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Velocity Curve Analysis of the Spectroscopic Binary Stars PV Pup, HD 141929, EE Cet and V921 Her by Nonlinear Regression

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Abstract. We use the method introduced by Karami & Mohebi (2007), and Karami & Teimoorinia (2007) which enable us to derive the orbital parameters of the spectroscopic binary stars by the nonlinear least squares of observed *vs.* curve fitting (o–c). Using the measured experimental data for radial velocities of the four double-lined spectroscopic binary systems PV Pup, HD 141929, EE Cet and V921 Her, we find both the orbital and the combined spectroscopic elements of these systems. Our numerical results are in good agreement with those obtained using the method of Lehmann-Filhés.

Key words. Stars: binaries: eclipsing—stars: binaries: spectroscopic.

1. Introduction

Determining the orbital elements of binary stars helps us to obtain necessary information such as the mass and radius of stars which play important roles during the evolution of stellar structures. Analyzing both the light and radial velocity curves deduced from the photometric and spectroscopic observations respectively, helps to derive the orbital parameters. One of the usual rules to analyze the velocity curve is the method of Lehmann-Filhés, see Smart (1990). Karami & Mohebi (2007), and Karami & Teimoorinia (2007) introduced a new method to derive the orbital parameters by the nonlinear regression of observed *vs.* curve fitting, hereafter (o–c). Here we use their method to obtain the orbital elements of the four double-lined spectroscopic binary systems PV Pup, HD 141929, EE Cet and V921 Her which have the following properties.

PV Pup is a detached double-lined late A-type eclipsing binary of fairly short period $P = 1.67$ days and small orbital eccentricity ($e = 0.05$). The spectral type is A0V and A2V for the primary and secondary component, respectively. The mean effective temperature for primary is 6920 K and for secondary is 6931 K. The angle of inclination is $83.09 \pm 18^\circ$ (Vaz & Andersen 1984). HD 141929 is a double-lined spectroscopic binary with a period of $P = 49.699$ days. The orbit is eccentric ($e = 0.393$). The effective temperature for both components is estimated to 9500 ± 250 K. Both components have the same spectral type A0/1V, however, the secondary is rotating slower than the primary and the inclination of the orbit is about 11° (Carrier 2002). EE cet is contact double-lined spectroscopic binary with a period of $P = 0.339917$ days (Rucinski *et al.* 2002a,b). V921 Her belongs to the A type of contact binaries, with the more massive component eclipsed during the deeper minimum. The spectral type is A7IV with a period of $P = 0.877$ days (Rucinski *et al.* 2003a,b).

This paper is organized as follows. In section 2, we reduce the problem to solving an equation which is a nonlinear function in terms of the orbital parameters. In section 3, the nonlinear regression technique for estimating the orbital elements is discussed. In section 4, the numerical results implemented for the four different binary systems are reported. Section 5 is devoted to conclusions.

2. Formulation of the problem

The radial velocity of star in a binary system is defined as follows:

$$\mathrm{RV} = V_{cm} + \dot{Z}, \tag{1}$$

where V_{cm} is the radial velocity of the center of mass of system with respect to the Sun and

$$\dot{Z} = K[\cos(\theta + \omega) + e\cos\omega], \tag{2}$$

is the radial velocity of star with reference to the center of mass of the binary, see Smart (1990). In equation (2), the dot denotes the time derivative and θ, ω and e are the angular polar coordinate (true anomaly), the longitude of periastron and the eccentricity, respectively. Note that the quantities θ and ω are measured from the periastron point and the spectroscopic reference line (plane of sky), respectively. Also

$$K = \frac{2\pi}{P}\frac{a\sin i}{\sqrt{1-e^2}}, \tag{3}$$

where P is the period of motion and inclination i is the angle between the line of sight and the normal of the orbital plane.

