

## Observational Evidences for Multi-component Magnetic Field Structure in Solar Flares

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**Abstract.** Two solar flares of 25 July 1981 and 5 November 2004 of importance 2N and M4.1/1B, respectively, were investigated using observational data obtained with the Echelle spectrograph of the Kyiv University Astronomical Observatory. Stokes  $I$  and  $V$  profiles of the FeI lines 5233, 5247.1, 5250.2, 5250.6, 5576.1 and of CrI 5247.6 Å have been analyzed. We found several evidences for the existence of spatially unresolved magnetic field structures with kG strengths. In particular, the values of the measured average longitudinal field  $B_{\parallel}$  depend on the Lande factors  $g$  of the lines: in general,  $B_{\parallel}$  increases with increasing factor  $g$ . Analogously, the observed line ratio  $B_{\parallel}(5250.2)/B_{\parallel}(5247.1)$  is increasing with increasing distance  $\Delta\lambda$  from the line center. The observed Stokes  $V$  profiles show some deviations from that of an assumed homogeneous field, presented by the Stokes  $I$  gradient,  $dI/d\lambda$ . A comparison with the non-split line FeI 5576.1 Å shows that some of these deviations are real and indicate the presence of subtelescopic magnetic elements with discrete field strengths of several kG. The lines with large Lande factors have considerable broadenings of the Stokes  $I$  profiles, indicating a strong background magnetic field of mixed polarity. On the basis of all these data we conclude that a four-component magnetic field structure is a possible explanation. The field strengths are about  $\pm 1.05$  kG in the background field, and 1.3–1.5, 3.9–4.0, and 7.4–7.8 kG at level of middle photosphere ( $h \approx 300$  km) in the spatially unresolved, small-scale magnetic elements.

*Key words.* Solar flares—small-scale magnetic fields—multi-component structure.

### 1. Introduction

Solar flares are closely connected to magnetic fields and rapid reconnection of magnetic fields plays a key role in these events (see, e.g., Severny 1988; Priest 1986). However, Parker (2001) emphasized that “we do not understand why the magnetic field is in the intensely fibril state that appears at the surface”, and “the conditions for

reconnection between fibrils necessarily becomes involved with the dynamical state of the fibrils themselves, about which little is known . . .". Really, detailed magnetic characteristics of the flare volume are practically unknown due to essential problems with the magnetic field measurements in solar active regions and flares. This problem has been discussed by many authors, e.g., by Lozitska & Lozitsky (1982), Moore *et al.* (1984), and Staude & Hofmann (1988). It is due to the strong dependence of some parameters of the used magneto-sensitive spectral lines on both magnetic field structure and thermodynamic conditions. Some types of instruments, e.g., the traditional magnetographs of the Babcock (1953) type, are practically useless to measure reliably the magnetic field in flare, in the case of strong flare emission with essential profile changes in the line core in particular. On the other hand, instruments such as Fourier transform spectrometers (Brault 1978) are more suitable for this case, but they are too slow to investigate such rapid phenomena as flares. A successful investigation of a flare using such an instrument was made, but in one spectral line only (Deming *et al.* 1990). However, data of many spectral lines are needed to determine the true magnetic structure in flares. Traditional spectral observations using analyzers for circularly polarized light are also suitable for this purpose. Though such data are inferior to those of a Fourier transform spectrometer in sensitivity and precision, they allow faster recording of a wide spectral range, which is of basic importance for flares.

There exist spectral observations pointing out the small-scale magnetic field structure in flares. A two-component flare emission in metal lines has been found by Lozitska & Lozitsky (1982, 1988). The whole flare emission could be easily divided into polarized and non-polarized components. The relation between the equivalent widths of such emission and the measured magnetic field indicated the existence of many subtelescopic flux-tubes in the flare, which rapidly expanded with height. Qualitatively, such a field structure agrees with that derived by Keller *et al.* (1990) using an inversion of Stokes profiles from a plage and a network region. A more complicated case was found by Lozitsky *et al.* (2000) in another flare. A two-component field structure with a non-monotonous height distribution of the field strength in the atmosphere was discovered. A field maximum of 4 kG was found in the upper chromosphere or the temperature minimum. Penn & Kuhn (1995) observed the chromospheric line HeI 10830 Å in three flare kernels and found an emission line profile and a mean magnetic field of 0.74 kG; the exact height of formation of this line is unknown.

In this paper, we make a comparative study of two solar flares using several methods which could give some information about the local magnetic field strength at the photospheric level. A first study of that type was presented at the Polarization Workshop 3 and published in a short paper by Lozitsky & Staude (2003).

## 2. Observational data and selected lines

The observational data were obtained with the Echelle spectrograph of the horizontal solar telescope of the Kyiv University Astronomical Observatory (Kurochka *et al.* 1980). This instrument can record the solar spectrum simultaneously from 3800 to 6600 Å with a spectral resolution of nearly 200,000 in the green region and with a temporal resolution of about several seconds.

**Table 1.** List of selected spectral lines.

Number	Wavelength (Å)	Element and multiplet number	Equivalent width $W$ (mÅ) Rowland table	Excitation potential (eV)	Effective Lande factor $g_{\text{eff}}$	Height of formation (km)
1	5232.946	FeI-383	346	2.94	1.261	–
2	5247.052	FeI-1	59	0.09	1.998	328
3	5247.564	CrI-18	76	0.96	2.50	308
4	5250.212	FeI-1	62	0.12	2.999	324
5	5250.650	FeI-66	104	2.20	1.502	330
6	5576.097	FeI-686	113	3.42	–0.012	–

The flare of 25 July 1981 had the coordinates  $11^\circ\text{S}$ ,  $36^\circ\text{E}$  and the importance  $2N$ . Three photographic Zeeman spectrograms were obtained with a circular polarization analyzer at 12:48, 12:58, and 13:25 UT. The first spectrogram refers to the phase near the flare peak and the last one to the end of the flare. All spectrograms refer to the same position on the Sun where the heliocentric angle was  $\mu \approx 0.77$ . For all spectrograms the entrance slit of the spectrograph was crossing the position of two bright flare knots outside the large spot in an area of magnetic fields with both  $S$ - and  $N$ -polarities. In the analysis presented below, we shall discuss the place with the strongest flare emission with a diameter of about 3000 km and magnetic  $N$  polarity.

The second flare of 5 November 2004 occurred in AR 10696 at  $\mu \approx 1$  with an importance of M4.1/1B. Six Zeeman spectrograms were obtained between 11:35:35 and 11:49:00 UT. In the present study we analyze the spectra of 11:36:55 UT corresponding to the peak of the flare, considering a place in the tail part of the delta configuration, close to the sunspot of  $N$  polarity with a field strength of 2500 G in the umbra. All spectra were made with ORWO WP3 photo-emulsion.

Six spectral lines were used for the magnetic field diagnostics (Table 1). The lines, nos. 2–5 are well known, they are formed in the narrow range of middle photosphere – at a height of  $h = 308\text{--}330$  km (Gurtovenko & Kostik 1989). This circumstance is important for small-scale magnetic field diagnostics (Stenflo 1973). In this connection, it is important and useful that these lines have Lande factors in a wide range from 1.5 to 3.0, with discrete step of about 0.5. In addition, their temperature sensitivities are also mutually close that follows from relatively close values of excitation potentials (0.09–2.20 eV).

