Lessons for Asteroseismology from White Dwarf Stars

Travis S. Metcalfe

High Altitude Observatory, NCAR, P.O. Box 3000, Boulder CO 80307, USA.
e-mail: travis@hao.ucar.edu

Abstract. The interpretation of pulsation data for sun-like stars is currently facing challenges quite similar to those faced by white dwarf modelers ten years ago. The observational requirements for uninterrupted long-term monitoring are beginning to be satisfied by successful multi-site campaigns and dedicated satellite missions. But exploration of the most important physical parameters in theoretical models has been fairly limited, making it difficult to establish a detailed best-fit model for a particular set of oscillation frequencies. I review the past development and the current state of white dwarf asteroseismology, with an emphasis on what this can tell us about the road to success for asteroseismology of other types of stars.

Key words. Stars: interiors—stars: oscillations—white dwarfs.

1. Introduction

For the past six years, I have been trying to improve the way that we match theoretical models to observations of pulsating white dwarfs. I have made some progress transforming the art of model-fitting from a hands-on procedure to something more objective, more global, and more automated. The impact of this approach on the analysis of pulsating white dwarfs over the past few years suggests that it might also be able to improve our seismological modeling of other types of stars.

I am now interested in applying this same approach to the relatively recent (and much more difficult) observations of solar-like oscillations in other stars. Reading through the recent literature on this subject, I have been impressed by how the current state of model-fitting for these stars resembles the state of white dwarf modeling ten years ago. With this in mind, my goal here is to share some of the lessons I've learned while working on white dwarfs, with the hope that they will be useful for the future analysis of sun-like stars.

Before I go any further, I'd like to make it clear that I do not think the current analysis methods for sun-like stars are *wrong*. I do think that the current methods are limited in various ways, and these limitations may prevent us from learning all that we can learn from the observations. I hope to demonstrate this possibility with some anecdotes about our recent progress modeling the pulsating white dwarfs.
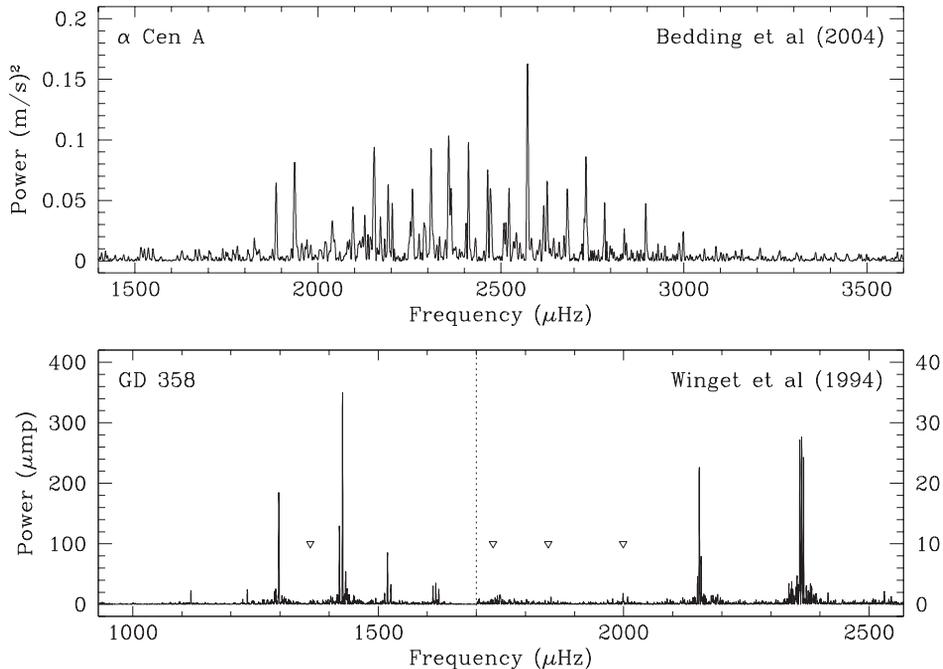
Figure 1. Power spectra of time-series measurements to detect solar-like oscillations in α Cen A published recently (top), compared to the pulsations observed in the white dwarf GD 358 published ten years earlier (bottom). The dotted line in the bottom panel divides the regions where the left and right vertical scales are applicable, and the open triangles mark the frequencies for several of the lower amplitude modes.