Observation shows that the photometric phase, ϕ, which is measured from the photometric reference point (line of sight), is a measurable quantity. Hence, one has to try to express θ appearing in equation (2), in terms of ϕ. But the main difficulty here is that this is not so easy in practice, unless in the two following cases:

i) Following Smart (1990) and Rucinski (2002), the photometric (orbital) phase, ϕ, is generally related to the eccentric anomaly, ψ, according to Kepler's equation as

$$\psi - e\sin\psi = 2\pi\left(\frac{t - T_0}{P}\right) = 2\pi\phi, \tag{4}$$

where T_0 is moment of the primary eclipse. Both T_0 and P is usually taken from literature sources and is fixed. Also θ and ψ satisfy the following relation

$$\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{\psi}{2}. \tag{5}$$

To obtain ψ for a given ϕ, one may solve equation (4) numerically. Then θ is derived from equation (5), however e should be fixed before. So we see that deriving θ in terms of ϕ depends on knowing the eccentricity. For a small eccentricity, $e < 1$, expanding equations (4) and (5) reduces to the following relation

$$\theta = 2\pi\phi + \left(2e - \frac{e^3}{4}\right)\sin(2\pi\phi) + \frac{5}{4}e^2\sin(4\pi\phi) + \frac{13}{12}e^3\sin(6\pi\phi) + O(e^4). \tag{6}$$

Equation (6) shows that when $e \ll 1$, then one can use $\theta = 2\pi\phi$ in equation (2).

ii) According to Budding (1993), for most normal eclipsing binaries, the line of sight inclines at a low angle to the orbital plane. This yields to the relationship between θ, ϕ and ω as follows:

$$\theta = 2\pi\phi - \omega \pm \frac{\pi}{2}, \tag{7}$$

where $\pm$ refers to the primary and the secondary components, and hereafter are shown by subscripts p and s, respectively. Therefore equations (1) and (2) for the two components reduce to

$$V_{r,p} = V_{cm} + K_p(e\cos\omega_p - \sin 2\pi\phi),$$
$$V_{r,s} = V_{cm} + K_s(e\cos\omega_s + \sin 2\pi\phi). \tag{8}$$

For a circle like orbit, in which its eccentricity is much less than unity, neglecting the term $e\cos\omega$ in equations (8) reduces to two simple sine curves. See equation (5) in Rucinski (2002).

To avoid the mentioned difficulties in obtaining θ in terms of ϕ, we try to remove it in our equations. To do this, first we take the time derivative of equation (2) as

$$\ddot{Z} = -K\sin(\theta + \omega)\dot{\theta}. \tag{9}$$

Then, using Kepler's second law and the relations obtained from the orbital parameters in the inverse-square field as

$$\dot{\theta} = \frac{h}{r^2}, \tag{10}$$

$$h = \frac{2\pi}{P}a^2\sqrt{1-e^2}, \tag{11}$$

$$r = \frac{a(1-e^2)}{1+e\cos\theta}, \tag{12}$$

one may show that equation (9), yields to

$$\ddot{Z} = \frac{-2\pi K}{P(1-e^2)^{3/2}}\sin(\theta+\omega)(1+e\cos\theta)^2, \tag{13}$$

where r, a and h are the radial polar coordinate, the semi-major axis of the orbit and the angular momentum per unit of mass, respectively.

Using equation (2), one can remove θ from equation (13) as follows:

$$P\ddot{Z} = \frac{-2\pi K}{(1-e^2)^{3/2}} \sin\left(\cos^{-1}\left(\frac{\dot{Z}}{K} - e\cos\omega\right)\right) \times \left\{1 + e\cos\left(-\omega + \cos^{-1}\left(\frac{\dot{Z}}{K} - e\cos\omega\right)\right)\right\}^2. \tag{14}$$

To simplify the notation further, we let $Y = P\ddot{Z}$ and $X = \dot{Z}$. Equation (14) describes a nonlinear relation, $Y = Y(X, K, e, \omega)$, in terms of the orbital elements K, e and ω. Using the nonlinear regression of equation (14), one can estimate the parameters K, e and ω, simultaneously.

One may show that the adopted spectroscopic elements are related to the orbital parameters. First, according to definition of the center of mass, the mass ratio in the system is obtained as

$$\frac{m_p}{m_s} = \frac{a_s \sin i}{a_p \sin i}. \tag{15}$$

According to Kepler's third law and equation (15), the following relation

$$m_p \sin^3 i = a_s \sin i \left(\frac{a_p \sin i + a_s \sin i}{P}\right)^2, \tag{16}$$

is obtained where a, P and m are expressed in AU, years and solar mass, respectively. A similar relation is obtained for the secondary component only by replacing p with s and *vice versa*, in equation (16). Note that in equations (15) and (16) parameter $a \sin i$ is related to the orbital parameters by the aid of equation (3).

