume azimuthal symmetric geometrical structure. To get the longitudinal part of the work function, we assume that an electron has come out from the cylindrically deformed cell through one of the plane faces, and is at the point P. The electrostatic potential at Q produced by this electron is given by

$$\phi^{(p)}(r, z) = \frac{e}{[r^2 + (z-h)^2]^{1/2}}, \tag{1}$$

where $AP = h$, $PD = r$ and $CQ = z$. This equation can also be expressed in the following form:

$$\phi^{(p)}(r, z) = e \int_{\xi=0}^{\infty} J_0(\xi r) \quad \exp[\mp(z-h)\xi] d\xi, \qquad \text{for } z > \text{ or } < h. \tag{2}$$

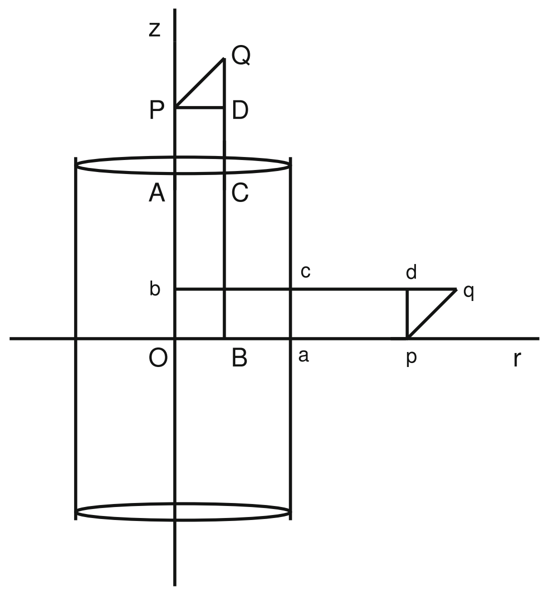

Figure 1. Schematic diagram for electron emission along and orthogonal to the direction of the magnetic field.

For charge distribution within the distorted WS cell, the Poisson's equation is given by

$$\nabla^2\phi = 4\pi e n_{\mathrm{e}}, \tag{3}$$

where ϕ is the electrostatic potential and n_{e} is the electron density, given by

$$n_{\mathrm{e}} = \frac{eB}{2\pi^2}\sum_{\nu=0}^{\nu_{\max}}(2-\delta_{\nu 0})\left(\mu^2 - m^2 - 2\nu eB\right)^{1/2}, \tag{4}$$

where ν is the Landau quantum number with $\nu_{\max}$ as the upper limit (it is finite for $T = 0$). The factor $2 - \delta_{\nu 0}$ within the sum takes care of singly degenerate zeroth Landau level and doubly degenerate other levels with $\nu \neq 0$ and μ is the chemical potential for the electrons, given by $\mu = E_{\mathrm{F}} - e\phi(r) =$ constant, this is the so called Thomas–Fermi condition, E_{F} is the Fermi energy for the electrons, given by $E_{\mathrm{F}} = (p_{\mathrm{F}}^2 + m_{\mathrm{e}}^2)^{1/2}$ (throughout this article we have chosen $\hbar = c = 1$). Hence we can re-write the Poisson's equation in the following form:

$$\nabla^2\phi = \frac{2e^2B}{\pi}\sum_{\nu=0}^{\nu\max}(2-\delta_{\nu 0})\left[(\mu + e\phi)^2 - m_{\mathrm{e}}^2 - 2\nu eB\right]^{1/2}. \tag{5}$$

To get an analytical solution for ϕ in the cylindrical coordinate system (r, θ, z), with azimuthal symmetry, we put, $m_{\mathrm{e}} = 0$, i.e., the kinetic energy is assumed to be high enough compared to the electron rest mass and $\nu_{\max} = 0$. The latter is a valid approximation, provided the magnetic field is too high ($\geq 10^{14}$ G), so that the electrons occupy only their zeroth Landau level. We expect, that the above assumptions are not so drastic for the type of physical system considered here, e.g., for the matter at the outer crust region of a magnetar. Under such approximation, the Poisson's equation reduces to

$$\nabla^2\phi \approx \frac{2e^2 B}{\pi}(\mu + e\phi). \tag{6}$$

Now defining $\psi = \mu + e\phi$ the Poisson's equation becomes

$$\nabla^2\psi \approx \frac{2e^3 B}{\pi}\psi = \lambda^2\psi. \tag{7}$$

In cylindrical coordinate (r, θ, z), the above equation can be written as

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial^2 z} = \lambda^2\psi. \tag{8}$$

