

## On Out of Plane Equilibrium Points in Photo-Gravitational Restricted Three-Body Problem

M. K. Das<sup>1</sup>, Pankaj Narang<sup>2</sup>, S. Mahajan<sup>3</sup> & M. Yuasa<sup>4</sup>

<sup>1</sup>*Institute of Informatics and Communication, University of Delhi South Campus, Benitojuarez Road, New Delhi 110 021, India.*

<sup>2</sup>*Department of Physics, A.R.S.D. College, University of Delhi, New Delhi 110 021, India.*

<sup>3</sup>*Department of Physics & Astrophysics, University of Delhi, New Delhi 110 021, India.*

<sup>4</sup>*RIST, Kinki University, Higashi-Osaka 577-8502, Japan.*

Received 2008 August 17; accepted 2009 September 1

**Abstract.** We have investigated the out of plane equilibrium points of a passive micron size particle and their stability in the field of radiating binary stellar systems *Krüger-60*, *RW-Monocerotis* within the framework of photo-gravitational circular restricted three-body problem. We find that the out of plane equilibrium points ( $L_i$ ,  $i = 6, 7, 8, 9$ ) may exist for range of  $\beta_1$  (ratio of radiation to gravitational force of the massive component) values for these binary systems in the presence of Poynting–Robertson drag (hereafter PR-drag). In the absence of PR-drag, we find that the motion of a particle near the equilibrium points  $L_{6,7}$  is stable in both the binary systems for a specific range of  $\beta_1$  values. The PR-drag is shown to cause instability of the various out of plane equilibrium points in these binary systems.

**Key words.** Radiation—Poynting–Robertson drag—binary stellar system—equilibrium points—stability.

### 1. Introduction

The photo-gravitational circular restricted three-body problem was first studied by Radzievskii (1950, 1953). In this work, besides the coplanar libration points  $L_i$ ,  $i = 1$  to 5, the effect of radiation was shown to result in the libration points  $L_{6,7}$  that exist in a plane perpendicular to the orbital plane of the radiating primaries. Since then several authors (cf. Chernikov 1970; Perezhogin 1976; Scheurman 1980; Simmons *et al.* 1985; Ragos & Zagouras 1988; Murray 1994; Ragos & Zafiroopoulos 1995; Ragos *et al.* 1995; Roman 2001; Kunitsyn & Chudayeva 2003; Kushvah & Ishwar 2004; Das *et al.* 2008a) extended the work to understand various issues related to the dynamics of a particle around radiating primaries. However, majority of these works involve the use of independent quantities  $q_1 = 1 - \beta_1$  and  $q_2 = 1 - \beta_2$ , where  $\beta_i$  corresponds to the ratio of radiation pressure force to the gravitational force of  $i$ -th binary component. Since  $\beta_i$  does depend on the size, density of the particle and mass and luminosity of the respective binary component, we apply a realistic relation connecting the parameters  $\beta_1$  and  $\beta_2$  and study the location and stability of out of plane equilibrium points

of a micron size particle moving around a radiating binary stellar system. Incorporating the PR-drag effect, we observe that the libration points  $L_{6,7}$  exist for certain range of values of  $\beta_1$  for the binary systems *Krüger-60* and *RW-Monocerotis*. Further, certain  $\beta_1$  values exist for which it is also possible to have four libration points, i.e.,  $L_i, i = 6, 7, 8, 9$  in these binary stars. Using linear stability analysis, it is observed that the stability of motion around any of these points depends on the parameter  $\beta_1$  and  $\beta_2$  involving physical parameters, i.e., mass and luminosity of the given binary system. For the binary stellar systems considered here, we find that all such equilibrium points are unstable. However, in the absence of PR-drag we observe that it is possible to have linearly stable motion around  $L_{6,7}$  for certain  $\beta_1$  values in both the binary systems.