3. Nonlinear least squares of (o–c)

To obtain the orbital parameters K, e and ω in equation (14), we use the nonlinear regression method. In this approach, the sum of squares of errors (SSE) for the number of N measured data is calculated as

$$\mathrm{SSE} = \sum_{i=1}^{N}(Y_i - \hat{Y}_i)^2 = \sum_{i=1}^{N}[Y_i - Y(X_i, K, e, \omega)]^2, \tag{17}$$

where Y_i and $\hat{Y}_i$ are the real and the predicted values, respectively. To obtain the model parameters, the SSE should be minimized in terms of K, e and ω as

$$\frac{\partial \mathrm{SSE}}{\partial K} = \frac{\partial \mathrm{SSE}}{\partial e} = \frac{\partial \mathrm{SSE}}{\partial \omega} = 0. \tag{18}$$

To solve equation (18), we use the SAS (Statistical Analysis System) software. Note that the nonlinear models are more difficult to specify and estimate than linear models.

For instance, in contrast to the linear regression, the nonlinear models are very sensitive to the initial guesses for the parameters. Because in practice, SSE might have to be minimized in the several points in the three dimensional parametric space including K, e and ω. However the final goal is finding the absolute minimum. Hence choosing the relevant initial parameters yields to the absolute minimum of SSE which is also stationary. This means that if one changes the initial guesses slightly, then the result reduces to the previous values for the parameters. But if SSE converged at the local minimum, the model would not be stationary. See Sen & Srivastava (1990) and Christensen (1996). Whatever the method used to determine the elements, it will always be necessary to test the elements as derived and make small changes in them before they can be regarded as a satisfactory representation of the observations. To do this a test is quickly computed for ten or twelve points of the RV curve by substituting the derived orbital elements in equations (1), (2), (4) and (5). See Karami & Teimoorinia (2007).

4. Numerical results

Here we use the method introduced by Karami & Mohebi (2007), and Karami & Teimoorinia (2007) to derive both the orbital and combined elements of the four different double-lined spectroscopic systems PV Pup, HD 141929, EE Cet and V921 Her. Using the measured experimental data for radial velocities of the two components of these systems obtained by Vaz & Andersen (1984) for PV Pup, Carrier (2002) for HD 141929, Rucinski *et al.* (2002a,b) for EE Cet and Rucinski *et al.* (2003a,b) for V921 Her, the suitable fitted velocity curves are plotted in terms of the photometric phase in Figs. 1, 2, 3 and 4, respectively.

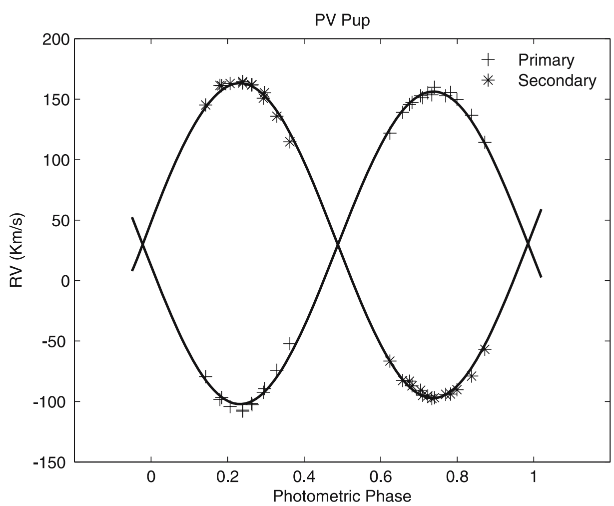

Figure 1. Radial velocities of the primary and secondary components of PV Pup plotted against the photometric phase. The observational data have been derived from Vaz & Andersen (1984).

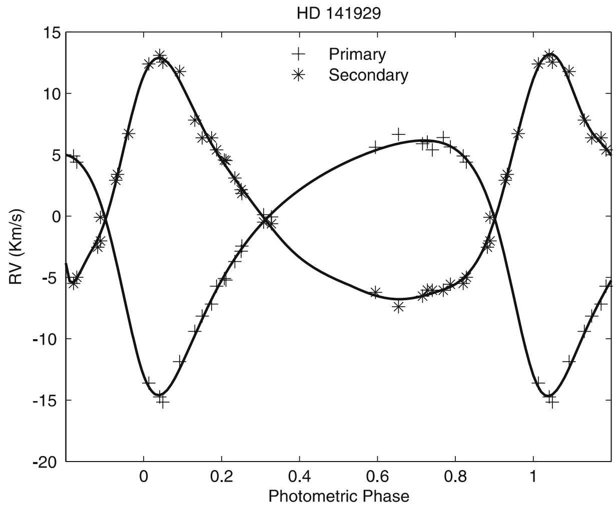

Figure 2. Same as Fig. 1 but for HD 141929. The observational data have been derived from Carrier (2002).

