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Report on the Dynamical Evolution of an Axially Symmetric Quasar Model

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Abstract. The role of the angular momentum in the regular or chaotic character of motion in an axially symmetric quasar model is examined. It is found that, for a given value of the critical angular momentum L_{zc}, there are two values of the mass of the nucleus M_n for which transition from regular to chaotic motion occurs. The $[L_{zc}–M_n]$ relationship shows a linear dependence for the time independent model and an exponential dependence for the evolving model. Both cases are explained using theoretical arguments together with some numerical evidence. The evolution of the orbits is studied, as mass is transported from the disk to the nucleus. The results are compared with the outcomes derived for galactic models with massive nuclei.

Key words. Galaxies: orbits—regular and chaotic motion.

1. Introduction

High resolution data obtained by the Hubble Space Telescope (HST) indicate that nuclear activity may be a common phenomenon during the lifetime of a galaxy. Furthermore, it is generally believed that the accretion episodes and the emitted nuclear power strongly depend on the mass of the system. One of the unsolved problems in Astronomy is the problem of the energy of quasars. Most astronomers believe that quasars are supermassive stellar objects (SMOs) with huge luminosity. This immense luminosity is the result of the accretion of matter outside those SMOs (see e.g., Cavaliere & Padovani 1988; Dobrzycki & Bechtold 1991; Siemiginowska 1991; Canizzo 1993; Peng & Chou 1997, 2001).

In the present paper we use a simple dynamical model for a quasar, in order to study the properties of motion. We believe that this is of interest because such studies are not very common in galactic dynamics. This study will give useful information for the dynamical properties and evolution of quasars. On the other hand, we will be able to compare the dynamical behavior of these systems with the results earlier obtained for galaxies and non-axially symmetric quasar models (see Papadopoulos & Caranicolas 2005).

We are particularly interested to find the nature of motion (regular or chaotic) and to seek for relationships connecting chaos and important physical parameters such as mass and angular momentum. As this was already done for galaxies, we will be able to compare the present results with the already well known results for galaxies (see Caranicolas & Innanen 1991; Caranicolas & Papadopoulos 2003).

Our dynamical model has two components. A disk and a massive nucleus and it is given by the following equation:

$$V(r,z) = -\frac{M_d}{[(\alpha + (z^2 + h^2)^{1/2})^2 + r^2]^{1/2}} - \frac{M_n}{(r^2 + z^2 + c_n^2)^{1/2}}, \tag{1}$$

where r, z are cylindrical co-ordinates, M_d is the mass, α is the scale length of the disk and h corresponds to the disk scale height. M_n is the mass, while c_n represents the scale length of the nucleus. Here again, the well known system of galactic units is used, where the unit of length is 1 kpc, the unit of time is 0.97746×10^8 yr and the unit of mass is $2.325 \times 10^7 M_\odot$. The velocity and the angular velocity units are 10 km/s and 10 km/s/kpc respectively while G is equal to unity. Therefore, the energy unit (per unit mass) is $100\,(\text{km/s})^2$. In these units, we use the values $c_n = 0.01$ kpc, $\alpha = 1.5$ kpc, $h = 0.02$ kpc, $M_d = 1000 - M_n$.

2. Results for the time independent model

In order to study properties of motion and the general dynamical behavior of the system, we shall use the two dimensional Hamiltonian,

$$H = \frac{1}{2}(p_r^2 + p_z^2) + V_{\text{eff}}(r,z) = E. \tag{2}$$

Here p_r, p_z are the momenta, per unit mass, conjugate to r and z, E is the star's energy while

$$V_{\text{eff}} = \frac{L_z^2}{2r^2} + V(r,z) \tag{3}$$

is the effective potential, L_z being the conserved component of the angular momentum.