For flare-active regions, values of formations can be somewhat different, but we do not expect strong shifts of the height of line formation because in the lower and middle photosphere, the thermodynamic conditions for flares of importance 1 and 2 are practically similar to those in the undisturbed atmosphere (Baranovsky *et al.* 1991; Lozitsky & Baranovsky 1993).

The spectral line FeI 5232.9 is much stronger than the other lines, its core is formed near the temperature minimum ( $h \approx 500$  km), although the formation height of the whole line is close to that of the other lines, nos. 2–5. We add this line to check some fine spectral effects discovered first in the other magneto-sensitive lines. Analogously, we have investigated the FeI 5576.1 line to study the non-magnetic effects in the line profiles. Its level of formation is also close to lines nos. 2–5.

The effective Lande factors  $g_{\text{eff}}$  for all lines excluding line no. 3 have already been determined in the laboratory (Zemanek & Stefanov 1976; Landi Degl’Innocenti 1982),

which is important for reliable magnetic field measurements (Stenflo *et al.* 1984). The value of  $g_{\text{eff}}$  for line no. 3 in Table 1 corresponds to the theoretical case of LS coupling.

### 3. Magnetic field diagnostics

#### 3.1 Averaged longitudinal magnetic field $B_{\parallel}$

The simplest test of the presence of unresolved magnetic fields consists of a comparison of the longitudinal magnetic fields  $B_{\parallel}$  measured in several spectral lines with different Lande factors but formed practically at the same depths and having similar temperature sensitivities. Such magnetic field strengths  $B_{\parallel}$  averaged over the aperture can be measured as the relative shift of the ‘centers of gravity’ of the  $I + V$  and  $I - V$  profiles. The physical meaning of this parameter is close to that of the longitudinal magnetic flux, when  $\Delta\lambda_H/\Delta\lambda_D \ll 1$ , where  $\Delta\lambda_H$  is the Zeeman splitting and  $\Delta\lambda_D$ , the Doppler half width. If the magnetic flux is intermittent in the form of flux-tubes with a field strength  $B$  and a filling factor  $\alpha$ , then:

$$B_{\parallel} \approx \alpha B \cos \gamma, \quad (1)$$

where  $\gamma$  is the angle of field inclination with respect to the line-of-sight.

If the flux-tube magnetic field is very strong,  $\Delta\lambda_H/\Delta\lambda_D \geq 1$ , then the observed field,  $B_{\text{obs}}$  is

$$B_{\text{obs}} < B_{\parallel} = \alpha B \cos \gamma, \quad (2)$$

due to the magnetic saturation effect (Howard & Stenflo 1972). Exactly,  $B_{\text{obs}} < B_{\parallel}$  is real only in the case of measurements with solar magnetograph which have fixed position of the exit slits. On the contrary, if we measure using photographic method (as in the present work), we can obtain  $B_{\text{obs}} = B$  due to the direct observations of fully-splitting Zeeman components.

Our spectral measurements in the lines, nos. 2–5 for flares are given in Fig. 1 together with similar data for weak magnetic fluxes,  $B_{\parallel} < 100\text{--}200$  G (Gopasyuk *et al.* 1973) and with spectral data for moderate fields ( $B_{\parallel} \approx 500$  G) in an active region outside flares (Lozitsky 2003). We can see very different dependences: in general, increasing relative fields  $B(g_i)/B(g = 3.0)$  with decreasing Lande factors in first and second cases, but opposite tendency – decreasing relative values  $B(g_i)/B(g = 3.0)$  with decreasing Lande factors – in the case of both flares. Note that measured magnetic fluxes in both flares were also moderate, in the range of 250–700 G.

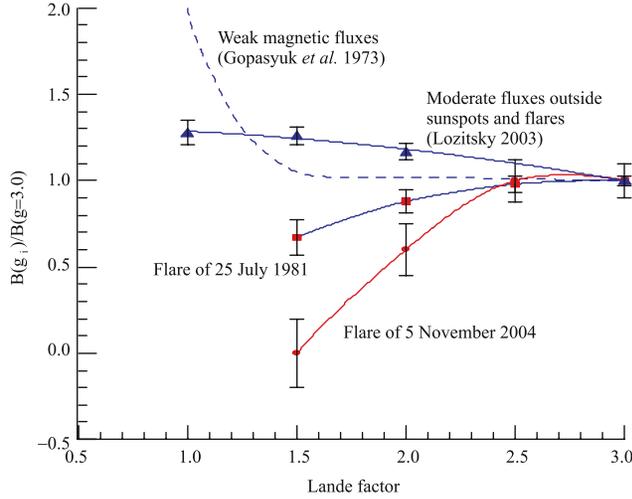
Beginning the interpretation of results presented in Fig. 1, let us put the question: which shape of dependence “ $B_{\parallel}$  versus  $g$ ” must be in a case of weak or moderate (<500 G) homogeneous field?

In this case, Stokes  $V$  parameter could be presented *via* a formula

$$V \propto \left( \frac{dI}{d\lambda} \right) \Delta\lambda_H, \quad (3)$$

where  $dI/d\lambda$  is the Stokes  $I$  gradient and  $\Delta\lambda_H$  is Zeeman splitting.

$$\Delta\lambda_H = 4.67 \times 10^{-13} g\lambda^2 B, \quad (4)$$



**Figure 1.** Comparison of the relative observed magnetic field values  $B(g_i)/B(g = 3.0)$  for weak magnetic fluxes ( $B_{\parallel} < 100\text{--}200$  G) from magnetographic measurements (Gopasyuk *et al.* 1973), for moderate fluxes ( $B_{\parallel} \approx 500$  G) measured in active regions outside flares (Lozitsky 2003), and for studied flares.

where  $\Delta\lambda_H$  and  $\lambda$  are in  $\text{\AA}$ , and  $B$  in gauss (G). In practice, measured magnetic field strength  $B_{\parallel}$  can be found as:

$$B_{\parallel} \propto \frac{V}{g}. \quad (5)$$

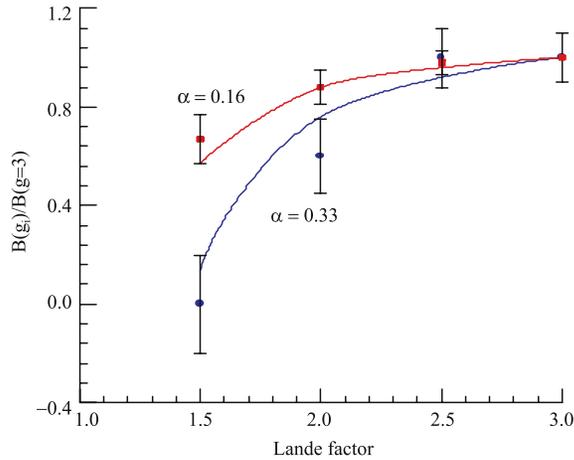
From equation (3)–(5) it follows that *in case of weak or moderate field, values of  $B_{\parallel}$  do not depend on Lande factor  $g$ .*

Second possible situation is the following: two-component field contains weak background field and small-scale moderate field. In this case, observed Stokes  $V_{\text{obs}}$  picture forms as a result of interflowing two Stokes  $V$  ‘waves’: first from background field  $V_{\text{backgr}}$  and the second from small scale (perhaps flux-tube) field  $V_{\text{fluxtube}}$

$$V_{\text{obs}} = V_{\text{backgr}} + V_{\text{fluxtube}}. \quad (6)$$

Both these ‘waves’ have very similar spectral distributions, but different amplitudes. Amplitude of each ‘wave’ is proportional to filling factor of each magnetic component, and also Lande factor  $g$  of line. If we use different spectral lines with different Lande factors, measured value of  $B_{\parallel}$  should be, as first approximation, *the same for different lines, independently from Lande factor*. In this case we can expect  $B(g_i)/B(g = 3.0) \approx 1$  in Fig. 1.