## 2. The location and stability of out of plane equilibrium points

Following Ragos & Zafiroopoulos (1995) and Ragos *et al.* (1995) the equation of motion of an infinitesimal mass moving in the radiation and gravitational field of the binary system, in a rotating barycentric co-ordinate system (cf. Szebeheley 1967; Hénon 1983), could be written as:

$$\ddot{X} - 2\dot{Y} = \Omega_x, \quad \ddot{Y} + 2\dot{X} = \Omega_y, \quad \ddot{Z} = \Omega_z, \quad (1)$$

where

$$\begin{aligned} \Omega_x = & X - \frac{Q_1(X + \mu)}{r_1^3} - \frac{Q_2(X + \mu - 1)}{r_2^3} \\ & - \frac{W_1}{r_1^2} \left[ \frac{X + \mu}{r_1^2} ((X + \mu)\dot{X} + Y\dot{Y} + Z\dot{Z}) + \dot{X} - Y \right] \\ & - \frac{W_2}{r_2^2} \left[ \frac{X + \mu - 1}{r_2^2} ((X + \mu - 1)\dot{X} + Y\dot{Y} + Z\dot{Z}) + \dot{X} - Y \right] \end{aligned} \quad (2)$$

$$\begin{aligned} \Omega_y = & Y \left( 1 - \frac{Q_1}{r_1^3} - \frac{Q_2}{r_2^3} \right) \\ & - \frac{W_1}{r_1^2} \left[ \frac{Y}{r_1^2} ((X + \mu)\dot{X} + Y\dot{Y} + Z\dot{Z}) + \dot{Y} + X + \mu \right] \\ & - \frac{W_2}{r_2^2} \left[ \frac{Y}{r_2^2} ((X + \mu - 1)\dot{X} + Y\dot{Y} + Z\dot{Z}) + \dot{Y} + X + \mu - 1 \right] \end{aligned} \quad (3)$$

$$\begin{aligned} \Omega_z = & \left[ -\frac{Q_1}{r_1^3} - \frac{Q_2}{r_2^3} \right] Z \\ & - \frac{W_1}{r_1^2} \left[ \frac{Z}{r_1^2} ((X + \mu)\dot{X} + Y\dot{Y} + Z\dot{Z}) + \dot{Z} \right] \\ & - \frac{W_2}{r_2^2} \left[ \frac{Z}{r_2^2} ((X + \mu - 1)\dot{X} + Y\dot{Y} + Z\dot{Z}) + \dot{Z} \right] \end{aligned} \quad (4)$$

$$\mu = \frac{M_2}{M_1 + M_2}, \quad Q_1 = q_1(1 - \mu), \quad Q_2 = q_2\mu, \quad (5)$$

$$W_1 = \frac{(1 - q_1)(1 - \mu)}{C_d}, \quad W_2 = \frac{(1 - q_2)\mu}{C_d}, \quad (6)$$

$$r_1^2 = (X + \mu)^2 + Y^2 + Z^2, \quad r_2^2 = (X + \mu - 1)^2 + Y^2 + Z^2. \quad (7)$$

Here,  $M_1$  and  $M_2$  refer to masses of the respective binary component;  $C_d = c/v_{in}$  corresponds to the dimensionless velocity of light and depends on the physical masses of primaries and distance between them;  $q_{1,2} = 1 - \beta_{1,2}$  corresponds to radiation parameters from the respective primaries;  $r_1$  and  $r_2$  correspond to the distances between the third body and primaries. Further,  $\beta_i$  corresponds to the ratio of force due to radiation and the gravitational force of the  $i$ -th binary component (cf. Das *et al.* 2008b) from the  $i$ -th binary component. It is important to note that for solar dust particles less than a  $\mu\text{m}$  comprising spherical silicate BPCA, carbon BPCA, silicate compact, asteroidal dust, young and cometary dust grains,  $\beta$  may vary in the range  $\sim 10^{-2}$ – $5.0$  (cf. Wilck & Mann 1996; Krivov *et al.* 1998; Kimura *et al.* 2002). Therefore in a real situation, it is possible to have  $\beta_i \geq 1$ . However, there exists a relation:

$$\beta_2 = \beta_1 \frac{L_2 M_1}{L_1 M_2}, \quad (8)$$

which connects the radiation parameters of respective binary components in terms of their luminosities and masses. We use the above relation to fix the value of the parameter  $\beta_2$  in terms of the mass and luminosity of the binary components for a given  $\beta_1 > 1$ . Therefore, the quantities  $q_1$  and  $q_2$  are not independent. It may be noted that several authors (cf. Ragos & Zafropoulos 1995; Ragos *et al.* 1995 and references quoted therein) have used the radiation parameters  $q_1$  and  $q_2$  as independent. It is in this sense their results are of limited applicability to the motion of a particle in stellar binary systems in general. Besides the classical coplanar equilibrium points (cf. Ragos & Zafropoulos 1995; Das *et al.* 2008b), it is possible to have out of plane equilibrium points exclusively due to radiation from binary components. Such points do not have any classical analogue. In the following, we discuss the effect of radiation on the location and stability of possible equilibrium points in two steps. First only the major radial component of the pressure force is considered so that the problem is reduced to that of a central force only (cf. Radzievskii 1950, 1953). In fact this approach to the problem is already an approximate one: for particles with velocity  $\mathbf{v}$ , terms of order  $\mathbf{v}/c$  and higher in the general radiation force term are neglected. In fact, due to radiation, the radiation force  $F$  on a particle may be written as  $F = F_p + F_{PR}$  (cf. Robertson 1937) where  $F_{PR}$  is the Poynting–Robertson drag and corresponds to a first order term in  $\mathbf{v}/c$ . For a  $1 \mu\text{m}$  dust particle at a distance of 1 AU from sun  $F_{PR}/F_p \sim 10^{-4}$  and therefore significant changes in the location of various equilibrium points are unlikely by the inclusion of  $F_{PR}$  terms. However, the inclusion of  $F_{PR}$  drag term change the nature of problem from purely a central force to a dissipative one. Therefore, in the second step, we incorporate  $F_{PR}$  term as well and show that the various out-of plane equilibrium points of the binary system become unstable.

### 2.1 Location and stability of equilibrium points in the absence of PR-drag

In the absence of PR-drag, the stationary solutions of equation (1), for the case  $Z \neq 0$ , results in the following conditions:

$$X_0 - \frac{Q_1(X_0 + \mu)}{r_{10}^3} - \frac{Q_2(X_0 + \mu - 1)}{r_{20}^3} = 0, \quad (9)$$

$$Y_0 = 0, \quad (10)$$

$$\left[ \frac{Q_1}{r_{10}^3} + \frac{Q_2}{r_{20}^3} \right] = 0, \quad (11)$$

where

$$r_{10}^2 = (X_0 + \mu)^2 + Z_0^2, \quad r_{20}^2 = (X_0 + \mu - 1)^2 + Z_0^2 \quad (12)$$

and the subscript '0' is used to denote the equilibrium values. Since  $Z_0 \neq 0$ , we observe that equations (9)–(11) are satisfied if:

$$X_0 = \frac{Q_1}{r_{10}^3} = -\frac{Q_2}{r_{20}^3}. \quad (13)$$

Obviously the last equation results in either  $Q_1 < 0$ ,  $Q_2 > 0$  or  $Q_1 > 0$ ,  $Q_2 < 0$ , i.e.,  $Q_1 Q_2 < 0$ . In view of the relation  $q_{1,2} = 1 - \beta_{1,2}$ , we observe that  $Q_1 Q_2 < 0$  implies that either  $\beta_1 > 1$ ,  $\beta_2 < 1$  or  $\beta_1 < 1$ ,  $\beta_2 > 1$ . Since for particles around  $0.1 \mu\text{m}$  it is possible to have  $\beta_1 > 1$ , we find the possibilities of having  $(1 - \beta_1) < 0$  and  $(1 - \beta_2) > 0$  in some binary stellar system.

On elimination of  $Z_0$ , we may rewrite equation (13) as:

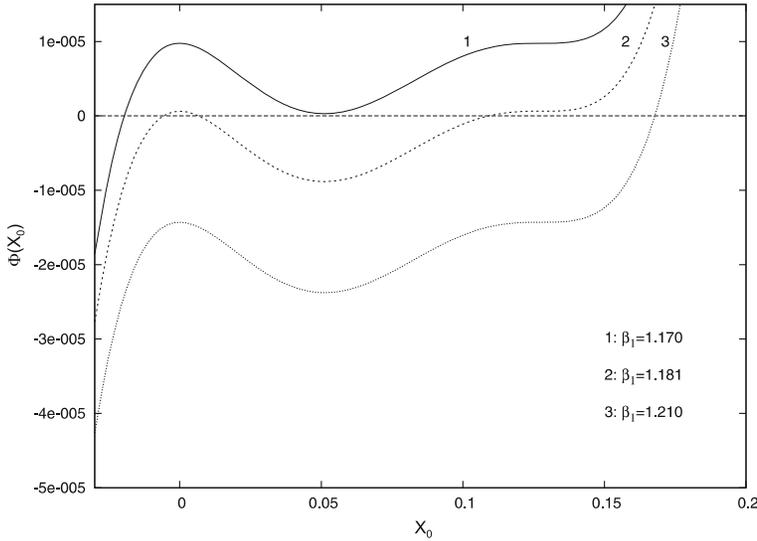
$$\begin{aligned} \Phi(X_0) &= 8X_0^5 + 12(2\mu - 1)X_0^4 + 6(2\mu - 1)^2 X_0^3 \\ &+ (2\mu - 1)^3 X_0^2 - (Q_1^{2/3} - Q_2^{2/3})^3 = 0. \end{aligned} \quad (14)$$

The solution of equation (14) for real  $X_0$  along with real  $Z_0$  obtained from:

$$Z_0 = \pm \left[ \left( \frac{Q_1}{X_0} \right)^{2/3} - (X_0 + \mu)^2 \right]^{1/2}, \quad (15)$$

using equation (12), provides the equilibrium point in the  $X$ - $Z$  plane.

It is readily observed that  $X_0 = (1 - 2\mu)/2$  is a solution of equation (14) in case  $|Q_1| = |Q_2|$ . Further, from the plot of  $\Phi(X_0)$  vs.  $X_0$  (cf. Fig. 1), we observe that for certain  $\beta_1$  value, if  $|Q_1| < |Q_2|$  or  $|Q_1| > |Q_2|$ , only one real solution of equation (14) occurs in the region  $X_0 < 0$  and  $X_0 > 0$ , respectively for *Krüger-60* (curves 1 and 3 of Fig. 1). The equilibrium point  $(X_0, 0, \pm Z_0)$  lying in the region  $X_0 < 0$  and referred to as  $L_{6,7}$  (cf. Radzievskii 1953) was shown to exist for a range of values of  $\beta_1$  for binary system *RW-Monocerotis* and *Krüger-60* (Das et al. 2008b). However, it is interesting to observe that for certain  $\beta_1$  values three possible real root occurs (curve 2, Fig. 1). Of these three roots, one lies in the region  $X_0 < 0$ , and two roots in the region  $X_0 > 0$ . For such  $\beta_1$  values, the root of equation (14) lying in the region  $X_0 < 0$  results in



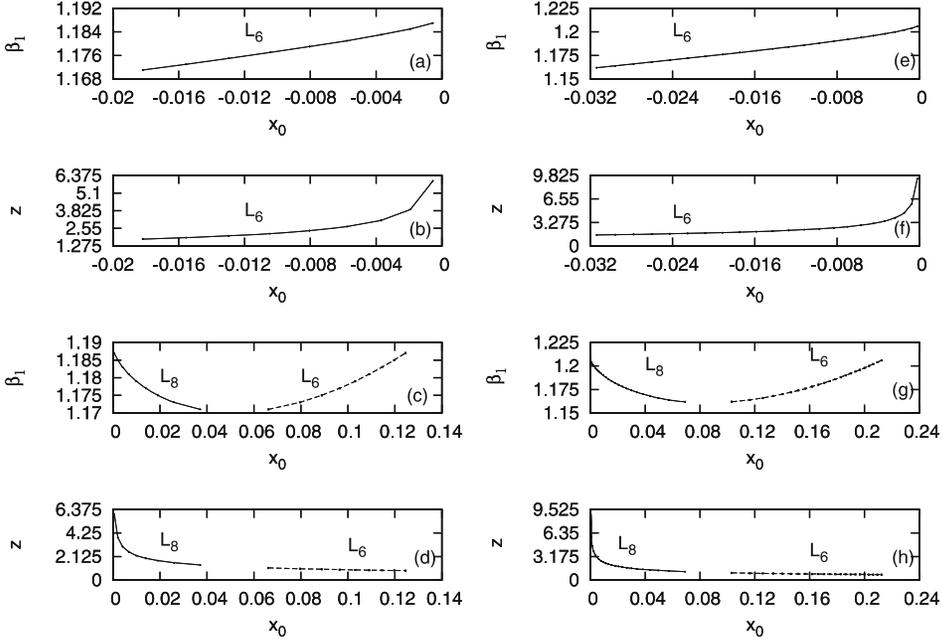
**Figure 1.** Variation of  $\Phi(X_0)$  with  $X_0$  in *Krüger-60*.

