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Markov Stochastic Technique to Determine Galactic Cosmic Ray Sources Distribution

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Abstract. A new numerical model of particle propagation in the Galaxy has been developed, which allows the study of cosmic-ray production and propagation in 2D. The model has been used to solve cosmic ray diffusive transport equation with a complete network of nuclear interactions using the time backward Markov stochastic process by tracing the particles' trajectories starting from the Solar System back to their sources in the Galaxy. This paper describes a further development of the model to calculate the contribution of various galactic locations to the production of certain cosmic ray nuclei observed at the Solar System.

Key words. Cosmic rays—ISM: galactic locations—stochastic processes.

1. Introduction

The analysis of elemental and isotopic composition of cosmic ray nuclei that reach the Solar System provides important information about their propagation and sources throughout the interstellar medium. Numerous numerical models have been developed for studying reacceleration, galactic halo size, antiprotons and positrons in cosmic rays, the interpretation of diffuse continuum gamma rays and even dark matter. The elemental and isotopic abundances ratio calculated using these techniques showed good agreement with observational data.

Many aspects, however, cannot be addressed using these models, e.g., the stochastic nature of the cosmic ray sources in space and time, which is important for high-energy electrons, and local inhomogeneities in the gas density that can affect radioactive secondary ratios in cosmic rays. Previous approaches to the nucleon propagation problem introduced by Jones (1979) and Bloemen *et al.* (1993) showed that energy losses are difficult to treat. Reacceleration was not considered in some of these approaches. The GALPROP development (Strong & Moskalenko 1998) has the advantage over the Leaky Box Model (Letaw *et al.* 1983) in having a simplified reaction network, describing the spatial distribution of cosmic rays and in the correct treatment of the radioactive nuclei and the realistic gas and source distribution, but on the other hand, neither can address the effects of small scale structures and the inhomogeneities in the interstellar gas on the radioactive secondary abundances in cosmic rays. These limitations motivated the need to develop a method that can handle some of these aspects.

We recently introduced a numerical model Farahat *et al.* (2008) and a corresponding computer code to study cosmic rays production and propagation in the Galaxy using the time backward Markov stochastic solution of the general diffusion transport equation (Gardiner 1983; Freidlin 1985; Øksendal 1992) starting from an observer's location described by Zhang (1999). The method follows the trajectory of random walking pseudo-particles that represent the particle number density. Instead of solving particle transport equations of one nuclear species at a time, we use a matrix to represent the cosmic ray composition so that number densities at a particular location and particle momentum for all nuclear species can be obtained together. In the first publication of the new method, we represented the results for some common elemental and isotopic abundance ratios, such as B/C, sub-Fe/Fe, ^{36}Cl/^{37}Cl and ^{54}Mn/^{55}Mn, made against benchmark calculations by Strong & Moskalenko (1998) and some observations by Connell (1998) and Lukasiak *et al.* (1999). The effect of the inhomogeneities in the interstellar medium was described using the new method. The effect of the Local Super Bubble surrounding the Solar System, on the production of ^{10}Be/^{9}Be was also simulated. The numerical method used to solve cosmic ray diffusive transport equation has been described in full detail elsewhere (Farahat *et al.* 2008); here we just summarize briefly its basic features.

2. Mathematical model

The model uses spatial distribution of points on a random trajectory to determine the elemental and isotopic abundances, which is computationally much easier and faster than using the classical Leaky Box Model (LBM) where the transport of CR is not controlled by diffusion but by a hypothetic leakage process at the imaginary boundaries. Using the backward stochastic solution of the general diffusion transport equation starting from an observer position described by Zhang (1999) we can calculate the elemental or isotopic abundance of single cosmic ray nuclei. This solution can be expanded to calculate the abundances of many elements and isotopes by solving a system of simultaneous diffusion transport equations each represents a particular element or isotope. In our study, we used realistic astrophysical parameters like the gas density distribution, elemental and partial cross sections.

The method depends on solving a group of diffusion transport equations each representing a particular species using the backward Markov Stochastic technique starting at an observer location in the Solar System and stopping at the galactic boundaries. Diffusion coefficient, halo size and rigidity are the main parameters contributing to this model and are chosen to best-fit observational data.