The third possible situation: two-component structure, with weak or moderate background field and small-scale (spatially unresolved) strong ( $>1$  kG) field. In this case, position of Stokes  $V$  peaks of background field,  $V_{\text{backgr}}$ , is practically the same for lines with different Lande factors. It corresponds, in general, to peaks of Stokes  $I$  gradient distribution,  $dI/d\lambda$ . On the contrary, position of Stokes  $V$  peaks of strong small-scale field,  $V_{\text{flux-tube}}$ , must be very different and should be close to the Zeeman splitting  $\Delta\lambda_H$  in the second component. For spectral lines with high Lande factors, e.g.,



**Figure 2.** Comparison of the observational and theoretical values of  $B(g_i)/B(g = 3.0)$  versus Lande factor for both flares. Squares and filled circles present observations for flares of 25 July 1981 and 5 November 2004, respectively; solid curves – theory for different filling factor (see text).

$g = 2.5$ – $3.0$ , two named Stokes  $V$  waves are mutually ‘splitted’, i.e., non-blended. If we use another line with some smaller Lande factor, for instance,  $g = 2.0$ , the Stokes  $V_{\text{flux-tube}}$  wave begins to blend far wings of first Stokes  $V$  wave, and it leads to change of measured magnetic field strength  $B_{\parallel}$ . If magnetic polarities in both components are the same, we should obtain  $B_{\parallel}(g = 2) > B_{\parallel}(g = 2.5$ – $3)$ . On the contrary, if these polarities are opposite, we obtain  $B_{\parallel}(g = 2) < B_{\parallel}(g = 2.5$ – $3)$  due to mutual cancellation of circular polarization of opposite signs. We can expect analogous results also in case, if these polarities are the same, but Zeeman manifestations are different; absorption type for one component and emission for second one. Also, we can expect the maximum effect of  $B_{\parallel}$  increasing (or decreasing) in the case when both above-mentioned Stokes  $V$  waves interflow fully, namely, in lines with very small Lande factors.

Figure 2 presents a comparison of our observations with calculations. A simple two-component magnetic model was assumed, with a field strength of  $B_{\text{backgr}} = +500$  G in the first background component, and  $B_{\text{fluxtube}} = -4000$  G in the second small-scale one. The filling factors of these components are  $\alpha_1$  and  $\alpha_2$ , respectively, where  $\alpha_1 + \alpha_2 = 1$ . The Stokes  $I$  and  $V$  profiles were calculated for a Milne-Eddington atmosphere model using Unno’s (1956) parameters  $\eta_0$  and  $\Delta\lambda_D$  close to those of a line such as FeI 5250.2, that is 2.5 and 40 mÅ, respectively. It was also assumed that in the area of the strong field component the Doppler half width of the line profiles is about 30% smaller than that in the first background field component. A similar effect of narrow line profiles has been observed earlier, e.g., by Lozitsky (1980) and Lozitzka & Lozitsky (1982). Magneto-optical effects were neglected because these effects are important in the central parts of the profiles only, i.e., for distances of less than 40 mÅ from the line centers (Lozitsky & Sheminova 1992). However, the most interesting effects for our study occur in the far line wings.

We can see in Fig. 2 that a filling factor of  $\alpha_2 \approx 0.16$  is fitting the first flare, and  $\alpha_2 \approx 0.33$  to the second one. Thus, the calculations give indications of roughly the same

magnetic field strength of  $B_2 \approx -4000$  G in the subtelescopic structures of both flares, but essentially different (about 2 times) filling factors. Note that the accuracy of our simple calculations is about 5–10%, and thus we neglect possible fine differences for these flares.

So, we can conclude that the described simple test based on  $B_{\parallel}(\lambda_i)$  values for different Lande factors gives an observational evidence to two possibilities:

- possible presence of strong unresolved magnetic fields of opposite polarity and
- presence of strong unresolved fields of same polarity, but in spectral manifestation of the emissive Zeeman effect.

Note that the last case was not visible directly: all investigated Stokes  $I \pm V$  profiles (for all lines listed in Table 1) were smooth, without emission peaks in their cores. If possible emission features in profiles are really existing, ones should have small intensities due to likely small filling factor.

### 3.2 The 5250.2/5247.1 line ratio

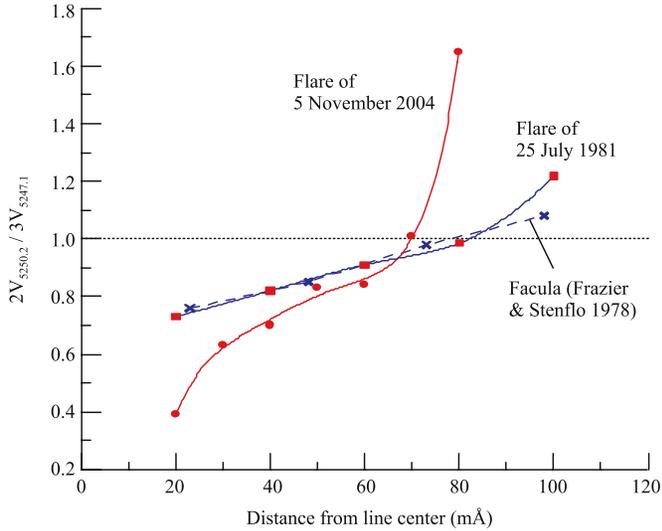
The magneto-sensitive lines FeI 5250.2 and 5247.1 have practically the same heights of formation and of temperature sensitivity, which is especially important for small-scale magnetic field diagnostics. If subtelescopic magnetic kG fields really exist, we can expect non-constant line ratio  $B_{\parallel}(5250.2)/B_{\parallel}(5247.1)$  at different distances,  $\Delta\lambda$ , from line center (Stenflo 1973). Because these lines have practically the same Stokes  $I$  profiles, instead of ratio  $B_{\parallel}(5250.2)/B_{\parallel}(5247.1)$  we can consider as equivalent Stokes  $V$  ratio  $2V_{5250.2}/3V_{5247.1}$  (Stenflo *et al.* 1987). As to theory, we can expect  $2V_{5250.2}/3V_{5247.1} \approx 1$  in case of weak or moderate fields inside the aperture. On the contrary, if this ratio is far from unity, the magnetic fields should be strong.

Figure 3 displays our results for the flares in comparison with the analogous data for solar faculae by Frazier and Stenflo (1978). We can see surprisingly good agreement in both data for faculae and flare of 25 July 1981, but essential difference named data *versus* data for flare of 5 November 2004 (typical error  $\pm 0.05$ ).

So long as our results for flare of 25 July 1981 are very similar to the one for faculae, we can use the theoretical model by Frazier & Stenflo (1978) for the interpretation of our data. Frazier & Stenflo concluded that as pointed out in Fig. 2, dependence for faculae could indicate the presence of small-scale magnetic fields of 2.0 kG strength assuming non-rectangular field profile. In the case of rectangular profile magnetic field, strengths should be less in 1.5–2 times (Stenflo 1973; Lozitsky & Thap 1989).