2. Similarities and differences

In case you haven't noticed, the Hertzsprung–Russel diagram is absolutely filled with pulsating stars (see Christensen-Dalsgaard 1998, Fig. 1). We believe that solar-like oscillations should be excited in just about any star with a convection zone. The basic idea is that convection generates sound waves at all frequencies ("white noise", like static on a radio). Some of these frequencies are resonant inside the spherical cavity of the star and become long-lived global pulsations (standing waves). Such pulsations have recently been detected in several main-sequence, subgiant, and giant stars. The oscillation amplitudes and the frequency of maximum power in these stars agree reasonably well with our theoretical expectations (see Bedding & Kjeldsen 2003 for a recent review).

There are several classes of pulsating white dwarfs, almost equally spaced in log T_{eff}. In each case, the pulsations seem to be excited by a similar mechanism (extra opacity from a near-surface partial ionization zone) but the surface chemical composition can be hydrogen, helium, or something else. Compared to other types of stars, white dwarfs are relatively simple from a physical perspective. Nuclear fusion is no longer active, and potentially complicating factors like fast rotation or strong magnetic fields are not a problem for most of these stars (Winget 1998).

The spherical symmetry of gravity allows us to describe stellar pulsations with spherical harmonic functions, the same mathematics that we use to describe electron

orbitals in quantum mechanics. We can assign each pulsation mode three quantum numbers: an l and m value to describe the pattern on the visible surface, and an n value (sometimes called k) for different radial overtones. For observations of stars, only modes with relatively low values of l can be detected, because the many bright and dark areas tend to cancel each other out for higher-l modes.

Asteroseismology of white dwarfs has advanced more quickly than that for other types of stars, in part because the pulsations are much less subtle. The total observed light variation is typically 10% or more, on very convenient timescales of 5–15 minutes. The light variations caused by solar-like oscillations are considerably smaller, so scintillation in the Earth's atmosphere severely limits our ability to detect them. However, the pulsations also cause variations in the spectral line profiles and equivalent widths, allowing us to detect them from the ground with time-series spectroscopic measurements.

From an observational standpoint, Fig. 1 illustrates just how similar the current status of sun-like stars is to the situation we had for white dwarfs ten years ago. In both panels we see a well-defined range of excited frequencies. For sun-like stars we see a series of almost equally spaced frequencies, which is characteristic of pressure (p-) modes. For white dwarfs the peaks are unevenly spaced in frequency, but evenly spaced in period. This is characteristic of the gravity (g-) modes.

From a theoretical standpoint, the observed pulsation periods are determined by the values of the sound speed (or Lamb frequency, L) and the buoyancy frequency (N) from the center of the star to the surface. The p-modes are excited at frequencies higher than both L and N, while the g-modes are excited at frequencies lower than both L and N (Unno *et al.* 1989). In a chemically uniform stellar model the pulsation modes are evenly spaced, but chemical stratification and variations in other relevant physical quantities cause discontinuities in L and N that lead to unevenly spaced modes. These deviations from uniform spacing (in frequency or period) contain detailed information about the interior structure of the star.

A fundamental difference between the observed modes in sun-like stars and white dwarfs is that we generally see modes of only one spherical degree in white dwarfs (mostly $l = 1$, but in some cases $l = 2$). In sun-like stars, we usually see radial ($l = 0$) and non-radial ($l = 1$, 2 and sometimes 3) modes excited simultaneously. Consequently, there is a historical difference in the way the pulsation data are usually presented. In Fig. 2, this difference has been circumvented by rotating the white dwarf plot. This allows us to see more readily how the current state of model-fitting for sun-like stars is also very similar to what was being done with white dwarfs ten years ago.