two equilibrium points  $L_{6,7}$  while the two roots in the region  $X_0 > 0$  provide four equilibrium points  $L_6, L_7, L_8$  and  $L_9$  for the two binary systems considered here. Earlier Lukyanov (1984) and Simmons *et al.* (1985) reported the possible existence of such equilibrium points in the general photo-gravitational restricted circular three-body problem in the absence of PR-drag. It may be noted that the application of equation (8) as a relation between  $\beta_i$ 's ( $i = 1, 2$ ) results in the location of various equilibrium points dependent on a single parameter  $\beta_1$  rather than two independent parameters  $\beta_1$  and  $\beta_2$  considered earlier by Lukyanov (1984), Simmons *et al.* (1985) and Ragos & Zagouras (1988). Further since  $\beta_2$  depends not only on  $\beta_1$  but also on the physical parameters like mass and luminosity of the binary components, the present computational results of Fig. 2 showing the relationship between various components of  $L_6$  and  $L_8$  along with their variation with  $\beta_1$  are expected to be more realistic for *Krüger-60* and *RW-Monocerotis*.

In this work we have confined ourselves to linear stability analysis. The characteristic equation used for computing the eigenvalues for a given binary system is same as in Das *et al.* (2008b). Table 1 listing the real and imaginary components of the characteristic equation for various  $\beta_1$  values for both the binary system clearly shows the possibility of having stable motion around  $L_{6,7}$  in the domain  $X_0 > 0$  (as all the eigen values are purely imaginary). However, the motion around  $L_{6,7}$  in the domain  $X_0 < 0$  is unstable. Further for all  $\beta_1$  values considered, the motion around  $L_{8,9}$  is also found to be unstable in these binary systems.

## 2.2 Location and stability of the out of plane equilibrium points in the presence of PR-drag

Following Ragos *et al.* (1995), we find the out of plane equilibrium points in the presence of PR-drag as solution of the following equations.



**Figure 2.** (a), (e) Variation of  $L_{6x}$  with  $\beta_1$  in the region  $X_0 < 0$ , (b), (f) variation of  $L_{6z}$  with  $L_{6x}$  in the region  $X_0 < 0$ , (c), (g) variation of  $L_{8x}$  and  $L_{6x}$  with  $\beta_1$  in the region  $X_0 > 0$ , (d)–(h) variation of  $L_{8z}$  and  $L_{6z}$  with  $L_{6x}$  in the region  $X_0 > 0$  for *Krüger-60* and *RW-Monocerotis*, respectively.

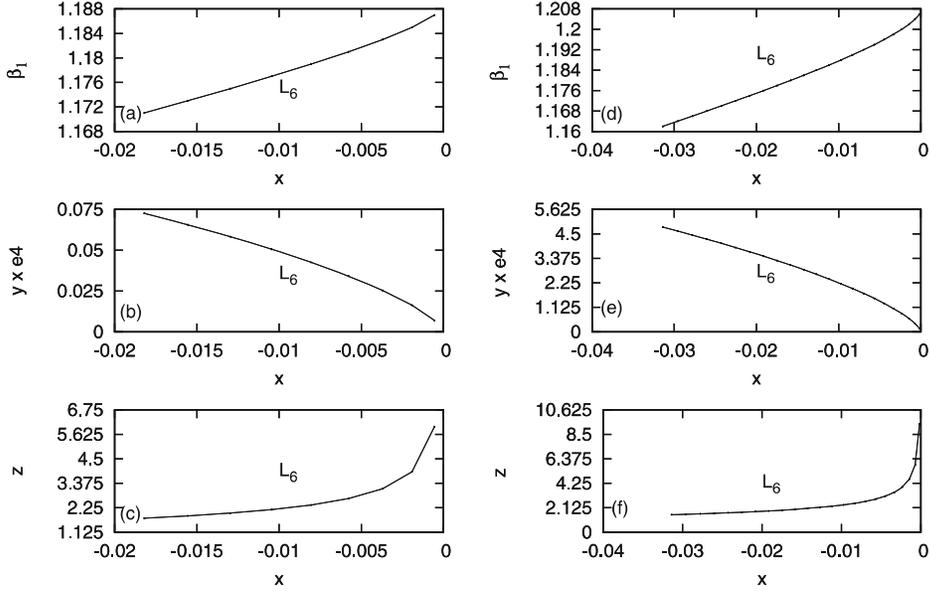
**Table 1.** Variation of eigenvalues of  $L_{6,7}$  with  $\beta_1$  for  $X_0 > 0$ .