The solution in the cylindrical coordinate system is given by

$$\psi(r, z) = \int_{\xi=0}^{\infty} J_0(\xi r)a(\xi)\exp\left[\left(\xi^2 + \lambda^2\right)^{1/2} z\right]\mathrm{d}\xi \quad \text{with } z < 0. \tag{9}$$

Here $\lambda^2 = 2e^3 B/\pi$ (in this connection, one should note that the parameter λ used in Schieber (1984) has a completely different physical meaning) and $a(\xi)$ is some unknown spectral function. Then following Schieber (1984), we assume that there exists a fictitious secondary field in vacuum. The work function is defined as the work done by this field in pulling out an electron from just inside the material surface to infinity. The secondary field, as introduced in Schieber (1984), is expressed in coherence with $\phi^{(\mathrm{p})}(r, z)$ and $\phi(r, z)$, and is given by

$$\phi^{(\mathrm{s})}(r, z) = e\int_{\xi=0}^{\infty} J_0(\xi r) f(\xi)\exp(-\xi z)\mathrm{d}\xi \quad \text{with } z > 0, \tag{10}$$

where $f(\xi)$ is again some unknown spectral function. To obtain $f(\xi)$, we follow Schieber (1984) and use the continuity conditions for tangential and transverse components of electric field and the displacement vector respectively, given by

$$E^{(\mathrm{t})} = E^{(\mathrm{p})\mathrm{t}} + E^{(\mathrm{s})\mathrm{t}} \quad \text{and} \quad D^{\perp} = D^{(\mathrm{p})\perp} + D^{(\mathrm{s})\perp} \tag{11}$$

where for the case of electron emission along the magnetic field direction, the tangential part of the electric field components are given by r derivatives of the corresponding potentials with a negative sign as the multiplicative factor, e.g., $E^{(\mathrm{t})} = -\partial\phi/\partial r$ and similarly for others. The axial part of the components for displacement vectors are obtained by taking z derivatives of the corresponding potentials multiplied by -1 and the dielectric constant of the medium, which are assumed to be unity, i.e., $D^{\perp} = -K\partial\phi/\partial z = -\partial\phi/\partial z = E^{\perp}$. Using the z derivatives for exponential functions and the relation $\partial J_0(\xi r)/\partial r = \xi J_0'(\xi r) = -\xi J_1(\xi r)$ for the r derivatives for Bessel functions, we finally have as follows (Schieber 1984):

$$f(\xi) = \left[\frac{1 - \left(1 + \frac{\lambda^2}{\xi^2}\right)^{1/2}}{1 + \left(1 + \frac{\lambda^2}{\xi^2}\right)^{1/2}}\right]\exp(-\xi h). \tag{12}$$

Then we have the secondary potential

$$\phi^{(s)}(r,z) = e\int_{\xi=0}^{\infty} J_0(\xi r)\left[\frac{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}{1+\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}\right]\exp[-\xi(h+z)]\mathrm{d}\xi \quad \text{for } z>0. \tag{13}$$

Hence the secondary field along the axial direction at $r = 0$ is given by

$$E_z^{(s)} = -\frac{\partial\phi^{(s)}}{\partial z} \tag{14}$$

$$= e\int_{\xi=0}^{\infty}\left[\frac{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}{1+\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}\right]\exp[-\xi(h+z)]\xi\mathrm{d}\xi. \tag{15}$$

Then the force acting on an electron at $z = h$, which is at the verge of emission, is given by

$$F_z^{(s)} = eE_z^{(s)} = e^2\int_{\xi=0}^{\infty}\left[\frac{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}{1+\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}\right]\exp(-2h\xi)\xi\mathrm{d}\xi. \tag{16}$$

The work done in pulling out an electron from just inside the metal surface to infinity is then given by

$$W_f = -\int_0^{\infty} F_z^{(s)}\mathrm{d}h \tag{17}$$

$$= -\frac{e^2}{2}\int_{\xi=0}^{\infty}\frac{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}{1+\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}\mathrm{d}\xi. \tag{18}$$

Substituting $\xi = \lambda \sinh u$, we have

$$W_f = \frac{e^2}{2}\lambda\int_0^{\infty}\exp(-2u)\cosh u\ \mathrm{d}u = \frac{\lambda}{3}e^2 = \frac{1}{3}\left(\frac{2e^3B}{\pi}\right)^{1/2}e^2. \tag{19}$$