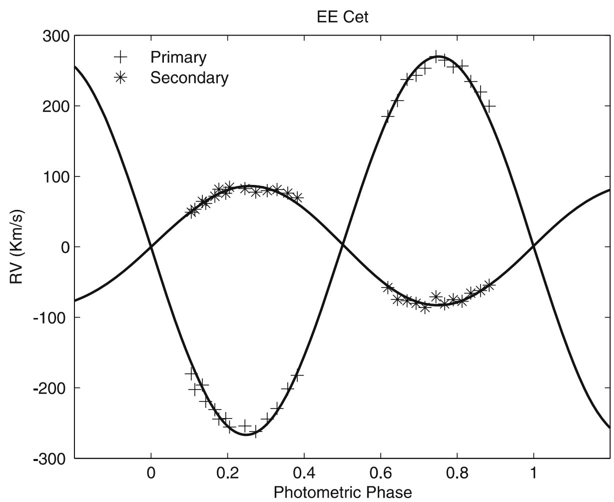

Figure 3. Same as Fig. 1 but for EE Cet. The observational data have been derived from Rucinski *et al.* (2002a,b).

The scaled radial acceleration values, $P\ddot{Z} = P\mathrm{d}\dot{Z}/\mathrm{dt} = \mathrm{dRV}/\mathrm{d}\phi$, corresponding to the radial velocity data in equation (14), are obtained only by taking the phase derivative of the RV without knowing the period P. To do this we implemented two methods:

1) A numerical differentiation method such as *Forward*, *Central* and *Extrapolated* differences, depending on desirable precision, the $P\ddot{Z}$ data corresponding with the RV data are obtained.

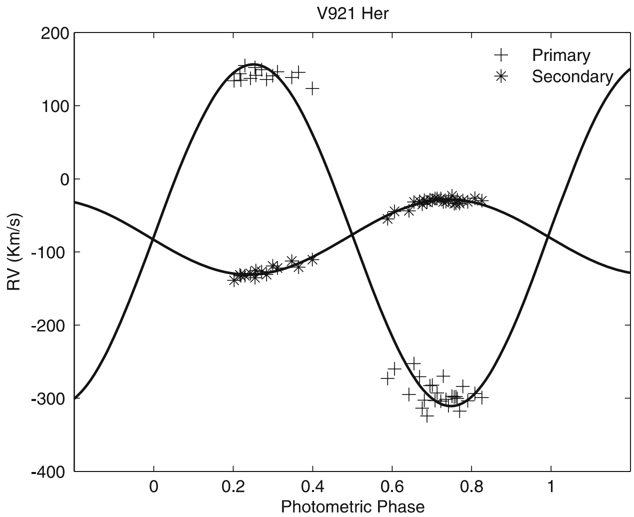

Figure 4. Same as Fig. 1 but for V921 Her. The observational data have been derived from Rucinski *et al.* (2003a,b).

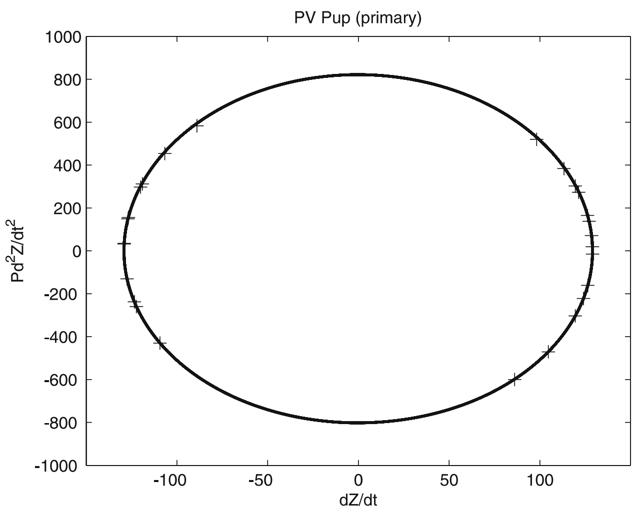

Figure 5. The radial acceleration scaled by the period *versus* the radial velocity of the primary component of PV Pup. The solid curve is obtained from the nonlinear regression of equation (14). The plus points are the experimental data.