In white dwarf models, the mean period spacing is related to the total stellar mass, while the deviations from the mean period spacing (dP) tell us about the thickness of the stratified surface layers (Bradley *et al.* 1993). In models of sun-like stars, the large frequency spacing ($\Delta\nu$) is related to the mean density of the star. The deviations that split modes with odd or even spherical degree (the so-called small frequency separation, $\delta\nu$) tell us about chemical gradients in the interior (see Brown & Gilliland 1994). The current level of sophistication for model-fitting of sun-like stars allows us to match the overall patterns of the oscillations, but there is room for improvement in the detailed fit to the individual modes. This is just where white dwarf modeling was in 1994.

The story of how we improved our approach to white dwarf modeling contains some important lessons for the modeling of sun-like stars (see also Metcalfe 2003a).
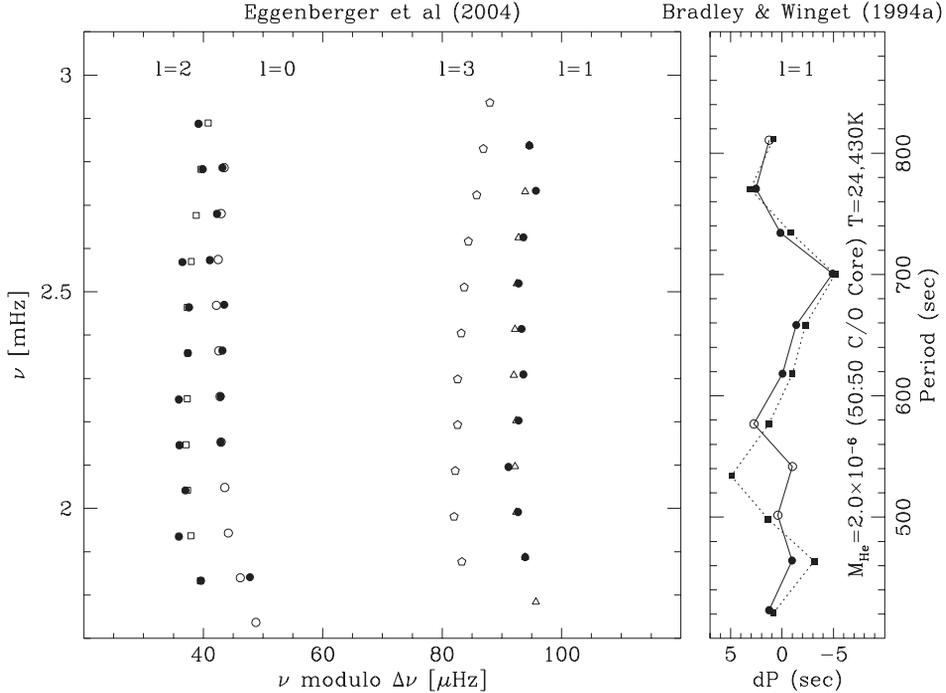
Figure 2. A recent diagram of the frequencies observed in α Cen A plotted against their deviations from the large frequency spacing (left), and a similar diagram from ten years earlier showing the periods observed in GD 358 plotted against their deviations from the mean period spacing (right). In both cases the theoretical models match the overall patterns, but some improvement is still needed to match the fine details.

This is particularly relevant now, since the next few years promise to unleash a flood of new observations of solar-like oscillations in other stars. We have already seen what is possible with a relatively modest space mission devoted to asteroseismology, the Canadian MOST satellite (Walker *et al.* 2003). Forthcoming missions with larger apertures, such as COROT (Baglin *et al.* 2002) and Kepler (Borucki *et al.* 2003) are likely to extend these successes and bring new surprises. Unfortunately, none of these missions will contribute new data on the faintest pulsating stars, so future advances for white dwarfs and variable sdB stars will most likely continue to come from ground-based observations.