As to flare of 5 November 2004, we can expect also kG fields in small-scale features, as it follows from comparison of these observations with theoretical dependences published in a paper by Rachkovsky *et al.* (2005). However, true amplitude of small-scale magnetic field should be here on about 300–500 G more, as at first flare.

The third test of small-scale magnetic fields is based on FeI 5247.1 and 5250.2 lines too. The particular drawback of these lines is their relatively high temperature sensitivity (Harvey & Livingston 1969; Staude 1970a, b). Lozitska & Lozitsky (1994) have shown that the temperature sensitivity of the pair 5250.2/5247.1 is not an obstacle for reliable magnetic field measurements if we do not use the Stokes  $V$  amplitude (such as in a solar magnetograph), but the Stokes  $V$  peak separation,  $\Delta\lambda_V$ . Also, the Stokes  $V$  peak half-width,  $\Delta\lambda_{1/2,V}$  and the Stokes  $I$  half-width,  $\Delta\lambda_{1/2,I}$ , could be successfully used for small-scale magnetic field diagnostics.



**Figure 3.** Comparison of Stokes  $V$  ratio  $2V_{5250.2}/3V_{5247.1}$  in FeI 5250.2 and FeI 5247.1 lines for the flares (present study) and faculae (Frazier & Stenflo 1978). We can see that, in general,  $2V_{5250.2}/3V_{5247.1} \neq 1$  and this circumstance indicates the presence of small-scale kG fields in solar flares.

**Table 2.** Observed Stokes  $I$  half-width, Stokes  $V$  peak separation, and Stokes  $V$  peak half-width.

Line	$\Delta\lambda_{1/2,I}$	$\Delta\lambda_V$	$\Delta\lambda_{1/2,V}$
FeI 5247.052	133	106	79
FeI 5250.212	161	126	80

The observed values of the mentioned parameters (in mÅ) for the flare of 25 July 1981 are given in Table 2.

Let us consider a simple model according to which small-scale flux-tubes with a field strength,  $B_f$  are embedded in a background field,  $B_b$ . The calculation scheme for such a model was described in detail by Lozitska & Lozitsky (1994) and Lozitsky *et al.* (2000).

If we assume a relative absorption coefficient,  $\eta_0$ , close to 2.5 and an angle of inclination with respect to the line of sight,  $\gamma$ , between  $0^\circ$  and  $90^\circ$ , then from the ratio

$$\frac{\Delta\lambda_{1/2,I}(5250.2)}{\Delta\lambda_{1/2,I}(5247.1)} = 1.21, \quad (7)$$

follows  $\Delta\lambda_D = 50$  mÅ,  $\Delta\lambda_H = 40$  mÅ for  $\gamma = 0^\circ$ , and  $\Delta\lambda_D = 48$  mÅ,  $\Delta\lambda_H = 50$  mÅ for  $\gamma = 90^\circ$ . The first case corresponds to  $B_b = \pm 1.05$  kG, and the second one to  $B_b = 1.3$  kG. The first case means that we observe a dispersion of field directions, with spatially unresolved contacts of background magnetic elements of both ( $N$  and  $S$ ) polarities. We can compare the derived values with the typical dispersion of about  $\pm 0.5$  kG for non-spot and non-flare regions (Harvey *et al.* 1972), and  $\pm 0.9$  kG for a 2B-flare (Lozitsky *et al.* 2000).

Taking into account the Stokes  $V$  peak ratio

$$\frac{\Delta\lambda_V(5250.2)}{\Delta\lambda_V(5247.1)} = 1.19, \quad (8)$$

we have  $B_f = 1.45$  kG, if  $\gamma = 45^\circ$ , i.e., close to the heliocentric distance for the studied flare. It is interesting to note that the derived value of  $B_f = 1.45$  kG is close to the value  $B_1 \approx 1.3$  kG obtained from the  $(BS - \Delta\lambda/g\lambda^2)$  diagram (see sections 3.5 and 3.6).

However, the Stokes  $V$  peak half-widths,  $\Delta\lambda_{1/2,V}$ , could not be explained on the basis of such a two-component model. Theoretically, we have  $\Delta\lambda_{1/2,V} = 76.5$  mÅ for FeI 5250.2, but 80 mÅ from observations. For the FeI 5247.1 line, the theoretical value is 67 mÅ, whereas the observed one is 79 mÅ. This difference (12 mÅ) is too large and needs some modification of the present model. One possible solution is an additional third magnetic field component which broadens the Stokes  $V$  peak. In fact, if the Zeeman splitting  $\Delta\lambda_H$  in such a third component is close to 150 mÅ (i.e.,  $B \approx 3.9$  kG for FeI 5250.2), then for FeI 5247.1 the same field strength products  $\Delta\lambda_H \approx 100$  mÅ due to the Lande factor ratio of 1.5. So long as  $\Delta\lambda_H \approx 100$  mÅ corresponds to the middle level of the Stokes  $V$  peak for lines like FeI 5250.2, we can expect an essential influence on the peak extension, in this case. On the contrary, the value of  $\Delta\lambda_H \approx 150$  mÅ corresponds to the Stokes  $V$  wing (no middle level), that would not cause an increase of the  $\Delta\lambda_{1/2,V}$  value.

Thus, the Stokes  $V$  peak half-width data give evidence of the existence of *more than two* structure components in the flare magnetic field.

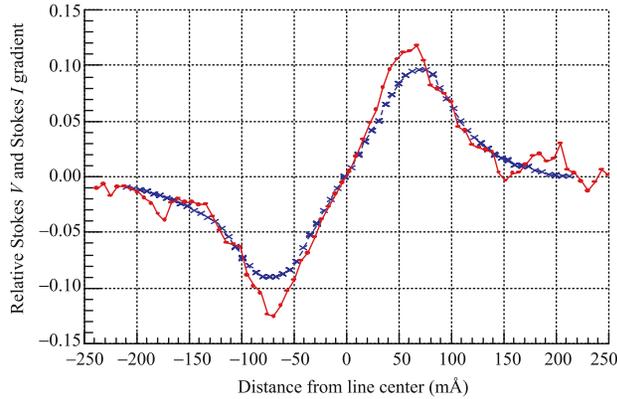
### 3.3 Stokes $V$ profiles

The fourth test of small-scale fields is based on the study of Stokes  $V$  profiles too. As it was pointed above (see equation 3), for really weak magnetic field ( $\Delta\lambda_H/\Delta\lambda_D \ll 1$ ) we can expect that  $V \propto (dI/d\lambda)\Delta\lambda_H$ . From this equation, it follows that the spectral distributions of the Stokes  $V$  parameter and the Stokes  $I$  gradient  $dI/d\lambda$  coincide, if the magnetic field is really weak (less than 1 kG).