2) A *Curve Fitting* program: the suitable velocity curves are fitted on the RV data and then one can obtain the corresponding $P\ddot{Z}$ data. Our experience shows that **the** accuracy of both approaches is very similar but the time consumed in the second approach is considerably shorter than in the first one.

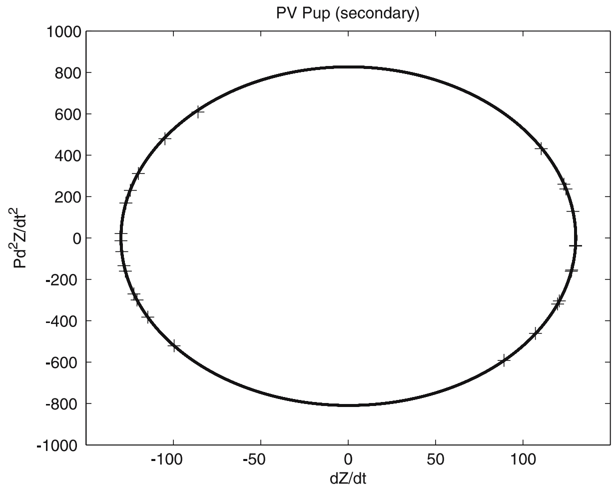

Figure 6. Same as Fig. 5 but for the secondary component of PV Pup.

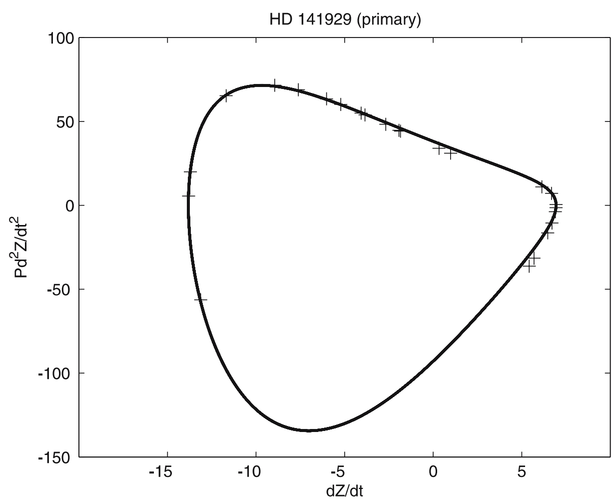

Figure 7. Same as Fig. 5 but for the primary component of HD 141929.

Figures 5–12 show the radial acceleration scaled by the period *versus* the radial velocity for the primary and secondary components of PV Pup, HD 141929, EE Cet and V921 Her, respectively. The solid closed curves are the result of the nonlinear regression of equation (14), which their good coincidence with the measured data yields to derive the optimized parameters K, e and ω. Figures show that also for PV Pup, EE Cet and V921 Her due to having small eccentricities, their radial velocity–acceleration curves display an elliptical shape. But in contrast for HD 141929 which is an eccentric system, its radial velocity–acceleration curve shows a noticeable deviation from an elliptic.

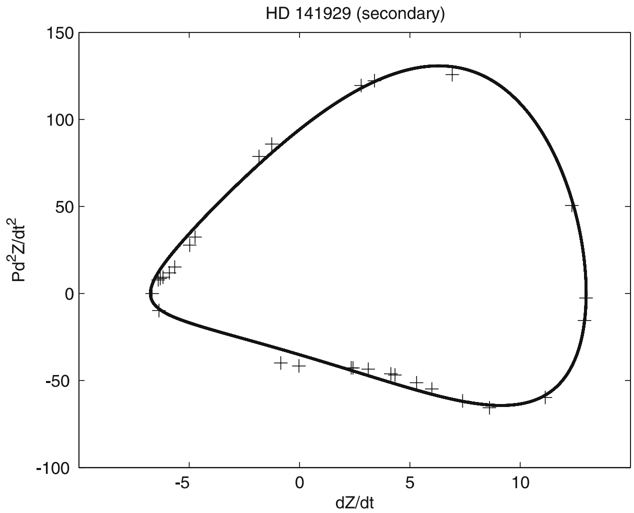

Figure 8. Same as Fig. 5 but for the secondary component of HD 141929.