Binary	$\beta_1$	$\lambda_1$	$\lambda_2$	$\lambda_3$
<i>RW-Monocerotis</i>	1.162	(0, 0.36217024)	(0, 0.84961841)	(0, 1.0709721)
	1.164	(0, 0.42888167)	(0, 0.76875324)	(0, 1.1068328)
	1.166	(0, 0.50137373)	(0, 0.68684667)	(0, 1.1299850)
<i>Krüger-60</i>	1.171	(0, 0.19666655)	(0, 0.92009171)	(0, 1.0558189)
	1.177	(0, 0.29345436)	(0, 0.79985195)	(0, 1.1287698)
	1.183	(0, 0.37515521)	(0, 0.70016981)	(0, 1.1700516)
	1.187	(0, 0.45360428)	(0, 0.61042658)	(0, 1.1923181)

$$X_0 - \frac{Q_1(X + \mu)}{r_1^3} - \frac{Q_2(X_0 + \mu - 1)}{r_2^2} + \left[ \frac{W_1}{r_1^2} + \frac{W_2}{r_2^2} \right] Y_0 = 0, \quad (16)$$

$$\left[ 1 - \frac{Q_1}{r_1^3} - \frac{Q_2}{r_2^3} \right] Y_0 - \frac{W_1(X_0 + \mu)}{r_1^2} - \frac{W_2(X_0 + \mu - 1)}{r_2^2} = 0, \quad (17)$$

$$\frac{Q_1}{r_1^3} + \frac{Q_2}{r_2^3} = 0. \quad (18)$$



**Figure 3.** (a), (d) Variation of  $L_{6_x}$  with  $\beta_1$ ; (b), (e) variation of  $L_{6_y}$  with  $L_{6_x}$ ; (c), (f) variation of  $L_{6_z}$  with  $L_{6_x}$  in the region  $X < 0$  for *Krüger-60* and *RW-Monocerotis*, respectively.

The foregoing equations could be easily obtained by considering  $\ddot{X} = \ddot{Y} = \ddot{Z} = 0$ ,  $\dot{X} = \dot{Y} = \dot{Z} = 0$  in equation (1). Using the last equation, we note that  $Q_1 Q_2 < 0$  or  $q_1 q_2 < 0$  must be satisfied for the existence of the out of plane equilibrium points. Further, it may be observed that the presence of PR-drag results in the possibility of having  $Y \neq 0$ , unlike the situation discussed in section 2.2. From equations (17)–(19), we find that:

$$P(r_1) = a_6 r_1^6 + a_4 r_1^4 + a_2 r_1^2 + a_1 r_1 + a_0 = 0, \quad (19)$$

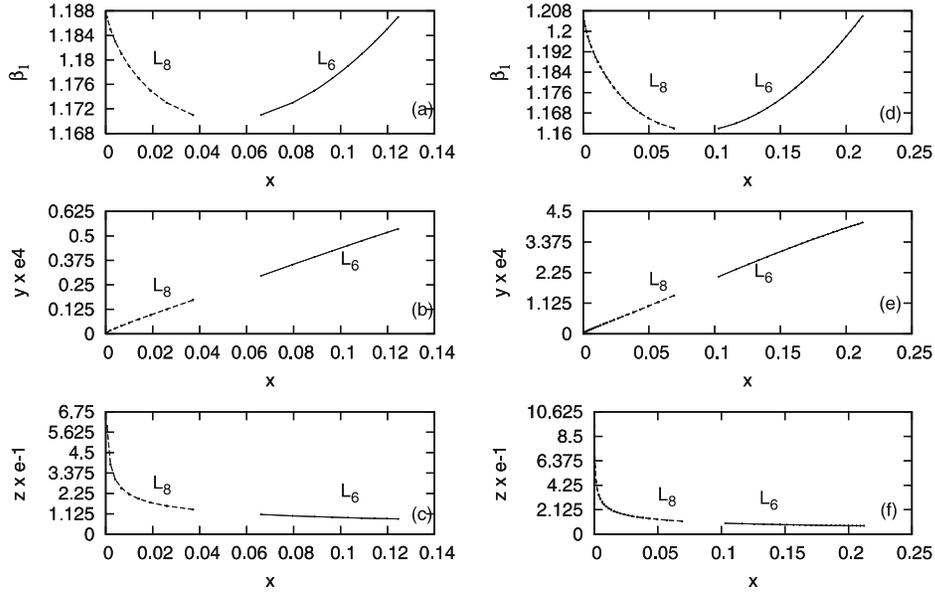
$$X_0 = \frac{1}{2} \left[ 1 - \left( \frac{Q_2}{Q_1} \right)^{2/3} \right] r_1^2 + \frac{1}{2} - \mu, \quad (20)$$

$$Y_0 = \frac{1}{2} \left[ W_1 - W_2 \left( \frac{Q_1}{Q_2} \right)^{2/3} \right] r_1^{-2},$$

$$+ \frac{1}{2} \left[ W_1 - W_2 - \left( W_1 \left( \frac{Q_2}{Q_1} \right)^{2/3} - W_2 \left( \frac{Q_1}{Q_2} \right)^{2/3} \right) \right], \quad (21)$$

$$Z_0 = \pm \sqrt{r_1^2 - (X_0 + \mu)^2 - Y^2}. \quad (22)$$

Using equation (8) and the foregoing equations with coefficients  $a_i$ ,  $i = 0, 1, 2, 4, 6$  as defined in Ragos *et al.* (1995), we have computed the co-ordinates  $X_0$ ,  $Y_0$ , and  $Z_0$  for the out of plane equilibrium points of a binary system. In the presence of PR-drag also we observe that a critical  $\beta_1$  exists for which it is possible to have two equilibrium



**Figure 4.** (a), (d) Variation of  $L_{6,x}$  and  $L_{8,x}$  with  $\beta_1$ ; (b), (e) variation of  $L_{8,y}$  and  $L_{6,y}$  with  $L_{6,x}$ ; (c), (f) variation of  $L_{8,z}$  and  $L_{6,z}$  with  $L_{6,x}$  in the region  $X > 0$  for *Krüger-60* and *RW-Monocerotis*, respectively.

points  $L_{6,7}$  in the region  $X_0 < 0$  and four equilibrium points, i.e.,  $L_{6,7,8,9}$  in the region  $X > 0$ . Figure 3(a) clearly shows that  $L_{6,x}$  increases with increase in  $\beta_1$  in the region  $X < 0$ . In such a region, for *Krüger-60*, an increase in  $X_0$  is accompanied by a decrease in  $Y$  while there is an increase in the  $Z$  values (Fig. 3b-c). Similar results are obtained for *RW-Monocerotis* (Fig. 3d-f). In the region  $X > 0$ , for *Krüger-60* an increase in  $\beta_1$  results in a decrease in the values of  $L_{8,x}$  while  $L_{6,x}$  increases (Fig. 4a). Further, it is observed that with increase in  $L_{6,x}$ ,  $L_{8,y}$  and  $L_{6,y}$  increases while  $L_{8,z}$  and  $L_{6,z}$  decrease (Fig. 4b-c) in *Krüger-60*. For the binary system *RW-Monocerotis* we observe similar trends in variation of  $L_6$  and  $L_8$  co-ordinates (Fig. 4d-f).

The inclusion of PR-drag induces instability in motion. This is evident from the fact that all the roots of the characteristic equation for equilibrium points  $L_{6,7,8,9}$  in entire domain considered here have finite (non-zero) real and imaginary components.