The model has cylindrical symmetry in the Galaxy, and the basic coordinates are R, z, p where R is the galactocentric radius, z is the distance from the galactic plane and p is the total particle momentum. We take the galactic wind convection velocity $\mathbf{V} = 20$ km/s, halo radius $R_h = 30$ Kpc, halo height $z_h = 4$ Kpc and the distance from the Sun to the center of the Galaxy $= 8.5$ Kpc. The He/H ratio is 0.11 by number. Isotopic cross sections are calculated using Webber *et al.* (1990) and Sihver *et al.* (1993). The solar modulation numerical model (SolMod, Fisk 1971) is used to modulate interstellar medium spectrum. The interstellar gas is filled with three gas components in the form of warm atomic gases, molecular clouds and hot plasma. We have developed a matrix method for the problem by defining a matrix presenting the number density of all cosmic ray species.

If all the species in the reaction network have the same diffusion tensor and drift coefficients, we can write the cosmic ray transport equation (Berezinskiĭ *et al.* 1990) in matrix format as:

$$\frac{\partial N}{\partial t} = \frac{1}{2}\sum_{\mu,\upsilon=1}^{4} \alpha^{\mu\upsilon}\frac{\partial^2 N}{\partial q^{\mu}\partial q^{\upsilon}} + \sum_{i=1}^{4} \beta^{\mu}\frac{\partial N}{\partial q^{\mu}} - CN + S(t,q), \tag{1}$$

where

$$\alpha^{\mu\upsilon}(t,q) = \begin{pmatrix} 2\boldsymbol{\kappa} & 0 \\ 0 & 2D \end{pmatrix}, \tag{2}$$

$$\beta^{\mu}(t,q) = \nabla \cdot \boldsymbol{\kappa} - \mathbf{V}p^2 \frac{\partial}{\partial p}\left(\frac{D}{p^2}\right) + \frac{p}{3}\nabla \cdot \mathbf{V} + b, \tag{3}$$

$$C(t,q) = n\upsilon\sigma + \frac{1}{\tau} - \frac{1}{3}\nabla \cdot \mathbf{V} + \frac{2\partial}{\partial p}\left(\frac{D}{p}\right) - \frac{\partial b}{\partial p}, \tag{4}$$

where b is the adiabatic momentum loss rate, τ is the decay time, $\mathbf{V}$ is the convection velocity, with a rigidity dependent spatial diffusion coefficient given by $\kappa = (\upsilon/c)k_0(\rho/\rho_0)^{\delta}$, $k_0 = 6 \times 10^{28}\,\mathrm{cm^2/s}$, $\rho_0 = 3\,\mathrm{GV}$ where

$$\delta = \begin{cases} 0.36 & \text{if } \rho > \rho_0; \\ -0.36 & \text{if } \rho < \rho_0. \end{cases}$$

The stochastic process starts from q at time $t\,(t_s = 0)$ and it steps back in time as the integration variable t_s increases. The solution to this problem is given as:

$$N(t,q) = \left\langle \int_0^{\tau^{t,q}} [S(Q_t^{\tau,q})] \exp\left\{-\int_0^{t} C(t - t_s, Q_{t_s}^{t,q})dt_s\right\} dt \right\rangle. \tag{5}$$

The parameter $C(t,q)$ in equation (5) allows the stochastic process to be created at an exponential rate as a function of time; $\tau^{t,q}$ is the time needed for the stochastic process to run backward from q at time t until it gets to the boundary. The above time backward stochastic solution can be used to calculate the elemental or isotopic abundance of a single cosmic ray nucleus. In case of several nuclei the problem is expanded to solve a system of simultaneous diffusion transport equations each represents a particular element or isotope. The solution in equation (5) can be used to calculate the abundance of ^{64}Ni, for example, however if the goal to calculate the abundances of both ^{64}Ni and ^{62}Ni we need to solve two simultaneous equations each represents one element and includes all sources and sinks for that element. The final abundance solution in this case will have the vector form $\begin{pmatrix} ^{64}\mathrm{Ni} \\ ^{62}\mathrm{Ni} \end{pmatrix}$. The same technique can be applied to higher number of elements as:

$$N(t,q) = \begin{pmatrix} N_1(t,q) \\ N_2(t,q) \\ \vdots \\ N_n(t,q) \end{pmatrix} \tag{6}$$

and

$$C(t,q) = \begin{pmatrix} C_{11} & 0 & \cdots & 0 \\ C_{21} & C_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{pmatrix}, \qquad (7)$$

where $C_{ii} = C_i$ and $C_{ij} = -(nv\sigma_{ij} + (1/\tau_{ij}))$ for $j < i$ and the cosmic ray source term is a vector in the form:

$$S(t,q) = \begin{pmatrix} S_1(t,q) \\ S_2(t,q) \\ \vdots \\ S_n(t,q) \end{pmatrix} = \begin{pmatrix} S_{10} \\ S_{20} \\ \vdots \\ S_{n0} \end{pmatrix} S^*(t,q) = S_0 S^*(t,q), \qquad (8)$$

where $S^*(t,q)$ denotes the spatial and time variation of all sources and S_0 is the composition of sources, σ_{ij}, the partial production cross-section from nucleus type j to i.