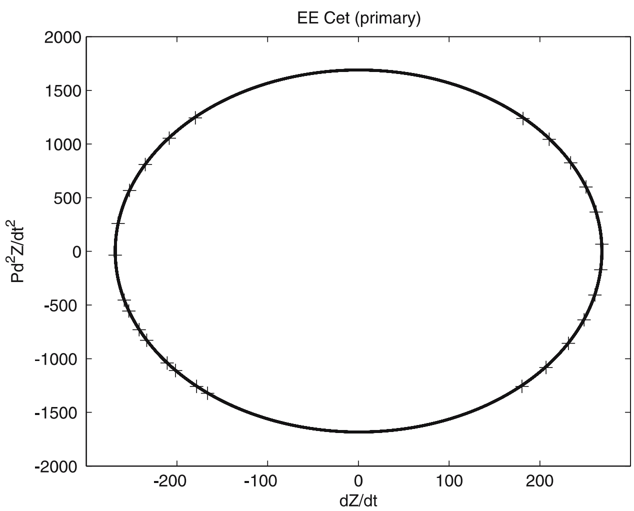

Figure 9. Same as Fig. 5 but for the primary component of EE Cet.

The orbital parameters, K, e and ω, obtained from the nonlinear least squares of equation (14) for PV Pup, HD 141929, EE Cet and V921 Her, are tabulated in Tables 1, 3, 5 and 7, respectively. The velocity of center of mass, V_{cm}, is obtained by calculating the areas above and below the radial velocity curve. Where these areas become equal to each other, the velocity of center of mass is obtained. Tables 1, 3, 5 and 7 show that the results are in good accordance with those obtained by Vaz & Andersen (1984) for PV Pup, Carrier (2002) for HD 141929, Rucinski *et al.* (2002a,b) for EE Cet and Rucinski *et al.* (2003a,b) for V921 Her.

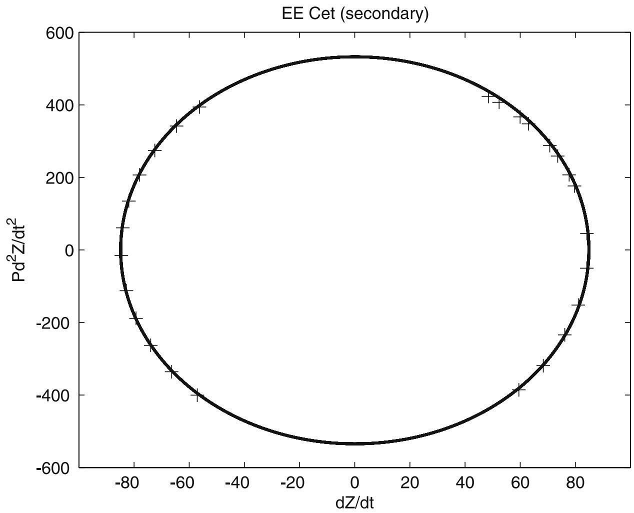

Figure 10. Same as Fig. 5 but for the secondary component of EE Cet.

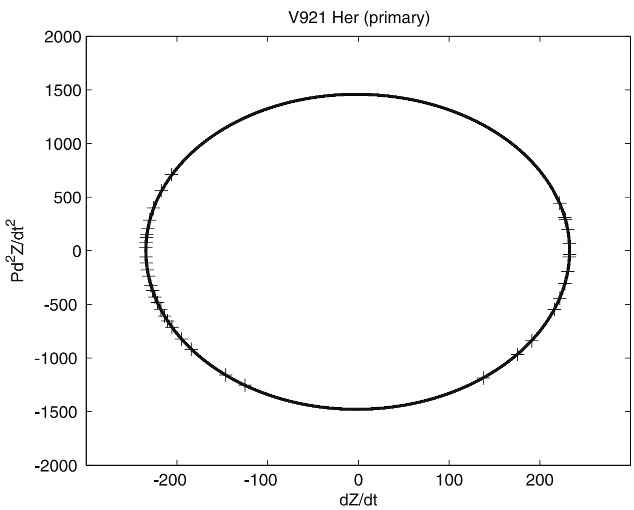

Figure 11. Same as Fig. 5 but for the primary component of V921 Her.