Expressing the magnetic field in terms of the critical field strength, we get

$$W_f = \frac{\lambda}{3}e^2 = \frac{1}{3}\left(\frac{2B}{\pi B_c^{(e)}}\right)^{1/2} m_e e^3 \tag{20}$$

where $B_c^{(e)} \approx 4.43\times10^{13}$ G, the typical field strength for electrons to populate their Landau levels in the relativistic scenario. This equation gives the variation of work function with the strength of the magnetic field ($\sim B^{1/2}$) associated with the emission of electrons along the direction of the magnetic field. From this expression it is also obvious that for a given magnetic field strength, if m_e is replaced by m_p, the proton mass or m_I, mass of the ions, then, since $m_I \gg m_p \gg m_e$, the work functions associated with their emissions along the z-direction can also be expressed by the same kind of inequalities. Then it is quite obvious that a high temperature is needed for thermo-ionic emission of protons or ions, high electric field for their field emissions and high frequency incident photons are essential for the photo-emission of these heavier components. Since the work function $\propto B^{1/2}$, the emission of electrons should decrease with the increase in the magnetic field strength. Physically, it means that the electrons become more strongly bound within the metals. In Fig. 2 we have shown the variation of W_f, the work function associated with the emission of electrons along the magnetic field direction, with B, the strength of the magnetic field. This graph clearly shows that the work function is an increasing function of the magnetic field strength and for low field values, W_f is a few eV in magnitude, which is of the same order of magnitude with the experimentally known values for laboratory metals. Since in this approximate calculation, the magnetic field is assumed to be extremely high, we are therefore unable to extrapolate this model to very low field values. Also, in this model we can not show the variation of W_f from one kind of metal to another. However, a comparison of our result with Fig. 4 of Medin & Lai (2007) shows that in the DFT calculation also the variation of work function from one kind of metal to another one is not so significant. Further, our results are consistent with the DFT calculation. In this context we would like to mention that the field emission from the metal surface may be assumed to be a kind of charge transport from the matter into the surrounding vacuum. The emission is caused by the presence of a strong electric field at the surface. Now in the presence of a strong magnetic field $> B_c$, the binding of the electrons within the material become strong and increases with the strength of magnetic field, which actually means, the Fermi energy of the electron gas gets lowered in the presence of a strong magnetic field, i.e., the system becomes more stable. As a result the emission of electrons will get suppressed in the presence of a strong magnetic field. The decrease in the emission current in the presence of strong magnetic field, in this particular physical problem

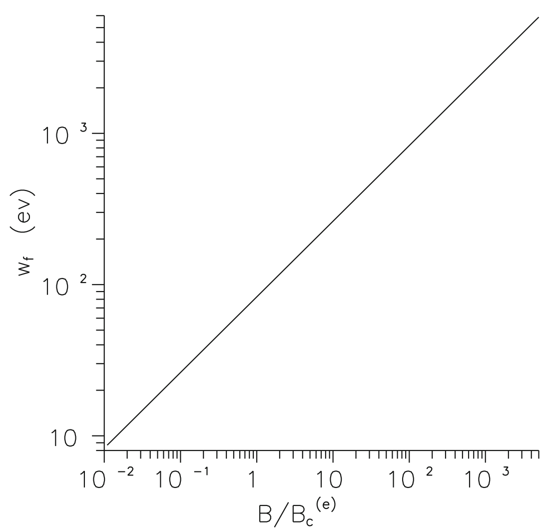

Figure 2. Variation of work function (W_f) with the magnetic field strength.

is controlled by the increase in the work function of the material with the magnetic field strength as given in this equation.

3. Transverse emissions

Let us now consider the emission of electrons in the transverse direction. As shown in Fig. 1, p the position of an electron, came out through the curved surface. Then following the analytical derivation as given in Appendix, we finally we get

$$W_f = -e^2 \int_{\xi=0}^{\infty} \frac{\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2} - \left[\frac{J_1(R\xi)}{J_0(R\xi)}\right]}{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}} \mathrm{d}\xi. \tag{21}$$

To evaluate W_f, let us consider part of this integral, given by

$$I = -\int_{0}^{\infty} \frac{\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}} \mathrm{d}\xi \tag{22}$$

which is the first part of the work function integral (eq. (A23)). To evaluate integral I, let us put $\xi = \lambda \sinh\theta$ and then it is trivial to show that

$$I = \frac{\lambda}{4} \int_{0}^{\infty} \frac{[\exp(\theta) + \exp(-\theta)]^2}{\exp(-\theta)} \mathrm{d}\theta. \tag{23}$$

Obviously, the numerical value of this integral is infinity.