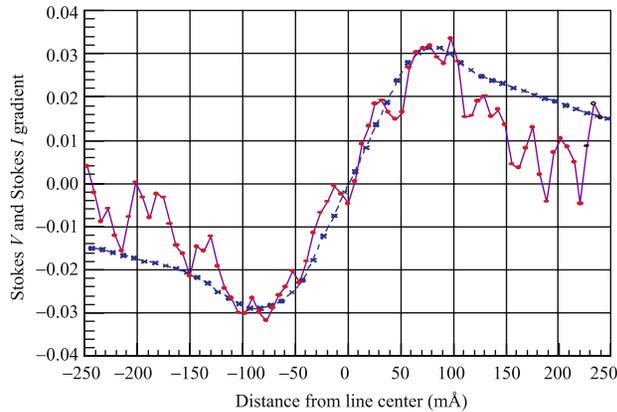
Some of the observed cases are given in Figs. 4 and 5. We can see some deviations between  $V$  and  $dI/d\lambda$  which have the tendency to be placed almost symmetrical to the line centers. Of course, this could be by chance, and we must check the noise level to obtain a more reliable conclusion.

To estimate these effects, we made photometry of spectral continuum in the range of 5248.5–5248.8 Å and found that typical intensity fluctuations connected with grain of photo-emulsion is relatively weak, average of 1%. So, we can expect that differences between  $V$  and  $dI/d\lambda$  exceeding 1% could be real.

On the other hand, some details presented in Figs. 3 and 4 cover a spectral range of about 30–50 mÅ, i.e., they are close to instrumental fluctuations. In principle, this does not exclude the reality of the effects under discussion. Lozitsky (1980) discovered similar narrow spectral features in an active region outside the flares using many different magneto-sensitive lines. Lozitsky *et al.* (1999) observed in a solar flare relatively weak (5–10%) emission peaks in FeI line cores which had observed half-widths of about 30–40 mÅ, but after reduction for instrumental broadening, it was about 10–20 mÅ only! Obviously, the lower limit of the half-widths of spectral lines in volumes with



**Figure 4.** Observed Stokes  $V$  profile (solid line) and Stokes  $I$  gradient  $dI/d\lambda$  (crosses and dashed line) for the FeI 5250.6 line in the flare. One can see the red–blue asymmetry of Stokes  $V$  and some differences between the parameters, in particular at  $\Delta\lambda = \pm 70$  and at  $\pm (180\text{--}200)$  mÅ.



**Figure 5.** Observed Stokes  $V$  profile (solid line) and Stokes  $I$  gradient (crosses and dashed line) for FeI 5233 line in the flare of 25 July 1981.

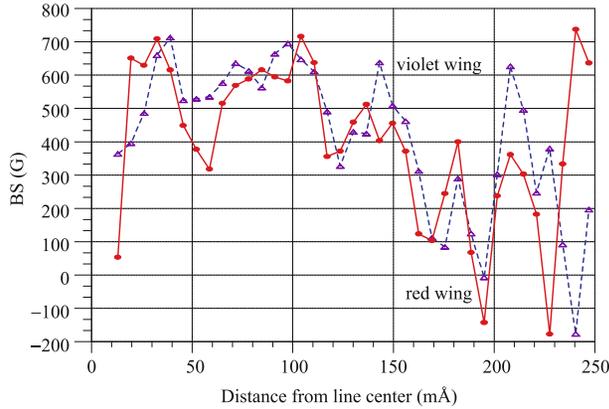
very strong and small-scale fields is practically unknown at present, and therefore we could continue the analysis for obtaining more clear conclusions.

### 3.4 Bisector splitting

For the flares and spectrograms under study, there is only one possible way to intensify the suspected magnetic effects in the line profiles. We could investigate a spectral line with a larger equivalent width  $W$ . In this case, the possible ‘magnetic’ spectral features should be more intensive, whereas the non-magnetic fluctuations are unchanged. A suitable line is FeI 5233, which has  $W = 346$  mÅ.

In addition, it is interesting to study the ratio

$$BS = \frac{V}{dI/d\lambda}. \quad (9)$$



**Figure 6.** Comparison of bisector splitting  $BS$  for violet (dashed) and red (solid) wings of FeI 5232.9 line for the flare of 25 July 1981. We can see the close correlation between named values, especially for distances less than 230 mÅ. The correlation coefficient  $r$  is 0.72, and the probability  $p$  of an accidental connection is 0.001.

This parameter  $BS$ , the ‘bisector splitting’, is similar to the Zeeman splitting  $\Delta\lambda_H$  for any given distance from the line center  $\Delta\lambda$ , see equation (3).

Figure 6 displays an example of the observed  $BS - \Delta\lambda$  distribution for the flare of 25 July 1981. We can see strong fluctuations of  $BS$  which display the tendency to be disposed by pairs with respect to the line center. In particular, there are pairs of maxima at  $\Delta\lambda = \pm (30-40)$ ,  $\pm 100$  and perhaps  $\pm 140$  mÅ. The minima of  $BS$  are disposed at  $\Delta\lambda = \pm 55$  and at  $\pm (120-130)$  mÅ.

Analogous effects were found in other lines, and in the flare of 5 November 2004 too. It is important to note that such effects are impossible in the case of weak or moderate field. If the magnetic field strength really equals 700 G, we can expect

$$BS(\Delta\lambda) \approx \text{const.} \quad (10)$$

for all investigated lines. Of course, in this case there should be no pairs of spectral features, and all fluctuations should be irregular.

### 3.5 ( $BS - \Delta\lambda/g\lambda^2$ ) diagram

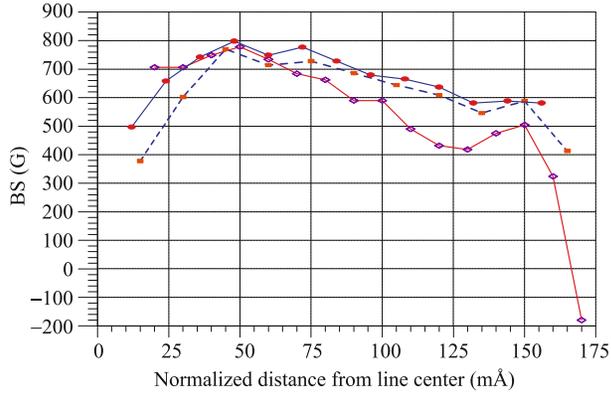
So long as the values of  $BS$  for the red and violet wings are mutually correlated, we can calculate the average values of  $BS$  for both wings of each line. In addition, if each statistically reliable maximum in the ( $BS - \Delta\lambda$ ) diagram disposed at a distance of  $\Delta\lambda_{\max}$  indicates a Zeeman  $\sigma$ -component related to a small-scale magnetic feature, then

$$\Delta\lambda_{\max} = \Delta\lambda_H = 4.67 \times 10^{-13} g\lambda^2 B, \quad (11)$$

where  $\Delta\lambda_H$  and  $\lambda$  are in mÅ, and  $B$  in gauss (G).

From equation (11) it follows for  $B = \text{const.}$ , that

$$\frac{\Delta\lambda_{\max}}{g\lambda^2} = \text{const.} \quad (12)$$



**Figure 7.** Bisector splitting  $BS$  versus the value  $82.7\Delta\lambda/g\lambda^2$  for three lines: FeI 5250.2 (rhombs and solid line), FeI 5247.1 (dashed line), and CrI 5247.6 (filled circles and solid line).

The criterion (12) can be used additionally to verify the magnetic nature of the spectral peculiarities (Lozitsky 1980).