### 3. Results

The problem concerning the location of out of plane equilibrium points in the photogravitational restricted three-body problem has been investigated. We have incorporated the effect of PR-drag in our analysis. Unlike the work of Ragos *et al.* (1995) (and references quoted therein), we considered a realistic relation connecting the parameters  $\beta_1$  and  $\beta_2$  with the physical parameters such as mass and luminosity of the binary components and investigated the locations of equilibrium points and their stability using linear analysis for the binary systems *Krüger-60* and *RW-Monocerotis*. In the absence of PR-drag unlike Roman (2001), we observe that the equilibrium points  $L_{6,7}$  exist for several values of  $\beta_1 > 1$  for  $X_0 < 0$  in both the binary systems in the  $X-Z$

plane. We also find that for  $|Q_1| < |Q_2|$ , there exists a range of  $\beta_1$  for which it is also possible to have four equilibrium points  $L_6, L_7, L_8$  and  $L_9$  for  $X_0 > 0$  in both the binaries. Increase in  $\beta_1$  results in an increase in  $L_{6_x}$  while  $L_{8_x}$  tends to decrease in the region  $X_0 > 0$  for both the binary systems in the absence of PR-drag. Further in the absence of PR-drag in both binary systems the components  $L_{8_z}$  and  $L_{6_z}$  show a decreasing trend with increase in  $L_{8_x}$  and  $L_{6_x}$  respectively. Similar trends are observed when PR-drag is incorporated in the analysis. However, inclusion of PR-drag in the analysis results in a small but finite  $y$ -component to the possible equilibrium points of the system. It is observed that both  $L_{8_y}$  and  $L_{6_y}$  increase with increase in  $L_{8_x}$  and  $L_{6_x}$ , respectively in both the binaries.

### References

- Chernikov, Yu. A. 1970, *Sov. Astronomy*, **14**, 176.  
 Das, M. K., Narang, P., Yuasa, M., Saha, L. M. 2008a, *Astrophys. Space Sci.*, **314**, 59.  
 Das, M. K., Narang, P., Mahajan, S., Yuasa, M. 2008b, *Astrophys. Space Sci.*, **314**, 261.  
 Hénon, M. 1983, Les Houches, Session XXXVI, p 54.  
 Kimura, H., Okamoto, H., Mukai, T. 2002, *Icarus*, **157**, 349.  
 Krivov, A., Kimura, H., Mann, I. 1998, *Icarus*, **134**, 311.  
 Kunitsyn, A. L., Chudayeva, A. M. 2003, *J. Appl. Maths. Mechs.*, **67**, 647.  
 Kushvah, B. S., Ishwar, B. 2004, *Bull. Cal. Math. Soc.*, **12**, 109.  
 Lukyanov, L. G. 1984, *Sov. Astron.*, **28**, 462.  
 Murray, C. D. 1994, *Icarus*, **112**, 465.  
 Perezhogin, A. A. 1976, *Sov. Astron. Lett.*, **2**, 174.  
 Radzievskii, V. V. 1950, *Astron. Zh.*, **27**, 250.  
 Radzievskii, V. V. 1953, *Astron. Zh.*, **30**, 265.  
 Ragos, O., Zafiroopoulos, F. A. 1995, *Astron. Astrophys.*, **300**, 568.  
 Ragos, O., Zafiroopoulos, F. A., Vrahatis, M. N. 1995, *Astron. Astrophys.*, **300**, 579.  
 Ragos, O., Zagourous, C. 1988, *CMeDA*, **44**, 135.  
 Robertson, H. P. 1937, *Mon. Not. Roy. Astron. Soc.*, **97**, 423.  
 Roman, R. 2001, *Astrophys. Space Sci.*, **275**, 425.  
 Schuerman, D. W. 1980, *ApJ*, **238**, 337.  
 Simmons, J. F. L., McDonald, A. J. C., Brown, J. C. 1985, *Cm*, **35**, 145.  
 Szebeheley, V. 1967, *Theory of Orbits*, Academic Press.  
 Wilck, M., Mann, I. 1996, *Planet. Space Sci.*, **44**, 493.  
 Zhou, H. N., Leung, K. C. 1988, *Astrophys. Space Sci.*, **141**, 257.