The other part of the work function integral (eq. (A23)) is also diverging in nature, but these two divergences will not cancel each other. It is also quite obvious that in the limiting cases, $R \longrightarrow \infty$, or $R \longrightarrow 0$, the diverging nature of work function for both the cases indicate that it is independent of the transverse dimension of the cylindrical cells. It is obvious that for $R \longrightarrow 0$, the second term on the numerator of eq. (A23) vanishes, whereas the rest can be integrated analytically and found to be diverging in the upper limit. On the other hand, for $R \longrightarrow \infty$, the second term becomes $\sim \tan(\xi R - \pi/4)$ (asymptotically), which itself also diverges for certain values of the argument, but again does not cancel with the infinity from the other part of this integral. Therefore in this limit also the work function becomes infinitely large. Such diverging nature of work function is associated with the electron emission along the transverse direction to the external magnetic field. Although we have considered here the extreme case of ultra-strong magnetic field, we do expect that even for low and moderate field values, the work function corresponding to the electron emission in the transverse direction will be several orders of magnitudes larger than the corresponding longitudinal values (see also Liu *et al.* 2008; Smoluchowski 1941). To elaborate this point a bit, we have noticed that in the extreme case, when the magnetic field is strong enough, so that all the electrons occupy only their zeroth Landau level, the work function associated with the emission of electrons in the transverse direction to the external magnetic field diverges. In this extreme situation, the transverse part of electron momentum $p_\perp = (2\nu eB)^{1/2} = 0$. However, in the non-extreme case, with moderate values for magnetic field strength, a large number of Landau levels for the electrons will be populated. In this case the transverse part of the electron momentum will be non-zero. Because of such finite values, we expect a large but not diverging value for the work function associated with the emission of electrons in the direction orthogonal to the direction of magnetic field, i.e., a very weak type electron emission current will be there in the transverse direction. However, in this case one has to solve the Thomas–Fermi equation (eq. (5)) numerically. Further, this value of the work function will no doubt be large compared to the isotropic case, when the magnetic field is $< B_c$, with no quantized Landau levels for the electrons.

4. Conclusion

In conclusion, we mention that the main purpose of this article is to show that (i) in the presence of a strong quantizing magnetic field the work function becomes anisotropic, (ii) the transverse part is infinitely large, (iii) the longitudinal part is finite but increases with the strength of the magnetic field and finally, (iv) low field values of work functions are more or less consistent with the tabulated values. In the present work, with such a simple model, the anisotropic nature of work function in the presence of strong quantizing magnetic field is predicted to the best of our knowledge for the first time. Further, the diverging character of work function associated with the electron emission in the transverse direction is also obtained for the first time, and as far as our knowledge is concerned, it has not been reported in any published work. We have also noticed that in the low magnetic field limit (within the limitation of this model) the numerical values of work function are of the same order of magnitude with the known laboratory data. However, we are not able to compare our results with the variation from one metal to another. The anisotropic nature of the work function is apparently coming from the deformed cylindrical nature of electron distribution around the iron nuclei caused by strong magnetic field at the outer crust region of a strongly magnetized neutron star. Now because of Ohmic decay, when the field strength becomes low enough ($< B_c^{(e)}$), we do expect that the spherically symmetric nature of the electron distribution around the nuclei will be restored. As a consequence, the electron emission processes will no longer be affected by the strong magnetic field through the work function of the metallic outer crust. If the diverging nature of the work function associated with the emission of electrons through the curved faces of the so called metallic atoms (they are in reality the electron distribution around the iron nuclei), is solely because of cylindrical deformation, which in the present model is caused by the presence of ultra-strong magnetic field, we therefore expect that for the emission of electrons through the curved surface of any object having cylindrical structure, the transverse component of work function will be much larger compared to the longitudinal part. As a consequence the electron emission current along the axial direction will be large enough compared to the transverse emission current (Liu *et al.* 2008). The work function associated with the emission of electrons along the direction of the magnetic field may be used to obtain electron field emission current at the polar region of strongly magnetized neutron stars, which will help us to understand the formation of magneto-spheric plasma.