A typical example of the observed  $(BS - \Delta\lambda/g\lambda^2)$  diagram for flare of 25 July 1981 is shown in Fig. 7. All  $BS$  values were calculated for normalized distances from the line center,  $\Delta\lambda_n$ , according to

$$\Delta\lambda_n = \frac{82.7\Delta\lambda}{g\lambda^2}. \quad (13)$$

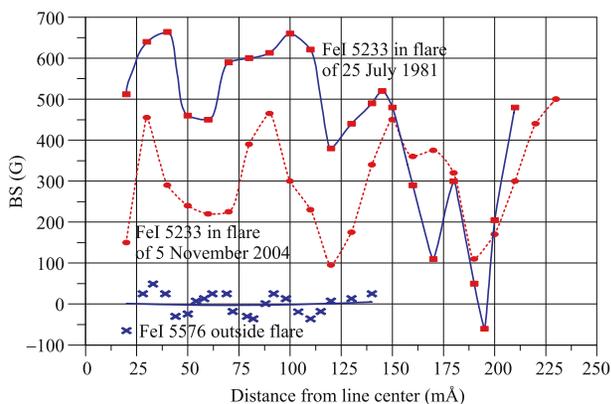
So long as the factor  $g\lambda^2 = 82.7 \times 10^6$  corresponds to the FeI 5250.2 line, all data were reduced to the case of that line.

We can see a good agreement for the three lines, nos. 2, 3, and 4 with the largest Lande factors (Fig. 6). The first common maximum corresponds to  $\Delta\lambda_{\max}(1) \approx 50 \text{ m}\text{\AA}$ , and the second one to  $\Delta\lambda_{\max}(2) \approx 150 \text{ m}\text{\AA}$ . The FeI 5250.6 line with Lande factor  $g = 1.5$  has also a similar distribution of bisector splitting, with deep minima on  $300 \text{ m}\text{\AA}$  of normalized distance from line center. This minima, reduced to the case of FeI 5233 line and non-normalized distance, corresponds to

$$\Delta\lambda_{\min}(5233) = 300 \times \left( \frac{1.261}{2.999} \right) \approx 126 (\text{m}\text{\AA}). \quad (14)$$

(Note that here values of 1.261 and 2.999 are Lande factors for FeI 5233 and FeI 5250.2 lines.) As to reality, one can see from Fig. 6, that FeI 5233 line, in fact, has  $BS$  minimum on close distance  $\approx 120 \text{ m}\text{\AA}$ . Thus, we can conclude that  $BS$  distributions for different lines are in satisfactory agreement, and this circumstance gives the evidence to the magnetic nature of common extremums in Fig. 7.

An additional argument to this point of view follows from the comparison of data for both flares (Fig. 8). We can see that two investigated flares give very similar and strong  $BS$  distributions, which are obviously greater than possible non-magnetic effects observed in ‘non-split’ FeI 5576 line. In the last case, it was formally supposed that its Lande factor is also 1.261, as in FeI 5233 line (really, it is  $-0.012$ , i.e., in  $\approx 100$  times less). One can see the close correlation between  $BS$  distributions for both



**Figure 8.** Comparison of bisector splitting in FeI 5233 for flares of 25 July 1981 (solid) and 5 November 2004 (dashed). For estimation of the possible instrumental effects, the equivalent  $BS$  fluctuations in ‘non-split’ FeI 5576 line (crosses) are presented too (see text).

flares, and common minima on the following distances  $\Delta\lambda = 60, 120$  and perhaps  $190 \text{ m}\text{\AA}$ .

Comparison of FeI 5233 and FeI 5576 indicates that at least two first minima on distances  $\Delta\lambda = 60, 120 \text{ m}\text{\AA}$  should be real and have no magnetic nature. In fact, possible standard deviation of measurements should be on a level about  $50 \text{ G}$ , as it follows from FeI 5576 line. But observed  $BS$  fluctuations in FeI 5233 reach out of  $200\text{--}300 \text{ G}$ , i.e., they are much stronger. So, we do not find any non-magnetic cause which could give similar perturbations.

### 3.6 Possible magnetic field strengths

In general, to obtain the true magnetic field strengths in unresolved magnetic structures, the magnetic polarity and type of the Zeeman effect, namely emission and absorption, should be made out. For each diagnostic diagram as shown in Fig. 7, we cannot conclude which type of extremum (minimum or maximum?) corresponds to the position of contribution of the Zeeman sigma-components in the spectra.

On the contrary, we can attempt a more reliable conclusion about the presence of *several* types of magnetic features having *different* magnetic field strengths. Also, we can point possible magnetic field range observing the spectral range of normalized distance from line center (see Fig. 7) where the correlated fluctuations of  $BS$  parameter were measured.

In principle, the determination of the magnetic polarity and type of Zeeman effect in such a complicated case could be determined from the comparison of Stokes  $V$  and Stokes  $I$  distribution. In particular, if we really observe the Zeeman effect in absorption for the same ( $N$ ) polarity, we should discover very weak depression in Stokes  $I$  for the same  $\Delta\lambda$  position where the  $BS$  maximums were found. On the other hand, if a very weak emission in Stokes  $I$  presents where the  $BS$  minimum exists, we have the emissive Zeeman effect for the same magnetic polarity.

For the 25 July 1981 flare, Lozitsky & Staude (2003) found a direct evidence of such a situation. The difference between the Stokes  $I$  profiles of the FeI 5250.2 line in the flare and at a place outside indicated the presence of *emissive* Zeeman sigma

components, while *the same* ( $N$ ) polarity existed at the mentioned spectral position. This spectral position corresponds to a magnetic field strength of 7.8 kG.

An application of criterion (11) to the above listed values of  $\Delta\lambda_{\max} = 50 \text{ m}\text{\AA}$  and  $\Delta\lambda_{\max} = 150 \text{ m}\text{\AA}$  (Fig. 7) gives the following magnetic field strengths:  $B_1 \approx 1.3 \text{ kG}$ ,  $B_2 \approx 3.9 \text{ kG}$ . Note that, second value is very close to the  $B_{\text{true}} \approx 4.0 \text{ kG}$  which follows from Fig. 2 (see section 3.1).

Let us put the question: which spectral peculiarities should correspond to this magnetic field strength (3.9–4.0 kG) in Fig. 8 for FeI 5233 line? First maximum on  $\Delta\lambda_{\max} \approx 35 \text{ m}\text{\AA}$  corresponds to 2.15 kG, but first minimum (on  $\Delta\lambda_{\min} \approx 60 \text{ m}\text{\AA}$ ) corresponds to  $B \approx 3.7 \text{ kG}$ . Likely, minimum on  $\Delta\lambda_{\max} \approx 60 \text{ m}\text{\AA}$  has a real physical meaning. In any case, results of three different methods are very close, if we consider that namely, minimum on  $\Delta\lambda_{\min} \approx 60 \text{ m}\text{\AA}$  (Fig. 6) indicates the position of very weak Zeeman  $\sigma$ -components from any subtelescopic structures with very strong field but small filling factor.

Third field value,  $B_3$ , could be close to 7.8 kG, as it follows from FeI 5250.6 line (see above). It is interesting to note that practically the same value, 7.4 kG corresponds to second minimum on Fig. 8,  $\Delta\lambda_{\min} \approx 120 \text{ m}\text{\AA}$ . Relatively small difference between these values (7.4 and 7.8 kG) could be due to the errors of measurements or vertical gradient of the magnetic field. Analogously to the second value  $B_2 \approx 4.0 \text{ kG}$ , we can suppose that magnetic elements with  $B_3 \approx 7.4\text{--}7.8 \text{ kG}$  also give the very weak spectral contribution in the form of *emissive Zeeman effect* for the same ( $N$ ) magnetic polarity.