In this paper we are introducing a further application of the new model where we apply the matrix method with the stochastic trajectory to calculate the contribution of various locations in the Galaxy to the production of certain cosmic ray nuclei. The simulation represents a probability distribution to the galactic locations that participate in the production of some nuclei we observe at the Solar System.

3. Results

A test particle is allowed to follow a stochastic path starting from the Solar System at time t and runs backward in time till hitting the Galaxy boundary as shown in Fig. 1. The simulation includes a few thousand stochastic trajectories and the average abundance for each element at the Solar System is estimated. We allow free escape at

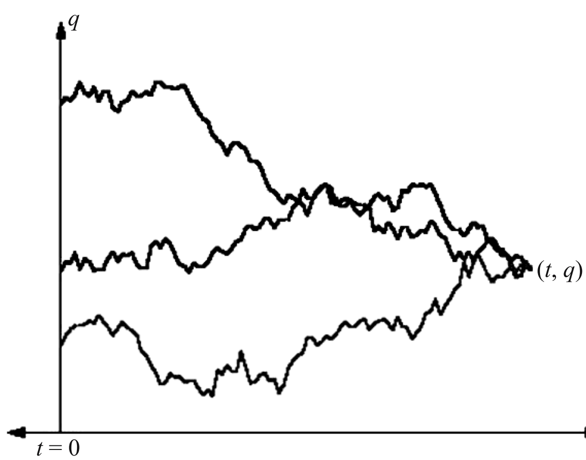

Figure 1. Graphical illustration of using backward stochastic processes to obtain the solution to general diffusion equations.

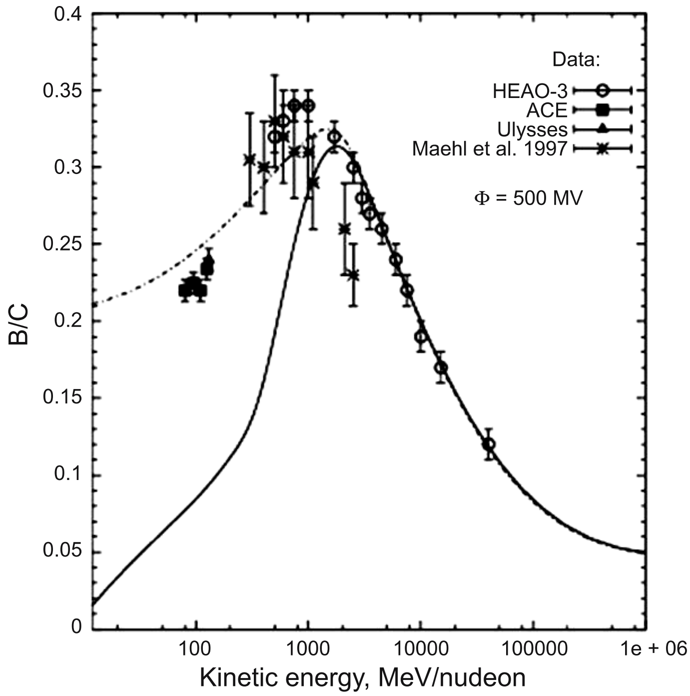

Figure 2. B/C ratio solid curve, no modulation; dash-dotted curve, with modulation, solar modulation = 500 MV.

galactic radius R_h and the halo size $\pm z_h$. The Galaxy is meshed into 1600 points where the smaller time step gives more detailed information about the location contribution. The abundance for each nucleus is recorded at certain energy and a single location in the stochastic path. Using equations (6), (7) and (8) we then determine the locations in Galaxy containing the sources of the production of a certain nuclide, this means if ^{12}C, ^{56}Fe, ..., etc., participate in the production of ^{10}B observed at the Solar System we determine the locations at which ^{12}C, ^{56}Fe, ..., etc., were produced. Different locations in the Galaxy were found unequally participating into the production of elements and isotopes observed in the Solar System.