The study is based on the numerical integration of the equations of motion

$$\ddot{r} = -\frac{\partial V_{\text{eff}}}{\partial r}, \qquad \ddot{z} = -\frac{\partial V_{\text{eff}}}{\partial z}. \tag{4}$$

In order to distinguish between regular and chaotic motion in the time independent model we use the $r - p_r$ ($z = 0$, $p_z > 0$) Poincare phase plane. Here we must make clear that with the term the "system shows chaotic orbits" we mean that, for the given values of the parameters, there is a part of the surface of section covered by chaotic orbits. Orbits were started at the plane of symmetry $z = 0$, with radial and vertical velocities ≤ 30 km/s.

First, we shall present a relationship between L_{zc} and M_n, where L_{zc} is the critical value of the angular momentum (that is the maximum value of the angular momentum for which chaotic regions are observed in the $r - p_r$ phase plane for a given value of

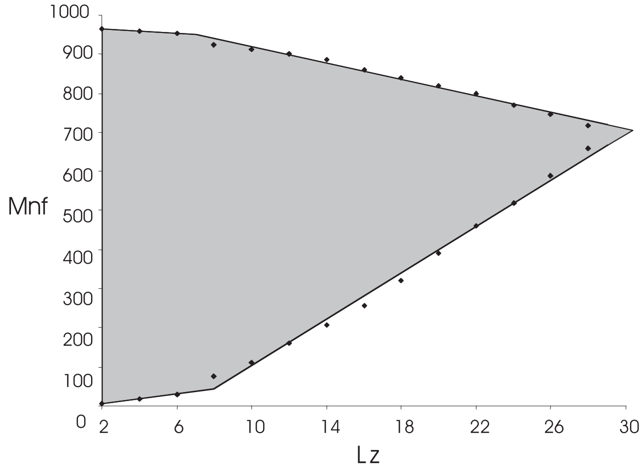

Figure 1. Relationship between L_{zc} and M_n. The values of the other parameters are given in text.

M_n, when all other parameters are kept constant). The results are shown in Fig. 1. As in the case of a disk galaxy, (see Caranicolas & Innanen 1991, Fig. 1) one observes that, a linear relationship exists between L_{zc} and M_n. What is interesting here is that, for a given value of the critical angular momentum there corresponds two values of the mass of the nucleus M_n, for which transition from regular motion to chaos occurs and not only one, as it was observed in disk galaxies. The two lines define an area in the $[L_{zc}–M_n]$ plane. For the values of the parameters within the two lines – including the lines – the $r–p_r$ phase plane shows areas of chaotic motion while, for values of the parameters outside these lines, all orbits are regular. It is interesting to observe that, as it was explained in Caranicolas & Innanen (1991), the slope of the straight lines is different for small values of L_{zc}.

Let us now explain the behavior of the system shown in Fig. 1. It is well known (see Caranicolas & Innanen 1991) that the responsibility for the appearance of chaotic orbit is the total F_z force. This force is written as:

$$F_z = -\frac{[(\alpha + (z^2 + h^2)^{1/2})^2](1000 - M_n)z}{(z^2 + h^2)^{1/2}\{[(\alpha + (z^2 + h^2)^{1/2})^2 + r^2]^{1/2}\}} - \frac{M_n z}{(r^2 + z^2 + c_n^2)^{3/2}}. \tag{5}$$

As one can see, there are two terms: the first term comes from the disk, while the responsibility for the second term is the nucleus. Figure 2 shows a plot of the two components *vs.* M_n for fixed values of r and z. We have taken $r = r_0 = 0.5$, $z = z_0 = 0.01$. Note that the $|F_z|$ force due to the nucleus increases as M_n increases while the other component due to the disk decreases as M_n increases. Furthermore, $|F_z|$ is proportional to M_n, while we know from Caranicolas & Innanen (1991) that $|F_z| \propto L_{zc}$. Thus, we conclude that M_n is proportional to L_{zc}, which is the result of Fig. 1.