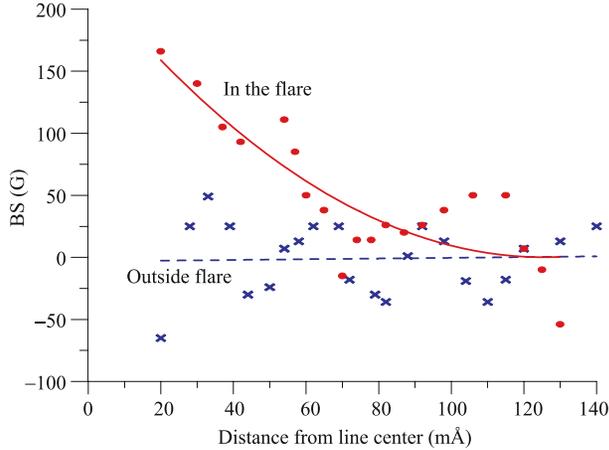
Three comments should be given to these values:

- Three different magnetic field values  $B_1$ ,  $B_2$ , and  $B_3$  mean a multi-component magnetic field structure, probably in the form of three types of magnetic features with different magnetic field strengths (Lozitsky 1980; Lozitsky & Staude 2003).
- Doppler and turbulent velocities in such structures should be close to zero, that follows from the negligible relative shift of the extrema in the violet and red wings (Fig. 6) and from very narrow (about 30–50 mÅ) spectral peculiarities.
- If such ‘super-strong’ magnetic fields (close to  $\approx 10 \text{ kG}$ ) really exist, we can observe very weak manifestation of the Zeeman effect *even in ‘non-split’ line* FeI 5576 because its Lande factor, strictly speaking, is non-zero,  $-0.012$ , i.e., about 105 times less than in FeI 5233. For instance, magnetic field of 10 kG should produce in FeI 5576 such Zeeman splitting, as magnetic field of about 100 G in FeI 5233.

Wonderfully, observations confirm this assumption (Fig. 9). We can see that weak splitting in FeI 5576 is observed really in the central part of line profile ( $\Delta\lambda < 60\text{--}80 \text{ m}\text{\AA}$ ), and its calibrated value, in fact, is close to 100–150 G. Note that analogous effect was discovered earlier also in other flares (Lozitsky 1993, 1998).

#### 4. Discussion and conclusion

Comparison of all the data derived above leads to the conclusion that a four-component magnetic field structure is possible, with field strengths of about  $\pm 1.05 \text{ kG}$  in the background field, and of 1.3–1.45,  $\approx 4.0$ , and 7.4–7.8 kG (at  $h \approx 300 \text{ km}$ ) in the small-scale, spatially unresolved magnetic elements.



**Figure 9.** Observational manifestation of the Zeeman effect in core of Fe I 5576 line in flare of 5 November 2004: crosses and dashed line – splitting of bisectors of  $I \pm V$  profiles in this line, formally calibrated to the case of Fe I 5233 line ( $g_{\text{eff}} = 1.261$ ), outside the flare; filled circles and solid line – observed  $BS$  splitting in bright knot of flare.

As to the fields in the 1.1–2.3 kG range, they were measured earlier by many authors, e.g., by Stenflo (1973); Wiehr (1978); Koutchmy & Stellmacher (1978); Lozitsky & Tsap (1989) and Lozitsky *et al.* (2000). On the contrary, the flux-tube mode of  $\approx 4.0$  kG strength is almost unknown and unstudied. Lozitsky (1980, 1986) found observational evidence of this mode in non-flare regions. Obviously similar data were also obtained by other authors, but they probably considered the data as insufficient or unreliable.

Arguments for the possible existence of a magnetic mode with  $B_3 \approx 8\text{--}10$  kG were given by Lozitsky (1979, 1980) for both flare and non-flare regions. At present we have only first preliminary data which could give hints at such ‘super-strong’ fields. In particular, it is unknown which is the upper field strength limit for the magnetic features. Observational evidences of flare fields of even 20–90 kG were found on the basis of a study of Stokes  $I \pm V$  and  $I \pm Q$  profiles in Fe I, Mg I and HI lines in a 2B-flare (Lozitsky 1993, 1998).

Why are these fields practically unknown?

Here are the main factors which make the diagnostics of such super-strong fields very difficult (see, e.g., Lozitsky 1993):

- A drop of the gas pressure in magnetic elements hampers their diagnostics, since it diminishes the optical thickness in the tubes. In the simplest case, when the flux-tube is homogeneous and untwisted, the condition

$$p + \left( \frac{B^2}{8\pi} \right) = p_{\text{ex}}, \quad (15)$$

should be met, where  $p$  and  $p_{\text{ex}}$  are the gas pressures inside and outside the tube, respectively;  $B$  is the magnetic field strength in the tube (the magnetic field pressure outside the tube is neglected). Clearly, the stronger  $B$  is inside the tube, the lower must  $p$  be when  $p_{\text{ex}}$  is fixed. We may therefore expect that magnetic

flux-tubes with the strongest fields will be most rarefied, i.e., *spectrally invisible*. Moreover, a magnetic field of 8 kG would require a  $p_{\text{ex}} > 2.5 \times 10^6 \text{ dyn cm}^{-2}$ ; that is,  $10^3$  times larger than  $p$  at line-forming heights of the quiet Sun. The time scale of a pressure adjustment of such a huge local increase of  $p$  (if it exists due to the flare) would be *extremely short* and make the observation very difficult.

- A small value of the filling factor  $\alpha$ : when  $\alpha \ll 1$ , the corresponding spectral features may be so feeble that it is practically impossible to separate them from the instrumental noise background or the emulsion grains. In this case, we must check carefully the spectral manifestation using special statistical methods.
- The dispersion of magnetic field strengths inside the flux-tubes can also mask the presence of super-strong fields. It should manifest itself in a smearing (diminishing depth and contrast) of the Zeeman  $\sigma$ -components.
- The diagnostics of super-strong magnetic fields can be made either difficult or easy depending on the local thermodynamic conditions. It is likely that the corresponding magnetic flux-tubes in flares are more conspicuous due to the increased gas pressure in them and to emission reversal of the Zeeman pattern. Flares might be the areas where super-strong fields are most likely to arise – there is empirical evidence that a local (vertically) intensification of the magnetic field (a ‘collapse’) occurs in flare areas (Lozitsky *et al.* 2000; Kurochka & Lozitsky 2005). Thus there is good reason to look for extra manifestations of super-strong fields just in flares. The diagnostics of the complicated magnetic structure in a flare could probably be improved by applying a Stokes profiles inversion procedure to the data (see, e.g., Carroll & Staude 2001).

Another interesting effect discovered two decades ago from observations (Lozitsky 1980, 2003) is a field strength ‘quantization’. It was shown that a theoretical interpretation of these phenomena can be offered within the frame of a linear force-free model (Soloviev & Lozitsky 1986). This model has a multi-polar periphery and a very strong field with discrete values near the tube axis. From a qualitative point of view, such a multi-polar periphery could give a spectral signature similar to a strong ‘mixed polarity’ background field. The theoretical values of discrete strengths near the tube axis are in satisfactory agreement with the observations.