The combined spectroscopic elements including $m_p \sin^3 i$, $m_s \sin^3 i$, $(a_p + a_s) \sin i$ and m_p/m_s are calculated by substituting the estimated parameters K, e and ω in equations (3), (15) and (16). The results obtained for the same four previous systems are tabulated in Tables 2, 4, 6 and 8. Tables show that our results are in good agreement with those obtained by Vaz & Andersen (1984), Carrier (2002), Rucinski *et al.* (2002, 2003) for PV Pup, HD 141929, EE Cet and V921 Her, respectively.

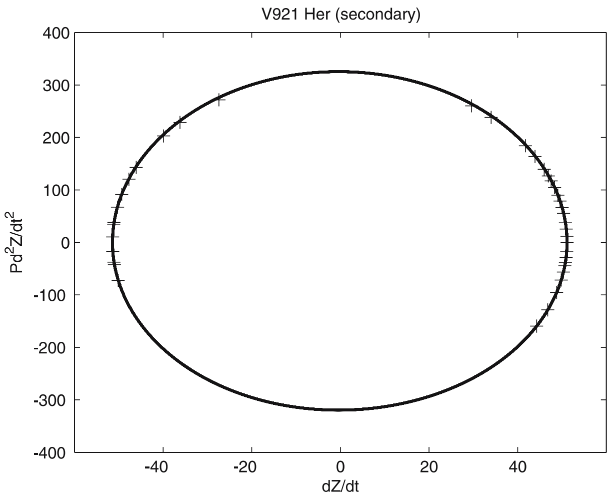

Figure 12. Same as Fig. 5 but for the secondary component of V921 Her.

Table 1. Spectroscopic orbit of PV Pup.

	This paper	Vaz & Andersen (1984)
Primary		
V_{cm} (km s^{-1})	27.16 ± 0.63	31 ± 0.5
K_p (km s^{-1})	129.14 ± 0.92	130 ± 0.5
e	0.01 ± 0.01	0.05 (fixed)
$\omega(^\circ)$	96.33 ± 0.24	149.7 (fixed)
Secondary		
V_{cm} (km s^{-1})	32.93 ± 0.85	29 ± 0.4
K_s (km s^{-1})	130.16 ± 0.39	130.9 ± 0.4
e	0.01 ± 0.01	0.05 (fixed)
$\omega(^\circ)$	278.27 ± 0.29	329.7 (fixed)

Table 2. Combined spectroscopic orbit of PV Pup.

Parameter	This paper	Vaz & Andersen (1984)
$m_p \sin^3 i/\mathrm{M}_\odot$	1.514 ± 0.019	1.531 ± 0.011
$m_s \sin^3 i/\mathrm{M}_\odot$	1.502 ± 0.026	1.52 ± 0.013
$(a_p + a_s) \sin i/\mathrm{R}_\odot$	8.55 ± 0.04	8.55 ± 0.022
m_s/m_p	0.992 ± 0.009	0.993 ± 0.005

Table 3. Same as Table 1 but for HD 141929.

	This paper	Carrier (2002)
Primary		
$V_{cm}\,(\mathrm{km\,s^{-1}})$	-0.44 ± 0.12	-0.33 ± 0.08
$K_p\,(\mathrm{km\,s^{-1}})$	10.38 ± 0.01	10.58 ± 0.16
e	0.391 ± 0.001	0.393 ± 0.008
$\omega\,(^\circ)$	148.04 ± 0.34	145.7 ± 1.7
Secondary		
$V_{cm}\,(\mathrm{km\,s^{-1}})$	-0.44 ± 0.12	-0.33 ± 0.08
$K_s\,(\mathrm{km\,s^{-1}})$	9.87 ± 0.01	9.95 ± 0.17
e	0.389 ± 0.001	0.393 ± 0.008
$\omega\,(^\circ)$	324.03 ± 0.34	325.7 ± 1.7

Table 4. Same as Table 2 but for HD 141929.

Parameter	This paper	Carrier (2002)
$m_p \sin^3 i/\mathrm{M}_\odot$	0.0163 ± 0.0001	0.01681 ± 0.00064
$m_s \sin^3 i/\mathrm{M}_\odot$	0.0171 ± 0.0001	0.01789 ± 0.00067
$a_p \sin i/10^6$ km	6.53 ± 0.01	6.65 ± 0.11
$a_s \sin i/10^6$ km	6.21 ± 0.02	6.25 ± 0.11
m_p/m_s	0.95 ± 0.004	0.94 ± 0.022

Table 5. Same as Table 1 but for EE Cet.