Appendix

The electrostatic potential at q in Fig. 1 is given by

$$\phi^{(p)}(r,z) = \frac{e}{[z^2+(r-r_0)^2]^{1/2}}, \tag{A1}$$

where $ap = r_0$, $cq = r$ and $pd = z$. It can also be expressed as

$$\phi^{(p)}(r,z) = e\int_{\xi=0}^{\infty} J_0[\xi(r-r_0)]\exp(\mp z\xi)\mathrm{d}\xi \quad \text{for } z>0 \text{ or } z<0 \tag{A2}$$

whereas the form of $\phi(r,z)$ (or the modified form $\psi(r,z)$) and $\phi^{(s)}(r,z)$ remain unchanged (eqs (6)–(7)). Using the relation

$$J_0(u-v) = \sum_{k=-\infty}^{+\infty} J_k(u)J_k(v), \tag{A3}$$

we can express

$$J_0(\xi r) = J_0[\xi(r+r_0-r_0)] = J_0[\xi(r+r_0)-\xi r_0] = \sum_{k=-\infty}^{+\infty} J_k[\xi(r+r_0)]\,J_k(\xi r_0). \tag{A4}$$

Then we have

$$\psi(r,z) = \sum_{k=-\infty}^{+\infty} J_k(\xi r_0)\int_{\xi=0}^{\infty} J_k[\xi(r+r_0)]\,a(\xi)\exp\left[-\left(\xi^2+\lambda^2\right)^{1/2}|z|\right]\mathrm{d}\xi. \tag{A5}$$

Similarly we have

$$\phi^{(p)}(r,z) = e\sum_{k=-\infty}^{+\infty} J_k(2\xi r_0)\int_{\xi=0}^{\infty} J_k[\xi(r+r_0)]\exp(-\xi\,|z|)\mathrm{d}\xi \tag{A6}$$

and

$$\phi^{(s)}(r,z) = e\sum_{k=-\infty}^{+\infty} J_k(\xi r_0)\int_{\xi=0}^{\infty} J_k[\xi(r+r_0)]f(\xi)\exp(-\xi\,|z|)\mathrm{d}\xi. \tag{A7}$$

Then from the continuity conditions on the curved surface along r direction, we have

$$\begin{aligned}
&\sum_{k=-\infty}^{+\infty} J_k(\xi r_0)\int_{\xi=0}^{\infty} J_k'[\xi(r+r_0)]\xi a(\xi)\exp\left[-\left(\xi^2+\lambda^2\right)^{1/2}|z|\right]\mathrm{d}\xi \\
&= e\sum_{k=-\infty}^{+\infty} J_k(2\xi r_0)\int_{\xi=0}^{\infty} J_k'[\xi(r+r_0)]\xi\exp(-\xi\,|z|)\mathrm{d}\xi \\
&\quad + e\sum_{k=-\infty}^{+\infty} J_k(\xi r_0)\int_{\xi=0}^{\infty} J_k'[\xi(r+r_0)]\xi f(\xi)\exp(-\xi\,|z|)\mathrm{d}\xi.
\end{aligned} \tag{A8}$$

Putting $z = 0$, $r = R$ and redefining the spectral function $a(\xi) = a(\xi)/e$, we have

$$\int_{\xi=0}^{\infty} \xi \mathrm{d}\xi \sum_{k=-\infty}^{+\infty} J_k'[(R + r_0)\xi]\{a(\xi) J_R(\xi r_0) - J_k(2\xi r_0) - J_k(\xi r_0) f(\xi)\} = 0. \tag{A9}$$

Since $\xi \mathrm{d}\xi$ is arbitrary, we have

$$\sum_{k=-\infty}^{+\infty} J_k'[(R + r_0)\xi]\{a(\xi) J_R(\xi r_0) - J_k(2\xi r_0) - J_k(\xi r_0) f(\xi)\} = 0. \tag{A10}$$

Similarly the continuity condition along the z-direction on the curved surface at $z = 0$ and $r = R$ is given by

$$\begin{aligned} \int_{\xi=0}^{\infty} \sum_{k=-\infty}^{+\infty} J_k(\xi r_0) J_k[(R + r_0)\xi] a(\xi) \left(\xi^2 + \lambda^2\right)^{1/2} \mathrm{d}\xi \\ = \int_{\xi=0}^{\infty} \sum_{k=-\infty}^{+\infty} J_k(2\xi r_0) J_k[\xi (R + r_0)] \xi \mathrm{d}\xi \\ + \int_{\xi=0}^{\infty} \sum_{k=-\infty}^{+\infty} J_k(\xi r_0) J_k[\xi (R + r_0)] f(\xi) \xi \mathrm{d}\xi. \end{aligned} \tag{A11}$$