3. Using observational constraints

In the current model-fitting approach for sun-like stars, it is common to use external observational constraints on the luminosity and effective temperature to restrict the search to a narrow range of model parameters (e.g., see Di Mauro *et al.* 2003, Fig. 1). This guarantees that the seismological model will be consistent with these constraints, but it eliminates the advantage of the pulsation data. Even if we find a reasonable fit to the pulsation modes, how do we know it couldn't be improved if we relaxed or removed the constraints? The power of asteroseismology is that it provides *independent* constraints on the stellar structure. If the stellar models are accurate representations
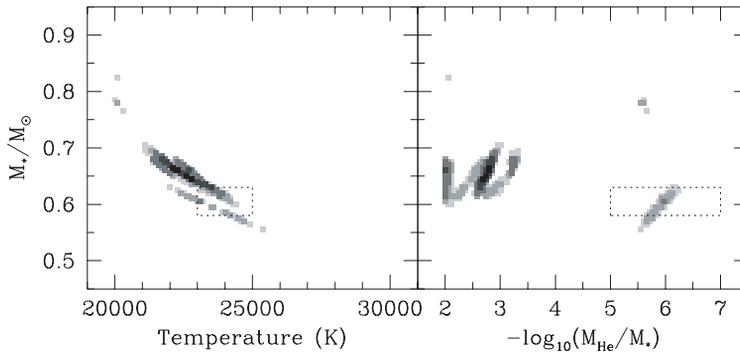
Figure 3. By searching a much broader range of parameter values compared to previous studies (dotted), we discovered a better match (darker) to the observations of GD 358. Our global search also revealed a strong sensitivity of the pulsations to the core composition, which eventually led to precise constraints on an important nuclear reaction rate.

of the real stars, then the pulsation modes should lead to the "observed" luminosity and effective temperature by themselves. If they do, this lends additional credibility to the resulting model. If instead the seismological data lead to a best-fit model that *disagrees* with the external constraints, then something is missing from the model; and that's exactly what we hope to learn through asteroseismology. We will never find a disagreement if we don't look for it.

To illustrate the importance of defining the range of the search as broadly as possible, let me tell you about the helium-atmosphere (DBV) white dwarf GD 358. The observations of this star came from the Whole Earth Telescope (Nather *et al.* 1990; Winget *et al.* 1994), and the initial asteroseismological analysis was done by Bradley & Winget (1994a). For our new approach using a parallel genetic algorithm (Metcalfe & Charbonneau 2003), we decided to define the range of the search based only on the physics of the model whenever possible.

For DBV stars, the three most important parameters are the stellar mass, the effective temperature, and the thickness of the surface helium layer. For our initial study we searched masses between 0.45 and 0.95 $M_\odot$. The observed mass distribution of white dwarfs is strongly peaked near 0.6 $M_\odot$ (Napiwotzki *et al.* 1999). Lower mass white dwarfs are theoretically expected to have a helium core, and take longer than the age of the universe to form through single-star evolution. Higher mass white dwarfs are extremely rare. We searched effective temperatures between 20,000 and 30,000 K, which easily encompasses the full range of the DBV instability strip whether or not trace amounts of hydrogen are allowed in the envelopes (Beauchamp *et al.* 1999). We allowed the surface helium layer to be between 10^{-2} and $\sim 10^{-8}$ in fractional mass. Thicker layers would initiate nuclear burning at the base of the layer (ruled out by the observations), and for thinner layers our models do not pulsate (Bradley & Winget 1994b).

The full range of our search is shown in Fig. 3, along with the range of the original search in 1994 (dotted). We found the same best-fit model as Bradley & Winget when we restricted ourselves to their search range, but our broader search revealed an even better solution (darker) outside of the range they considered (Metcalfe *et al.* 2000). This global approach also taught us that our models are much more sensitive to the core

composition than we had previously thought. This is significant because the oxygen mass fraction in the core of a white dwarf is largely determined by the relative rates of the 3α and the ^{12}C$(\alpha, \gamma)^{16}$O nuclear reactions during the red giant phase. The latter reaction is poorly constrained by laboratory data, which are extrapolated over six orders of magnitude in the cross-section to reach the energies relevant to red giant stars.