	This paper	Rucinski *et al.* (2002)
Primary		
$V_{cm}\,(\mathrm{km\,s^{-1}})$	1.52 ± 0.85	1.6(0.93)
$K_p\,(\mathrm{km\,s^{-1}})$	268.29 ± 0.52	266.92(1.54)
e	0.001 ± 0.001	–
$\omega\,(^\circ)$	305.59 ± 13.67	–
Secondary		
$V_{cm}\,(\mathrm{km\,s^{-1}})$	1.52 ± 0.85	1.6(0.63)
$K_s\,(\mathrm{km\,s^{-1}})$	84.91 ± 0.07	84.05(1.24)
e	0.001 ± 0.001	–
$\omega\,(^\circ)$	75.77 ± 4.1	–

Table 6. Same as Table 2 but for EE Cet.

Parameter	This paper	Rucinski *et al.* (2002)
$m_p \sin^3 i/\mathrm{M}_\odot$	0.417 ± 0.002	–
$m_s \sin^3 i/\mathrm{M}_\odot$	1.317 ± 0.007	–
$(a_p + a_s) \sin i/\mathrm{R}_\odot$	2.651 ± 0.004	–
m_p/m_s	0.316 ± 0.001	0.315(5)
$(m_p + m_s) \sin^3 i/\mathrm{M}_\odot$	1.73 ± 0.01	1.706(41)

Table 7. Same as Table 1 but for V921 Her.

	This paper	Rucinski *et al.* (2003)
Primary		
$V_{cm}\,(\mathrm{km\,s^{-1}})$	-78.05 ± 0.65	−79.04(1.05)
$K_p\,(\mathrm{km\,s^{-1}})$	233.62 ± 0.49	227.17(1.97)
e	0.005 ± 0.004	–
$\omega\,(^\circ)$	132.42 ± 1.16	–
Secondary		
$V_{cm}\,(\mathrm{km\,s^{-1}})$	-78.05 ± 0.65	−79.04(1.05)
$K_s\,(\mathrm{km\,s^{-1}})$	51.3 ± 0.7	51.45(0.89)
e	0.009 ± 0.004	–
$\omega\,(^\circ)$	237.2 ± 0.8	–

Table 8. Same as Table 2 but for V921 Her.

Parameter	This paper	Rucinski *et al.* (2003)
$m_p \sin^3 i/\mathrm{M}_\odot$	0.379 ± 0.008	–
$m_s \sin^3 i/\mathrm{M}_\odot$	1.724 ± 0.018	–
$(a_p + a_s) \sin i/\mathrm{R}_\odot$	4.94 ± 0.02	–
m_p/m_s	0.2169 ± 0.0035	0.226(5)
$(m_p + m_s) \sin^3 i/\mathrm{M}_\odot$	2.103 ± 0.027	1.971(61)

5. Conclusions

To derive the orbital elements of the spectroscopic binary stars, we used the the method introduced by Karami & Mohebi (2007), and Karami & Teimoorinia (2007). This method is applicable to orbits of all eccentricities and inclination angles. In this method, the time consumed is considerably shorter than in the method of Lehmann-Filhés. It should be more accurate as the orbital elements are deduced from all points of the velocity curve instead of four as in the method of Lehmann-Filhés. The present method enables one to vary all of the unknown parameters K, e and ω simultaneously instead of one or two of them at a time. It is possible to make adjustments in the elements before the final presentation. There are some cases, in which the method of Lehmann-Filhés is inapplicable, and in these cases the present one may be found useful. One such case would occur when observations are incomplete because certain phases could not have been observed. Another case in which this method is useful is that of a star attended by two dark companions with commensurable periods. In this case the resultant velocity curve may have several unequal maxima and the method of Lehmann-Filhés would fail altogether.

Using the measured experimental data for radial velocities of PV Pup, HD 141929, EE Cet and V921 Her obtained by Vaz & Andersen (1984), Carrier (2002), Rucinski *et al.* (2002) and Rucinski *et al.* (2003), respectively, we find the orbital elements of these systems by the mentioned method. Our numerical calculations show that the results obtained for both the orbital elements and combined spectroscopic parameters are in good agreement with the those obtained via the method of Lehmann-Filhés. In a subsequent paper we intend to test numerically our method for the other different systems.

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