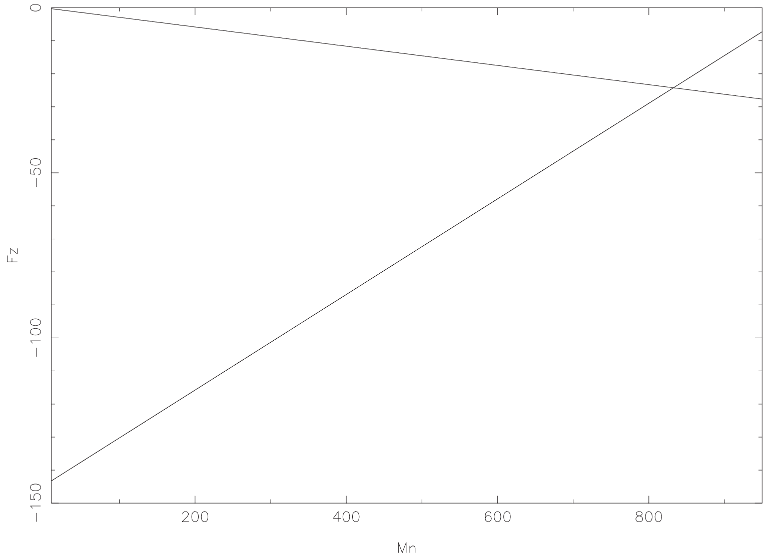

Figure 2. A plot of the two components of F_z *vs.* M_n for fixed values of r and z. The similarity between Figs. 1 and 2 is evident.

3. The evolving model

In this section we shall study the behavior of orbits when the system evolves with time. It is assumed that mass is transported from the disk to the nucleus. This phenomenon is described by the equations

$$M_d = M_{di} - m[1 - \exp(-kt)], \tag{6}$$

$$M_n = M_{ni} + m[1 - \exp(-kt)], \tag{7}$$

where M_{di}, M_{ni} are the initial values of the mass of the disk and nucleus, M_d, M_n are the values of mass of disk and nucleus at time t, m is the portion of the disk mass that is transported and $k > 0$ is a parameter. Note that in all cases we have $M_d + M_n = 1000$.

Again, orbits were started at the plane of symmetry $z = 0$, with radial and vertical velocities ≤ 30 km/s. In order to distinguish between regular and chaotic motion, we have calculated the maximal Lyapunov characteristic number (LCN) (see for details Lichtenberg & Lieberman 1983).

The numerical experiments show that a relationship exists between M_{nf} and L_{zc}, which is shown in Fig. 3. Here M_{nf} is the final mass of nucleus, that is, the limit of the current mass M_n as $t \to \infty$. The initial value of mass of the nucleus is $M_{ni} = 5$. One observes that the general characteristics are the same as those given in Fig. 1. Again there are two lines. For the values of the parameters in the area – including the lines – the system shows chaotic orbits, while for the values of the parameters outside the two lines all orbits are regular. Note that in this case we do not have straight lines. We believe that this is a result coming from the evolving potential, where we have an exponential evolution of the strength of the F_z force due to the exponential mass transportation.

This is justified if we look at Fig. 4, where a plot of the F_z force due to the nucleus and disk is given as a function of time t. In other words, we have replaced in (5) the value of the mass M_n with that given from equations (6) and (7). Thus, we have made the F_z force a function of time. The values of r and z are as in Fig. 2. We see that the F_z force that belongs to the nucleus increases exponentially, while that belonging to the disk decreases exponentially. Using the same arguments as in the time-independent case, we can replace F_z by M_n and t by L_z to get a similar figure to Fig. 3. Note that we do not expect to get the same results from Figs. 3 and 4. The comparison only shows that the numerically obtained results can be qualitatively reproduced using theoretical arguments together with some numerical evidence.