Figure 2 shows B/C ratio for testing our model as known to be one of the most accurately measured ratios covering a wide energy range and having well established cross sections. The figure shows the calculated local interstellar medium (LIS) (solid curve) and 500 MV modulated (dashed curve) B/C ratio compared with the observational data from HEAO3 (Webber *et al.* 1996), ACE (Davis *et al.* 2000), Ulysses (DuVernois *et al.* 1996; Maehl *et al.* 1977).

Figures 3–5 show contour plots for different locations contributing to the production of the sources of ^{12}C, ^{10}B and ^{10}Be, respectively. The results are calculated at 1 GeV/nucleon.

We investigated the contribution from several locations within a specified region around the Solar System. The model shows that most of the cosmic rays observed in the Solar System are from sources within 10 Kpc around the Solar System. The very low abundance regions appear in the ^{10}Be distributions near the center of the Galaxy is

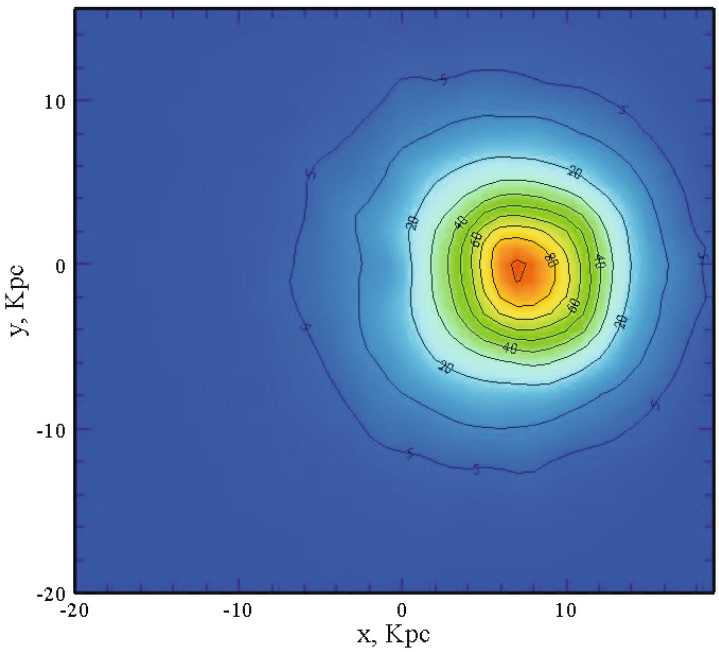

Figure 3. Contribution of various locations around the Galaxy to the production of ^{12}C sources; energy = 1 GeV/nucleon and no modulation.

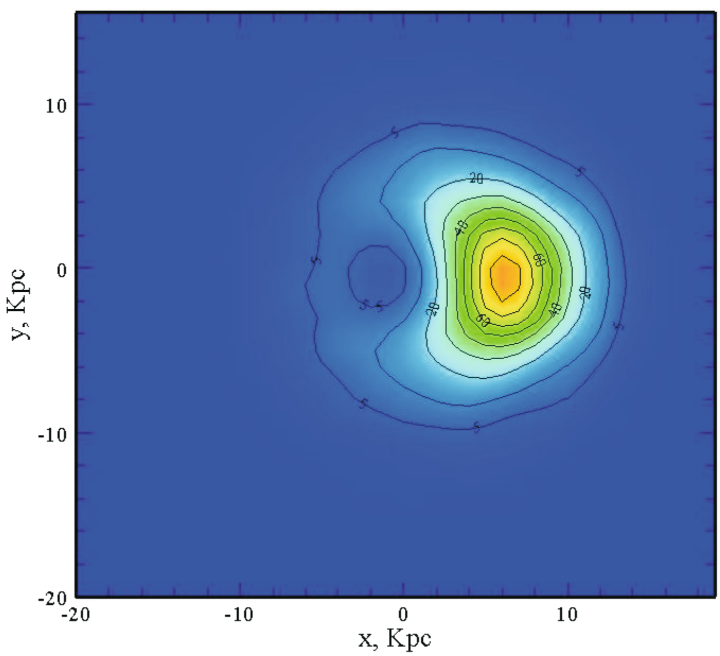

Figure 4. Contribution of various locations around the Galaxy to the production of ^{10}B sources; energy = 1 GeV/nucleon and no modulation.