Motivated by our newly-discovered sensitivity to the core composition, we added adjustable C/O profiles to our models and performed a new global search. While our match to the observed pulsation modes improved slightly by considering a broader range of model parameters, optimizing the C/O profile improved the match dramatically (see Metcalfe *et al.* 2001, Fig. 4d). Since then we have applied the same method to another DBV white dwarf, CBS 114, leading to two independent constraints on the reaction rate that agree with each other (Metcalfe 2003b). The two values also agree with recent extrapolations from laboratory measurements (Angulo *et al.* 1999), but are considerably more precise.

By letting the pulsation modes speak for themselves we improved our fit to the observations, and in the process we discovered a previously unknown sensitivity in the models that ultimately led to constraints on an important nuclear reaction rate. Along the way, we found inconsistencies between some of the derived model parameters and other observations (e.g., spectroscopic masses and temperatures) that also prompted us to improve the model physics. None of this would have been possible if we had defined our search too narrowly.

4. Individual modes and spacing

Early attempts to detect solar-like oscillations in other stars produced Fourier spectra that did not clearly resolve individual frequencies, but showed excess power roughly where the excited modes were expected. Using auto-correlation techniques, these observations could infer the large frequency spacing, which could then be used to estimate the mean density of the star (see Kjeldsen *et al.* 1995, Fig. 3). More recent observations can now confidently resolve the individual modes, but auto-correlation is often still used to determine the large frequency spacing. Because the models may suffer from systematic errors, some recent model-fitting attempts have concentrated on the frequency spacing rather than matching the individual frequencies. This may be useful for a preliminary analysis, but the individual modes contain additional information that is lost by matching only the spacing. Using the individual modes can also produce clues about the source of any systematic errors, allowing us to improve the input physics of our models.

To illustrate the benefit of matching the individual modes, and not just the mode spacing, let me tell you about our experience with the hydrogen-atmosphere (DAV) white dwarf BPM 37093. When a white dwarf gets cool enough, the C/O core eventually undergoes a phase transition from liquid to solid. It crystallizes, from the center outward. This is important because the crystallization process releases latent heat, which delays the gradual cooling of the star. Since we can use the coolest white dwarfs in any stellar population (the Galactic disk and halo, open and globular clusters) to determine the age, it is crucial that we calibrate this cooling delay. Otherwise, we will underestimate the true ages.

A typical white dwarf with a mass of $0.6\,M_\odot$ will begin to crystallize when it cools to an effective temperature between 6000 and 8000 K, depending on the core
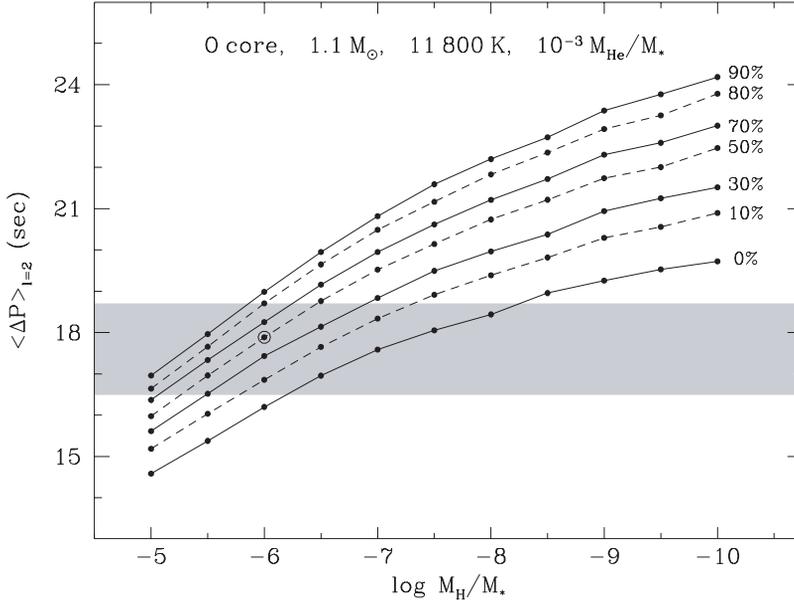
Figure 4. When our fit used only the *mode spacing* (shaded), we were unable to distinguish between thinner H layers and a larger crystallized mass fraction in BPM 37093 — both parameters increased the mode spacing. By exploiting the information in the *individual modes*, one model (circled) stands out as the best in this small grid. Of course, a simultaneous optimization of all of the parameters is needed to find the global solution.