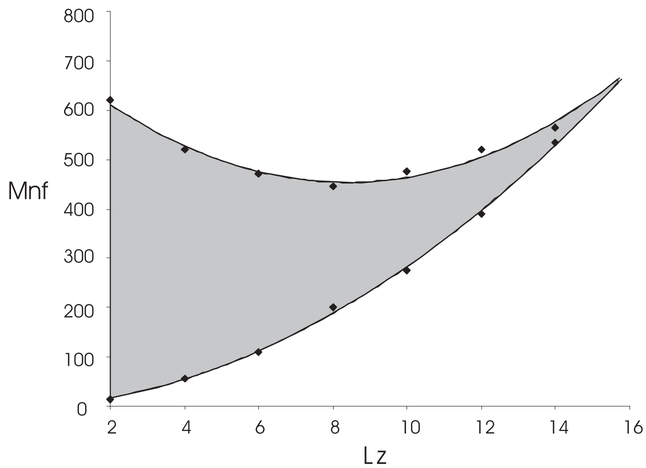

Figure 3. Same as Fig. 1 but for the evolving model. Note that the area where chaotic motion is observed is not defined by straight lines.

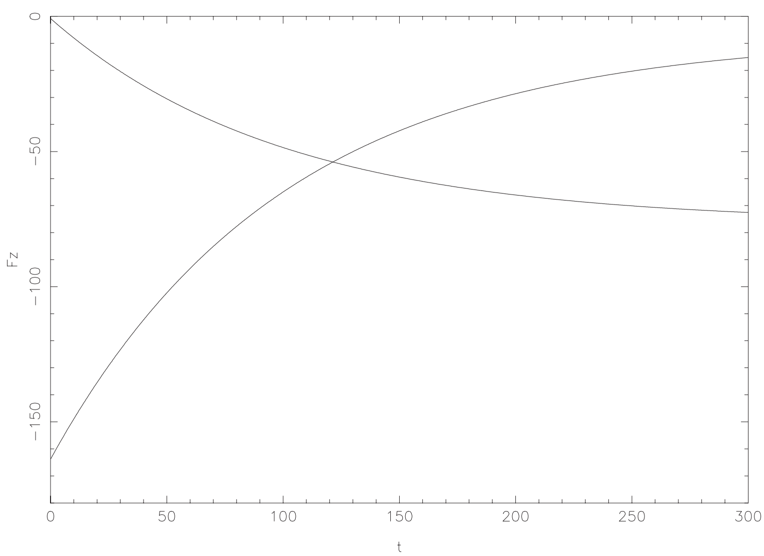

Figure 4. Same as Fig. 2 but for the evolving model. Note that the similarity to Fig. 3 is evident.