Here $a(\xi)$ is again the re-defined form. As before, from the arbitrariness of $\xi \mathrm{d}\xi$, we have

$$\sum_{k=-\infty}^{+\infty} J_k[\xi (R + r_0)] \left[a(\xi) \left(1 + \frac{\lambda^2}{\xi^2}\right)^{1/2} - J_k(2\xi r_0) - f(\xi) J_k(\xi r_0) \right] = 0. \tag{A12}$$

Hence we can write

$$a(\xi) = F(R, r_0, \xi) + f(\xi), \tag{A13}$$

where

$$F(R, r_0, \xi) = \frac{\sum_{k=-\infty}^{+\infty} J_k'[\xi (R + r_0)] J_k(2\xi r_0)}{\sum_{k=-\infty}^{+\infty} J_k'[\xi (R + r_0)] J_k(\xi r_0)} \tag{A14}$$

and similarly

$$a(\xi) \left(1 + \frac{\lambda^2}{\xi^2}\right)^{1/2} = G(R, r_0, \xi) + f(\xi), \tag{A15}$$

where

$$G(R, r_0, \xi) = \frac{\sum_{k=-\infty}^{+\infty} J_k[\xi (R + r_0)] J_k(2\xi r_0)}{\sum_{k=-\infty}^{+\infty} J_k[\xi (R + r_0)] J_k(\xi r_0)}, \tag{A16}$$

which further gives

$$\frac{F+f}{G+f} = \frac{1}{\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}. \tag{A17}$$

Solving for $f(\xi)$, we have

$$f(\xi) = \frac{F\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2} - G}{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}. \tag{A18}$$

Now using the identity

$$J_k'(u) = \frac{1}{2}[J_{k-1}(u) - J_{k+1}(u)] \text{ and } J_\nu(v-u) = \sum_{-\infty}^{+\infty} J_{\nu+k}(v) J_k(u) \tag{A19}$$

we have

$$F = \frac{J_1[(R-r_0)\xi]}{J_1(R\xi)} \text{ and } G = \frac{J_0[(R-r_0)\xi]}{J_0(R\xi)}. \tag{A20}$$

Then substituting $f(\xi)$ in the expression for $\phi^{(s)}(r,z)$ (eq. (A7)), we have

$$\phi^{(s)}(r,z) = e\int_{\xi=0}^{\infty} J_0(\xi r)\left[\frac{F\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2} - G}{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}\right]\exp(-\xi z)\mathrm{d}\xi. \tag{A21}$$

The corresponding electrostatic field at $(R, z=0)$ is given by

$$\begin{aligned} E^{(s)}(R,0) &= -\frac{\partial\phi^{(s)}(r,z)}{\partial r} \\ &= e\int_{\xi=0}^{\infty} \frac{J_1[(R-r_0)\xi]\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2} - \frac{J_0[(R-r_0)\xi]J_1(R\xi)}{J_0(R\xi)}}{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}}\xi\,\mathrm{d}\xi, \end{aligned} \tag{A22}$$

where we have substituted F and G from eq. (A20). The work function associated with the emission of electrons in the transverse direction is then given by Schieber (1984)

$$W_f = -e\int_R^{\infty} E^{(s)}(R,0)\mathrm{d}r_0. \tag{A23}$$

Using the relation

$$\int_0^{\infty} J_n(u)\mathrm{d}u = 1, \tag{A24}$$

we have

$$\int_R^{\infty} J_0[(R-r_0)\xi]\mathrm{d}r_0 = \int_R^{\infty} J_1[(R-r_0)\xi]\mathrm{d}r_0 = \frac{1}{\xi}. \tag{A25}$$

Then finally we get

$$W_f = -e^2 \int_{\xi=0}^{\infty} \frac{\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2} - \left[\frac{J_1(R\xi)}{J_0(R\xi)}\right]}{1-\left(1+\frac{\lambda^2}{\xi^2}\right)^{1/2}} \mathrm{d}\xi. \tag{A26}$$

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