composition. More massive white dwarfs have higher central pressures, so they will begin to crystallize at higher temperatures. BPM 37093 is massive enough ($\sim 1.1\,M_\odot$) that it has theoretically started to crystallize while at an effective temperature inside the DAV instability strip ($\sim 12,000\,K$). The pulsations do not penetrate the solid core, so the inner boundary for each mode is located at the edge of the crystallized core rather than near the center of the star. This changes the period of each mode, compared to what it would have been in the absence of crystallization. As a consequence, we can calibrate the theory of crystallization empirically by modeling the pulsations in this star.

   The observations of BPM 37093 were obtained by the Whole Earth Telescope (Kanaan *et al.* 2000, 2005), and the initial theoretical study came from Montgomery & Winget (1999). This early analysis concentrated on the average period spacing of the modes, since the main effect of the crystallized core is to spread the pulsation periods farther apart. However, they found a troubling degeneracy between changes to the crystallized mass fraction and changes to the thickness of the surface hydrogen layer, leading to ambiguous results from the average period spacing (see Fig. 4). By exploiting the information contained in the *individual periods*, we were able to lift this degeneracy and determine a unique best-fit model. Of course, in the small grid of models shown in Fig. 4, we have fixed several other important parameters, so the true best-fit model requires a more global exploration.

   This is a huge computational problem, since we need to consider the stellar mass, the effective temperature, the thickness of the helium mantle and the surface hydrogen layer, the core composition, and the crystallized mass fraction, all simultaneously. Our initial fits to the individual periods confirmed that BPM 37093 is substantially

crystallized (Metcalfe *et al.* 2004), but the search was too limited to determine the exact fraction. Our latest results look promising (Metcalfe *et al.* 2005), and have recently been confirmed by Brassard & Fontaine (2005). But this expanded search still required that we fix the mass and core composition, and there remain many locally optimal models, so our search for a global solution continues.

By using the individual modes, and not just the spacing, we lifted the degeneracy between the crystallized mass fraction and the surface hydrogen layer mass in models of the DAV white dwarf BPM 37093. This led to the first direct confirmation of crystallization theory, and opened the door to a more global exploration of the models which is still in progress. If we had confined ourselves to fitting the average period spacing, we would never have obtained this first glimpse inside of a crystallized star.

5. Summary

Let me summarize the main points.

- The future of asteroseismology for sun-like stars is primarily in space.
  We soon expect many new observations from current and planned satellite missions. It will be a distinct advantage to have in place the analysis methods that will help us make sense of these data efficiently.

- *Lesson 1*: Let the pulsations speak for themselves.
  To interpret these data, it is important that we avoid introducing our subjective biases by defining the range of our model-fitting search too narrowly. The asteroseismic models should agree with independent measurements without coercion.

- *Lesson 2*: Use the individual modes, not just the spacing.
  The individual frequencies contain additional information that is lost if we focus only on the overall pattern of frequency spacing. This could unnecessarily limit what we are ultimately able to derive from the observations.

- The future of asteroseismology for fainter stars is multi-site.
  Upcoming space missions will not affect the future of the fainter pulsators. Advances in our understanding of white dwarfs and variable subdwarf B stars will continue to come from multi-site campaigns, possibly with larger telescopes.

This is a very exciting time for asteroseismology. It is the beginning of a new era, when we will improve our knowledge of solar physics by studying sun-like stars. This will allow us to explore the past and future of our own sun by observing pulsations in solar analogues, calibrating our models at different evolutionary stages and under various physical conditions. For the white dwarf stars, we are closer to the end of an era. Over the past decade we have steadily improved our models and our analysis methods, charting a course that is just beginning for sun-like stars. If we are clever enough, we can use what we have learned along the way to accelerate our progress on other types of stars.

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