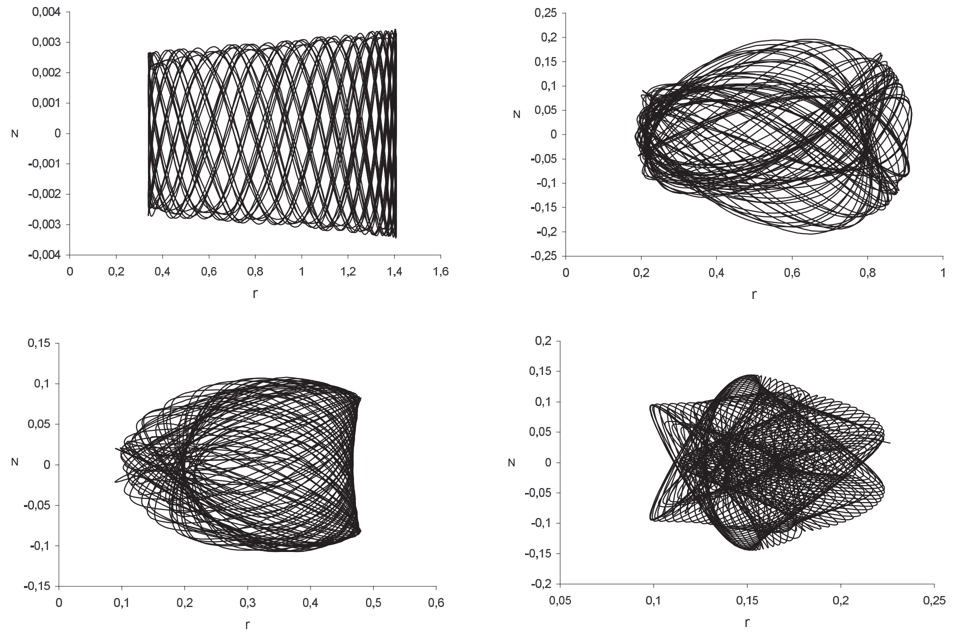

Figure 5. Evolution of an orbit in the time dependent model. It is interesting to note that the orbit switches between regularity and chaos and *vice-versa*.

It is interesting to follow the evolution of orbits in the time dependent potential. According to the diagram shown in Fig. 3, an orbit with a given value of angular momentum and a small initial value of the mass of the nucleus starts as regular, then becomes chaotic, when the mass of nucleus evolves between the limits given by the two lines in Fig. 3, and later becomes again regular. The results are shown in Fig. 5. The orbit has initial conditions $r = 1.536$, $z = 0 = p_r = 0$, while the value of $p_z \leq 30$ km/s is found using the initial value of energy (at $t = 0$). The initial value of energy, which is not conserved, is $E = -450$, $k = 0.01$ while the value of angular momentum is $L_z = 8$. The upper left shows the orbit when $5 \leq Mn \leq 55$, the upper right when $200 \leq Mn \leq 250$, down left $440 \leq Mn \leq 490$, down right $705 \leq Mn \leq 755$. Figure 6 shows the LCN of the orbit shown in Fig. 5. The numbers indicate the areas of regularity 1, 4 and the areas where the orbit is chaotic 2, 3.

From all the above, one can see that, in our time-dependent quasar dynamical model, the regular or chaotic nature of orbits strongly depends on the mass of nucleus with two transition points for a given value of critical angular momentum. On the contrary, in dynamical models for disk galaxies with active nuclei things are different where for a low value of the critical angular momentum there corresponded only one value of mass of nucleus for which transition from regular to chaotic motion occured (see Fig. 1 in Caranicolas & Papadopoulos 2003).

This significant difference is explained if we make a plot of the F_z-force *vs.* M_n. Figure 7 shows such a plot. In other words, the diagram in Fig. 7 is similar to that in Fig. 4 but for the disk galaxy described in Caranicolas & Papadopoulos (2003). Note that the F_z-force due to the disk is practically constant while the attractive nuclear

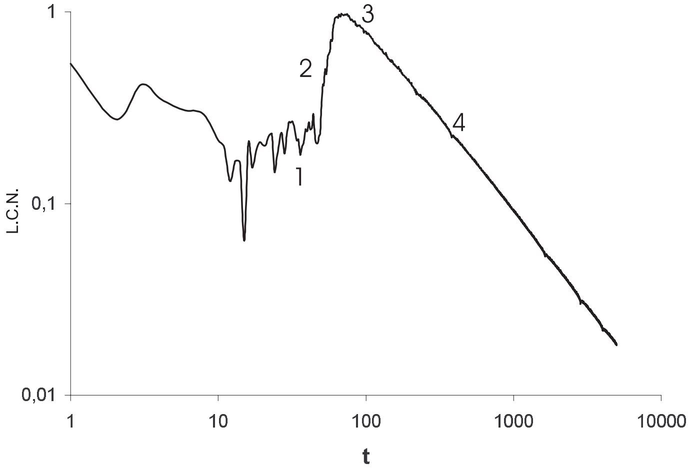

Figure 6. LCN for the orbit shown in Fig. 5. The areas of regularity are indicated as 1, 4 and the areas where the orbit is chaotic are indicated as 2, 3.