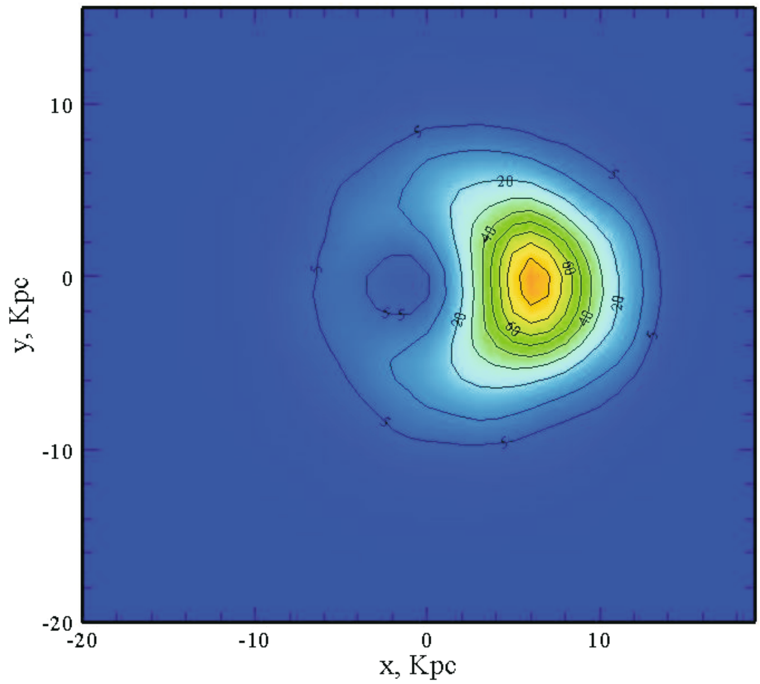

Figure 5. Contribution of various locations around the Galaxy to the production of ^{10}Be sources; energy = 1 GeV/nucleon and no modulation.

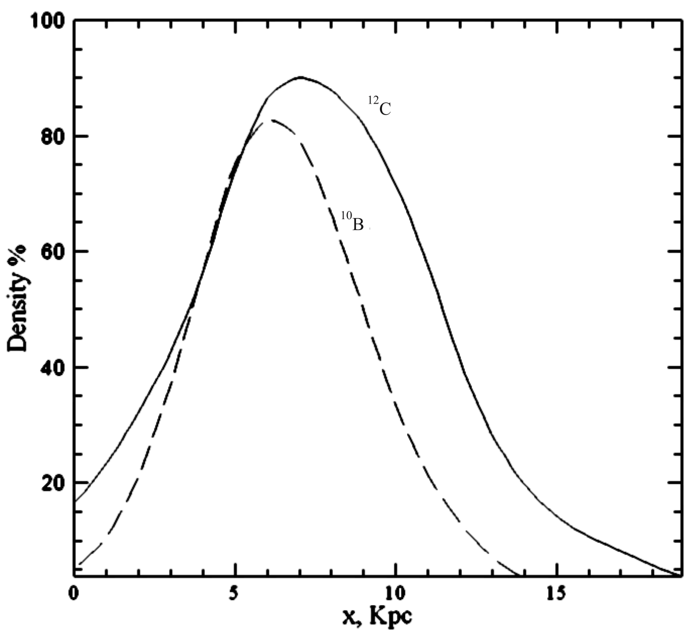

Figure 6. Comparison of the contribution of various locations around the Galaxy to the production of ^{12}C, ^{10}B sources; energy = 1 GeV/nucleon and no modulation.

due to the interstellar medium density distribution (Moskalenko *et al.* 2002) with lower number density around the galactic center which will be reflected on the number of interactions with the interstellar medium producing ^{10}Be. The source/Solar System abundances have been taken from Anders & Grevesse (1989). Figure 6 shows a comparison of the contribution of various locations around the Galaxy to the production of ^{12}C and ^{10}B sources at 1 GeV/nucleon. The atomic hydrogen (HI) distribution derived from the measurement of 21 cm emission can be found in Gordon & Burton (1976) and Cox *et al.* (1986). The distribution of molecular hydrogen H_2 is taken from Bronfman *et al.* (1988) assuming that the ratio of H_2 to CO is roughly constant.

4. Conclusion

The source contribution to a particular cosmic ray species observed at the Solar System is calculated using the backward Markov Stochastic method and the solution of a group of diffusion transport equations each representing a particular species. The stochastic path starts at an observer location in the Solar System and stop at the galactic boundaries. The source contribution to a particular cosmic ray species observed at the Solar System is calculated at each time step. The model shows that most of the cosmic rays observed in the Solar System are from sources within 10 Kpc around the Solar System. The diffusion coefficient, halo size and radius are the main parameters of this model and are chosen to provide the best fit to observational data.

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