It seems very much like that idea about possible magnetic field strength discretization in small-scale magnetic elements begins to be in the air. Recently Socas-Navarro & Lites (2004) found evidences for mixed field strengths in the quiet Sun, with two discrete values at 300 and 1700 G. Socas-Navarro (2004) obtained several magnetic strength ‘nodes’ in small-scale magnetic elements. He applied multi-line Stokes analysis using observational material from three modern spectropolarimeters (THEMIS, SPINOR and TIP + POLIS). Perhaps, some general peculiarities of the small-scale magnetic fields (including magnetic field strength discretization) are similar for flare and non-flare places on the Sun.

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## References

- Babcock, H. W. 1953, *Astrophys. J.*, **118**, 387.
- Baranovsky, E. A., Lozitska, N. I., Lozitsky, V. G. 1991, *Kinematika i Fizika Neb. Tel.*, **7**, 52.
- Brault, J. W. 1978, In: Proc. JOSO Workshop *Future Solar Optical Observations – Needs and Constraints*; Godoli, G., Noci, G., Righini, A. (eds.), *Osserv. Mem. Oss. Astrofis. Arcetri* **106**, 33.
- Carroll, T. A., Staude, J. 2001, *Astron. Astrophys.*, **378**, 316.
- Deming, D., Hewegama, T., Jennings, D. E., Osherovich, V., Wiedemann, G., Zirrin, H. 1990, *Astrophys. J.*, **364**, L49.
- Frazier, E. N., Stenflo, J. O. 1978, *Astron. Astrophys.*, **70**, 789.
- Gopasyuk, S. I., Kotov, V. A., Severny, A. B., Tsap, T. T. 1973, *Solar Phys.*, **31**, 307.
- Gurtovenko, E. A., Kostik, R. I. 1989, *Fraunhof. Spectr i Sist. Soln. Sil Oscillatorov, Nauk. Dumka, Kiev*, 200 pp. (in Russian).
- Harvey, J., Livingston, W. 1969, *Solar Phys.*, **10**, 283.
- Harvey, J., Livingston, W., Slaughter, C. 1972, In: Proc. Conf. on *Line Formation in Magnetic Field*, NCAR, Boulder, Colorado, p. 227.
- Howard, R., Stenflo, J. O. 1972, *Solar Phys.*, **22**, 402.
- Keller, C. U., Solanki, S. K., Steiner, O., Stenflo, J. O. 1990, *Astron. Astrophys.*, **233**, 583.
- Kouchmy, S., Stellmacher, G. 1978, *Astron. Astrophys.*, **67**, 93.
- Kurochka, E. V., Lozitsky, V. G. 2005, *Kinem. and Physics of Celestial Bodies, Suppl.*, **5**, 143.
- Kurochka, E. V., Kurochka, L. N., Lozitsky, V. G., Lozitska, N. I., Ostapenko, V. A., Polupan, P. N., Romanchuk, P. R. 1980, *Vestn. Kiev. Univ., Ser. Astronomii*, **22**, 48.
- Landi Degl'Innocenti, E. L. 1982, *Solar Phys.*, **77**, 285.
- Lozitska, N., Lozitsky, V. 1994, *Solar Phys.*, **151**, 319.
- Lozitska, N. I., Lozitsky, V. G. 1982, *Soviet Astron. Lett.*, **8(4)**, 270.
- Lozitska, N. I., Lozitsky, V. G. 1988, In: Proc. Int. Workshop *Solar Maximum Analysis*, Irkutsk, p. 80.
- Lozitsky, V. G., Baranovsky, E. A. 1993, *Izv. Krivsk. Astrofiz. Obs.*, **88**, 67.
- Lozitsky, V. G., Tsap, T. T. 1989, *Kinematika i Fizika Neb. Tel.*, **5**, 50.
- Lozitsky, V. G., Sheminova, V. A. 1992, *Kinematika i Fizika Neb. Tel.*, **8**, 12.
- Lozitsky, V. G., Staude, J. 2003, *ASP Conf. Ser. 307, Solar Polarization Workshop 3* (eds.) Trujillo-Bueno, J., Sanches Almeida, J. (San Francisco: ASP), 378.
- Lozitsky, V. G., Baranovsky, E. A., Lozitska, N. I., Leiko, U. M. 2000, *Solar Phys.*, **191**, 171.
- Lozitsky, V. G., Kurochka, L. N., Lozitska, N. I. 1986, *Sov. Astron. J.*, **63**, 814.
- Lozitsky, V. G., Lozitska, N. I., Gordovsky, M. Yu. 1999, *Vestn. Kyiv. Univ. Astron.*, **35**, 17.
- Lozitsky, V. G., Lozitska, N. I., Kurochka, L. N., Vaculik, V. 1987, *Vestn. Kiev. Univ. Astron.*, **28**, 33.
- Lozitsky, V. G. 1979, *Astrometria i Astrofizika*, **38**, 13.
- Lozitsky, V. G. 1980, *Phys. Solariterr., Potsdam*, **14**, 88.
- Lozitsky, V. G. 1986, *Kinematic Phys. Celest. Bodies*, **2**, 28.
- Lozitsky, V. G. 1993, *Kinematika i Fizika Neb. Tel.*, **9**, 23.
- Lozitsky, V. G. 1998, *Kinematika i Fizika Neb. Tel.*, **14**, 401.
- Lozitsky, V. G. 2003, *DrSc Dissertation*, Kyiv, 299 pp.
- Moore, R., Hurford, G. J., Jones, H. P., Kane, S. R. 1984, *Astrophys. J.*, **276**, 379.
- Parker, E. N. 2001, *Chin. J. Astron. Astrophys.*, **1**, 99.
- Penn, M. J., Kuhn, J. R. 1995, *Astrophys. J.*, **441**, L51.
- Priest, E. R. 1986, *Solar Phys.*, **104**, 1.
- Rachkovsky, D. N., Tsap, T. T., Lozitsky, V. G. 2005, *J. Astrophys. Astr.*, **26**, 435.
- Severny, A. B. 1988, *Some problems of Solar Physics, Moskov, Nauka*, 224 p.
- Socas-Navarro, H., Lites, B. W. 2004, *Astrophys. J.*, **616**, 587.
- Socas-Navarro, H. 2004, *Astrophys. J.*, **613**, 610.
- Soloviev, A. A., Lozitsky, V. G. 1986, *Kinematika i Fizika Neb. Tel.*, **2**, 80.
- Staude, J., Hofmann, A. 1988, In: Proc. Int. Workshop on *Reconnection in Space Plasma*, Potsdam; *ESA SP-285*, Vol. **II**, 123.
- Staude, J. 1970a, *Solar Phys.*, **12**, 84.
- Staude, J. 1970b, *Solar Phys.*, **15**, 102.
- Stenflo, J. O., Solanki, S. K., Harvey, J. W. 1987, *Astron. Astrophys.*, **171**, 305.

- Stenflo, J. O., Harvey, J. W., Brault, J. W., Solanki, S. 1984, *Astron. Astrophys.*, **131**, 333.  
Stenflo, J. O. 1973, *Solar Phys.*, **32**, 41.  
Wiehr, E. 1978, *Astron. Astrophys.*, **69**, 279.  
Zemanek, E. N., Stefanov, A. P. 1976, *Vestn. Kiev. Univ. Astron.*, **18**, 20.