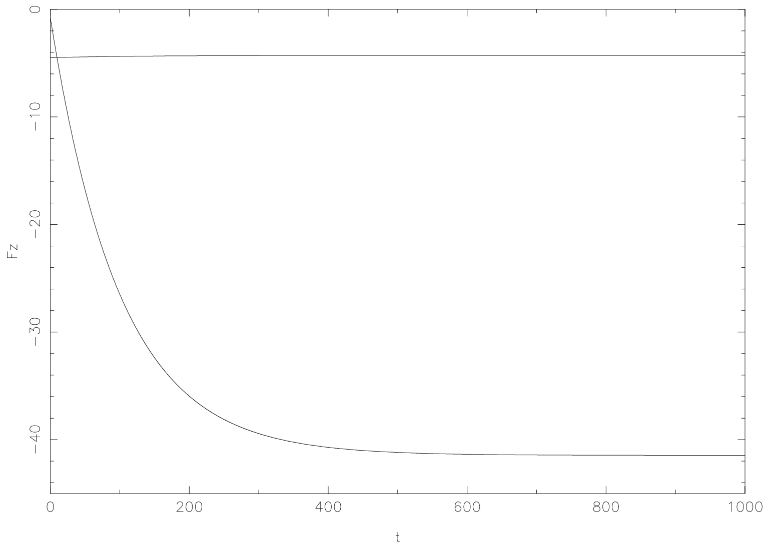

Figure 7. Same as Fig. 2 but for the disk-galaxy evolving model described in Caranicolas and Papadopoulos (2003). Details are given in text.

F_z-force increases rapidly as M_n increases. As one can see, the area defined between these two curves is meant for very small values of M_n. Therefore, it seems very possible that it is difficult to detect it through numerical experiments. Furthermore, this area is of no importance because, in any case, it is negligible.

4. Discussion

In this article we have adopted a simple axially symmetric model with a disk and a super-massive nucleus in order to describe the motion in a quasar. Using this model we tried to derive relationships connecting the physical parameters, such as the mass of the nucleus and the angular momentum, to the ability of the system to display chaotic motion. Our choice had two targets. As our knowledge for the dynamical behavior of those systems is poor, the first target was to derive results for their dynamical properties. The second target was to compare the outcomes with the already available results for galaxies.

A linear relationship was found to exist between the mass of nucleus and the value of critical angular momentum in order to observe chaotic motion. The same relationship, but not linear, seems also to exist for the evolving model. Both relationships are explained using semi-theoretical arguments together with a plot of the F_z-force which is responsible for the appearance of chaos.

Interesting results are obtained if we follow the evolution of orbits in the time-dependent model. Our numerical work shows that as mass is falling from the disk to the nucleus, an orbit can start as regular, then become chaotic and finally end up as a regular orbit.

The dynamical behavior of quasars is different from that observed in disk galaxies. The reason is that in the case of a quasar model, the transportation of mass from disk to the nucleus affects drastically the strength of the F_z-force coming from the disk, while this force remains practically constant in the case of a disk galaxy. On the other hand, the chaotic phenomena in axially symmetric quasars are seen to be smaller than those observed in non-axially symmetric quasars as the LCN (see Papadopoulos & Caranicolas 2005) in the latter case is much larger.

This paper is focused on orbital dynamics which applies to collisionless components such as stars. The effect of gaseous matter is not discussed. But, as this is a minor point, our work can be considered as a good approximation because in most galaxies the gaseous component is less than 10% of the total galactic mass.

To end up we would like to make clear that all the results of this work were based on a simple dynamical model, which was chosen for two basic reasons: the first reason was that we considered it difficult to construct, understand and handle a complicated model, while the second reason was that we wanted to start from a simple model which had similarities with the already well-known galactic disk models. On the other hand, we believe that, using the outcomes of the present work as a starting point, we will be able to make better efforts in the future.

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